

NASRULLAH

7870

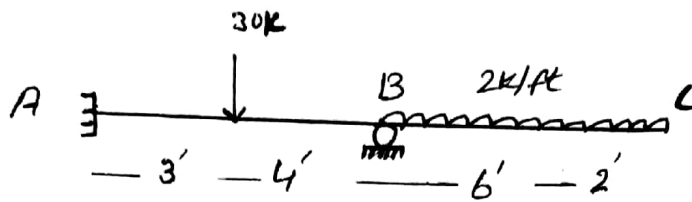
B

ENGR. ADEED KHAN

STRUCTURE ANALYSIS II

25-09-2020

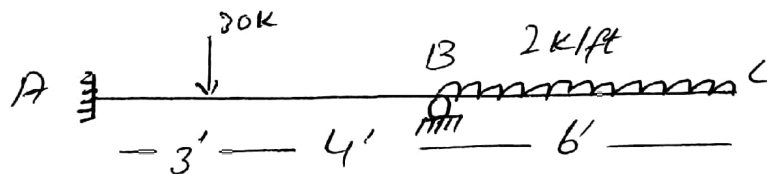
Q no # 01

SolutionStep # 01

Determining kinematic indeterminacy

$$K.I = 5^{\circ}$$

So we have to reduce the extended portion



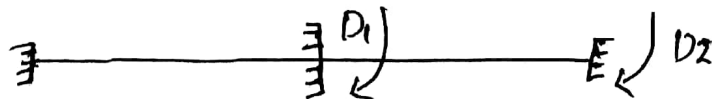
$$\Rightarrow \frac{2(2)}{1} = 4 \text{ k/ft}$$

NOW

$$K.I = 2^{\circ}$$

Step # 02

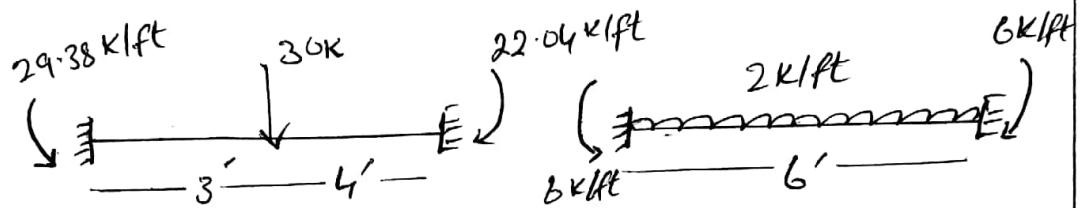
Determine unknown joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix} \quad \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step #03

compute  $\{ADL\}$  Matrix



$\Rightarrow$  For point load (not at mid) :-

For left end :-

$$\frac{pab^2}{L^2} = \frac{(30)(3)(4)^2}{(7)^2} = 29.38 \text{ k/ft}$$

For Right end :-

$$\frac{pa^2b}{L^2} = \frac{(30)(3)^2(4)}{(7)^2} = 22.04 \text{ k/ft}$$

$\Rightarrow$  For UDL :-

$$\frac{WL^2}{12} \rightarrow \frac{(2)(6)^2}{12} = 6 \text{ k/ft}$$

$$ADL_1 = +22.04 - 6 = 16.04 \text{ k/ft}$$

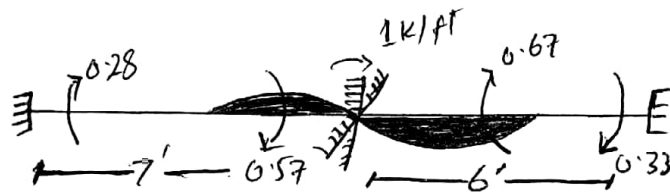
$$ADL_2 = 6 \text{ k/ft}$$

Step #04compute  $[S]$  matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

a)

$$D_1 = 1K, D_2 = 0$$



$$\frac{4EI}{7} = 0.57$$

$$\frac{4EI}{6} = 0.67$$

$$\frac{2EI}{6} = 0.33$$

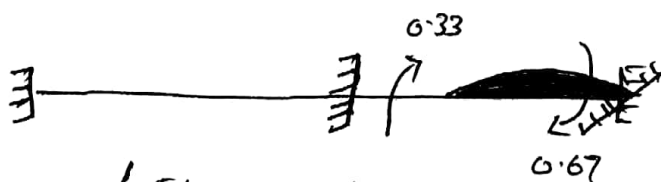
$$\frac{2EI}{7} = 0.28$$

$$S_{11} = 0.57 + 0.67$$

$$= 1.24EA$$

$$S_{21} = 0.33EA$$

$$b) D_1 = 0, D_2 = 1K$$



$$\frac{4EI}{6} = 0.67$$

$$\frac{2EI}{7} = 0.33$$

$$S = \begin{pmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{pmatrix}$$

Step #05

Compute (D) matrix

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} - \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$$= \frac{1}{\begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}} \times \text{Adj } A \times \begin{bmatrix} 0 \\ 4 \end{bmatrix} - \begin{bmatrix} 16.04 \\ 6 \end{bmatrix}$$

$$|S| = (1.24 \times 0.67) - (0.33 \times 0.33)$$

$$= 0.8308 - 0.1089$$

$$|S| = 0.7219$$

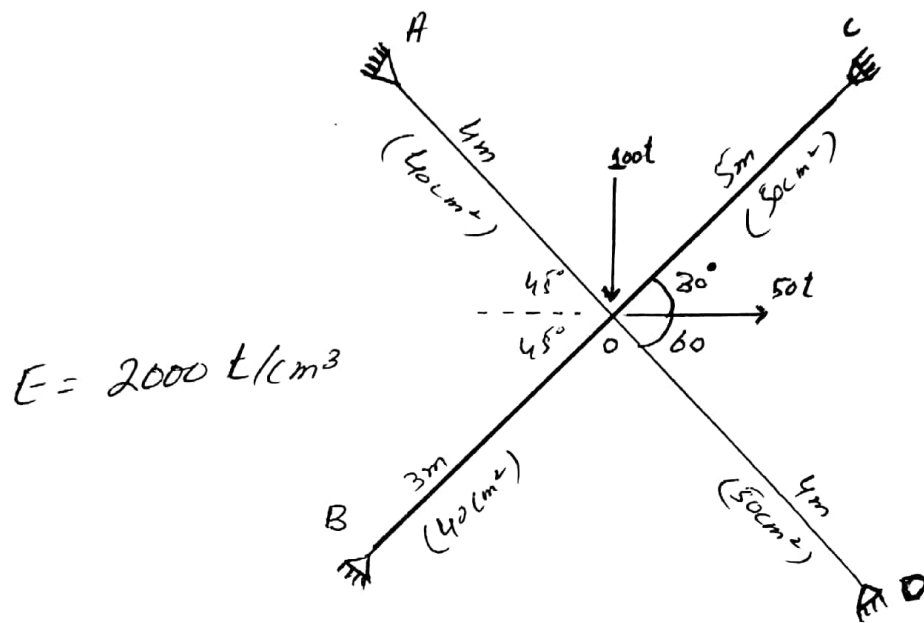
$$\text{Adj } A = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}$$

NOW

$$\begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix} = \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix} = \begin{bmatrix} 16.04 \\ -2 \end{bmatrix} E$$

$$\begin{bmatrix} D1 \\ D2 \end{bmatrix} = \begin{bmatrix} -13.915 \\ 3.894 \end{bmatrix}$$

Q no # 02

Solution :-For A

$$\sin 45^\circ = \frac{P}{h} = \frac{P}{4}$$

$$\Rightarrow P = 2.828 \text{ m}$$

$$\cos 45^\circ = \frac{b}{4}$$

$$\Rightarrow b = 2.828 \text{ m}$$

For B

$$\sin 45^\circ = \frac{P}{3}$$

$$\cos 45^\circ = \frac{b}{h}$$

$$\Rightarrow b = 2.12 \text{ m}$$

For C

$$\sin 30^\circ = \frac{P}{h=5}$$

$$\Rightarrow P \Rightarrow 2.5 \text{ m}$$

$$\cos 30^\circ = \frac{b}{5}$$

$$\Rightarrow b = 4.33 \text{ m}$$

Now

$$EA(A) = 2000 \times 40 = 80,000 \text{ t}$$

$$EA(B) = 2000 \times 40 = 80,000 \text{ t}$$

$$EA(C) = 2000 \times 50 = 100,000 \text{ t}$$

$$EA(D) = 2000 \times 50 = 100,000 \text{ t}$$

Step # 01

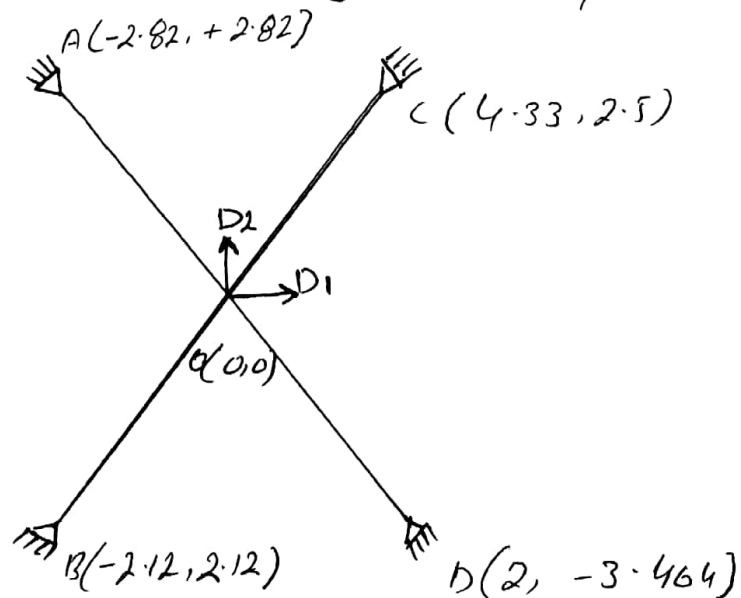
K.I

$$K.I = 2j - \gamma$$

$$= 2(5) - 8 = 2^\circ$$

Step # 02

Select unknown joint displacement.





$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Step #03

$$\{AMID\}_{4 \times 2} \quad \& \quad \{S\}_{2 \times 2}$$

$$i) \quad D_1 = 1, \quad D_2 = 0$$

$$AMD = \frac{EA}{L^2} (2k - x_j)$$

$$AMD_{11} = \frac{80,000}{(400)^2} \times (0 + 282) = 140$$

$$AMD_{21} = \frac{80,000}{(300)^2} \times (0 + 212) = 188.44$$

$$AMD_{31} = \frac{100,000}{(500)^2} \times (0 - 433) = -173.2$$

$$AMD_{41} = \frac{100,000}{(400)^2} \times (0 - 200) = -125$$

Now

$$S_{11} = \sum_{121}^m \frac{EA}{L^3} (x_k - x_j)^2$$

$$= \frac{80,000}{400^3} (282)^2 + \frac{80,000}{300^3} (212)^2 + \frac{100,000}{500^3} (-433)^2$$

$$+ \frac{100,000}{400^3} (-200)^2$$

$$S_{11} = 99.405 + 133.107 + 149.991 + 62.5$$

$$S_{11} = 445.003$$

$$S_{12} = S_{21} = \sum_{i=1}^n \frac{EA}{L^3} (x_k - x_j) (y_k - y_j)$$

$$= \frac{80,000}{400^3} (282)(-282) + \frac{80,000}{300^3} (212)(212) + \frac{100,000}{500^3} (-250)(-250)$$

$$+ \frac{100,000}{400^3} (-200)(0+346)$$

$$S_{12} = S_{21} = 12.237$$

ii)  $D_1 = 0$  ,  $D_2 = 1K'$

$$AM(1) = \frac{EA}{L^3} (y_k - y_j)$$

$$AM(1)_2 = \frac{80,000}{400^2} (-282) = -141$$

$$AM(1)_3 = \frac{80,000}{300} (212) = 188.44$$

$$AM(1)_4 = \frac{100,000}{500^2} (-250) = -100$$

$$AM(1)_{42} = \frac{100,000}{400^2} (346) = 216.25$$

NOW  $S_{22} = \sum_{i=2}^n \frac{EA}{L^3} (y_k - y_j)^2$

$$\frac{80,000}{400^3} (-282)^2 + \frac{80,000}{300^3} (212)^2 + \frac{100,000}{500^3} (250)^2$$

$$+ \frac{100,000}{400^3} (346)^2$$

$$\delta_{22} = 469.628$$

Step #04

$$[D] = [S]^{-1} \times [AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.063 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

Step #05

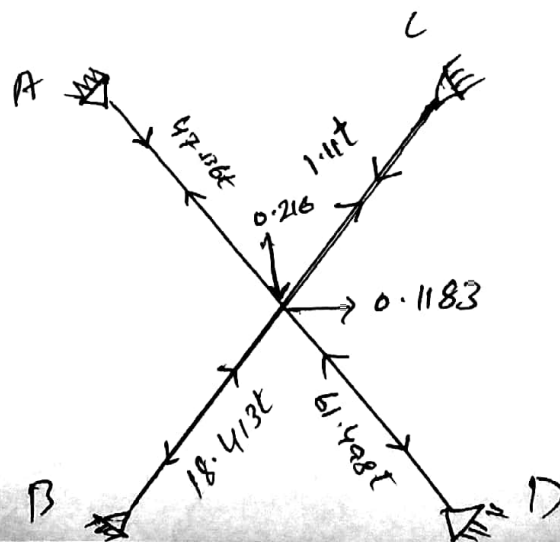
[AM]

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

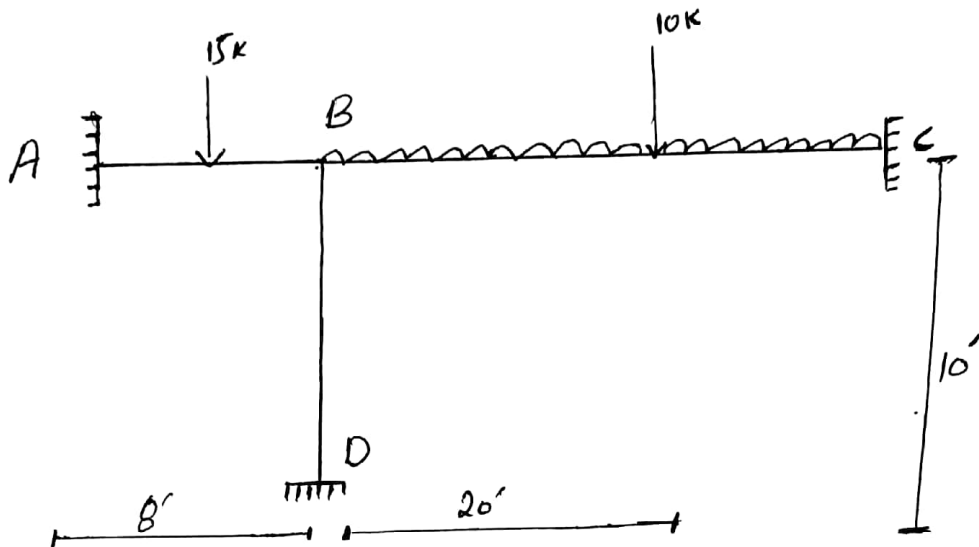
$$\begin{aligned} & [141 \times 0.1183 + (-141) \times (0.216)] \\ & 188.44 \times 0.1183 + 188.44 \times (-0.216) \\ & -173.24 \times 0.1183 + (-100) \times (-0.216) \\ & -125 \times 0.1183 + 216.25 \times (0.216) \end{aligned}$$

$$\begin{aligned} \begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} &= \begin{bmatrix} 16.68 + 30.46 \\ 22.29 - 40.70 \\ -20.49 + 21.6 \\ -14.79 - 46.71 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} &= \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ -61.498t \end{bmatrix} \end{aligned}$$



Qno # 03

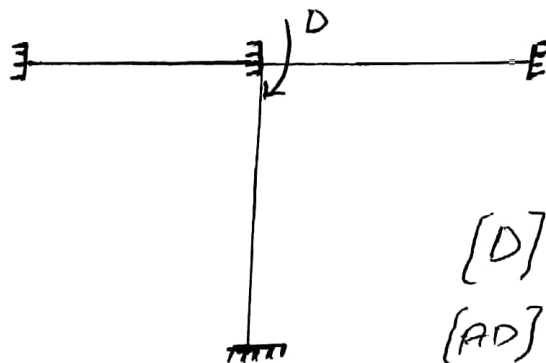


Solution

Step #1 Determine Kinematic Intermediary.

$$K \cdot I = 1^0$$

Step #02 Determine Unknown Joint Displacement

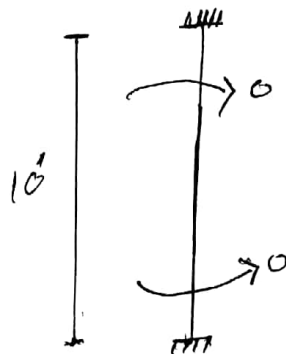
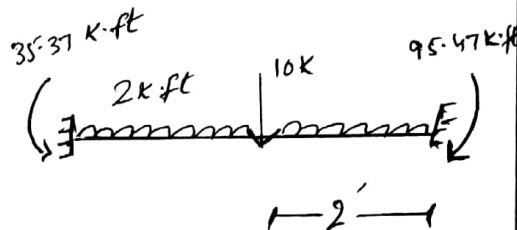
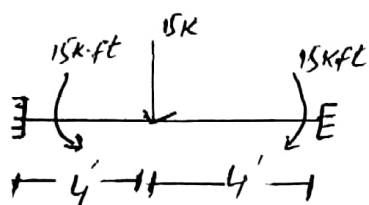


$$[D] = [?]$$

$$[AD] = [0]$$

Step #03

(compute  $[A]DL$  matrix

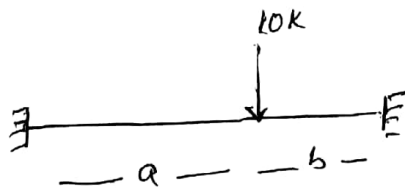


$\Rightarrow$  point load at center:  $\frac{PL}{8} = \frac{(15)(8)}{8} = 15K-ft$

$\Rightarrow$  Uniformly Distributed load:  $\frac{WL^2}{12} = \frac{2(20)^2}{12} = 66.67K-ft$

$\Rightarrow$  point load (not at mid):

Suppose



For left end:  $\frac{Pab^2}{L^2} = \frac{(10)(12)(8)^2}{(20)^2} = 19.2K-ft$

For right end:  $\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8K-ft$

So total moment at left end:

$$19.2 + 66.67 = 85.87 \text{ k}\cdot\text{ft}$$

Similarly at right end :-

$$28.8 + 66.67 = 95.47 \text{ k}\cdot\text{ft}$$

$$\text{So } [ADL] = -85.87 + 15 = -70.87 \text{ k}\cdot\text{ft}$$

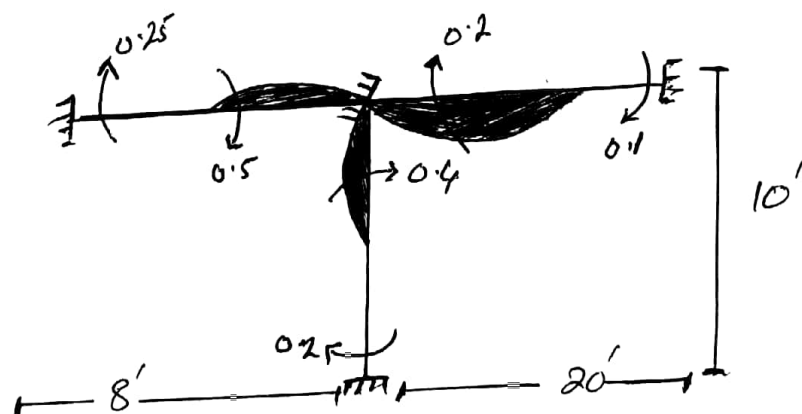
Step #04

Determine  $[S]$  matrix

$$[S] = [S_{ij}]$$

NOW

$$D = 1 \text{ k}$$



$$\Rightarrow \frac{4EI}{8} = 0.5$$

$$\frac{2EI}{8} = 0.25$$

$$\Rightarrow \frac{4EI}{10} = 0.4$$

$$\Rightarrow \frac{4EI}{20} = 0.2$$

$$\frac{2EI}{20} = 0.1$$

$$\Rightarrow \frac{2EI}{10} = 0.2$$

$$[S] = (0.5 + 0.4 + 0.2) EI$$

$$= 1.1 EI$$

$$[S] = 1.1 EI$$

Step # 05

compute [D] matrix

$$[D] = [S]^{-1} \times [AD] - [ADL]$$

$$[D] = \frac{1}{1.1} \times [0] - [-70.87]$$

$$= \frac{70.87}{1.1}$$

$$[D] = [64.42] / EI$$