

Department of Electrical Engineering

Final term exam

Date: 23/09/2020

Course Details

Course Title: Signals & Systems

Module: 04

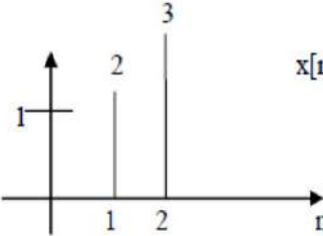
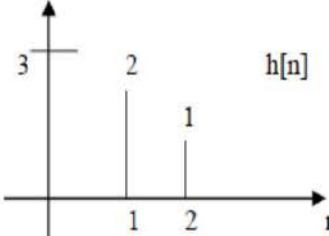
Instructor: _____

Total Marks: 50

Student Details

Name: TALHA KHAN

Student ID: 13845

Q1.	<p>Identify the basic difference between a periodic and an aperiodic signal using examples.</p>	<p>Marks 06</p>
		CLO 1
Q2.	$f(x) = \begin{cases} 0 & -\pi \leq x \leq 0 \\ \pi & 0 \leq x \leq \pi \end{cases}$ <p>Retrieve the Fourier series for the given function.</p>	<p>Marks 12</p>
		CLO 3
Q3.	<p>If $X(z) = \frac{2z^2 + 2z}{(z^2 + 2z - 3)}$</p> <p>Retrieve $x[n]$ using inverse Z-transform method.</p>	<p>Marks 10</p>
		CLO 3
Q4.	<p>If $x[n] = 4\delta[n] - 3\delta[n - 1] + 4\delta[n - 2]$ $h[n] = 2\delta[n - 1] - 3\delta[n - 2] + 2\delta[n - 3]$</p> <p>Produce $Y(z)$ and $y[n]$</p>	<p>Marks 10</p>
		CLO 3
Q5.	<p>Evaluate $y[n]$ using convolution summation.</p> <div style="display: flex; justify-content: space-around; align-items: flex-end;"> <div style="text-align: center;">  <p>$x[n]$</p> </div> <div style="text-align: center;">  <p>$h[n]$</p> </div> </div>	<p>Marks 12</p>
		CLO 2

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Ihsan

Q(1)

Ans Periodic Signal:-

A signal is considered periodic if it repeats itself after a specific interval of time.

Definition:- periodic signal is a signal which repeats itself after specific interval of time.

• Representation:-

Periodic can be represented by a mathematical equation.

• Determination of value:-

The value of periodic signal can be determined at any point in time.

• Type of signal:-

periodic signal are deterministic signals.

• Example:-

Sinusoidal wave, cosine wave, triangle wave and square wave are example of periodic signal.

▶ Aperiodic signal:-

A signal is considered aperiodic signal when it does not repeat its pattern over a period or interval of time.

▶ Definition:- Aperiodic signal is a signal which does not repeat itself after a specific interval of time.

▶ Representation:- Aperiodic signal cannot be represented by any mathematical equation.

• Determination of value:-

The value of aperiodic signal cannot be determined with certainty at any given point of time.

• Type of Signal:-

Aperiodic signals are random signals.

• Example:- Aperiodic signal is signal created by microphone or telephone when one or two words are pronounced. Also, signal propagated by AM radio station or FM radio stations are all aperiodic signals.

$$Q(2) \quad f(x) \begin{cases} 0 & -\pi \leq x \leq 0 \\ \pi & 0 \leq x \leq \pi \end{cases}$$

Sol :-

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^0 f(x) dx + \int_0^{\pi} f(x) dx \right]$$

$$= \frac{1}{2\pi} \left[\int_{-\pi}^0 0 dx + \int_0^{\pi} \pi dx \right]$$

$$= \frac{1}{2\pi} \left[0 + \pi x \Big|_0^{\pi} \right]$$

$$= \frac{1}{2\pi} \left[\pi(\pi - 0) \right]$$

$$a_0 = \frac{1}{2\pi} \times \pi^2$$

$$a_0 = \frac{\pi}{2}.$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^0 f(x) \cos nx \, dx + \int_0^{\pi} f(x) \cos nx \, dx \right]$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^0 (0) \cos nx \, dx + \int_0^{\pi} \pi \cos nx \, dx \right]$$

$$= \frac{1}{\pi} \left[0 + \int_0^{\pi} \pi \cos nx \, dx \right]$$

$$= \frac{1}{\pi} \left[\pi \int_0^{\pi} \cos nx \, dx \right]$$

$$= \frac{1}{n} \left[\sin n(\pi) - \sin n(0) \right]$$

$$= \frac{1}{n} [0 - 0]$$

$$a_n = 0.$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^0 f(x) \sin nx \, dx + \int_0^{\pi} f(x) \sin nx \, dx \right]$$

$$= \frac{1}{\pi} \left[\int_{-\pi}^0 f(x) \sin nx \, dx + \int_0^{\pi} f(x) \sin nx \, dx \right]$$

$$= \frac{1}{\pi} \left[0 + \pi \int_0^{\pi} \sin nx \, dx \right]$$

$$= \frac{1}{\pi} \left[\pi \left(\frac{\cos nx}{n} \right) \Big|_0^{\pi} \right]$$

$$= \frac{-\pi}{n\pi} \left[\cos nx \Big|_0^{\pi} \right]$$

$$= -\frac{1}{n} [\cos n(\pi) - \cos n(0)]$$

$$b_n = \begin{cases} 0 & \text{if } n \text{ even} \\ \frac{2}{n} & \text{if } n \text{ odd} \end{cases}$$

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$$f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + a_3 \cos 3x + \dots$$

$$b_1 \sin x + b_2 \sin 2x + b_3 \sin 3x + \dots$$

$$f(x) = \frac{\pi}{2} + 2 \sin x + \frac{2}{3} \sin 3x.$$

Q (3)

Ans: Given:-

$$X(z) = \frac{2z^2 + 2z}{(z^2 + 2z - 3)}$$

$$X(z) = \frac{2z(z+1)}{z^2 + 3z - z - 3} \quad \text{Taking "2z" common}$$

Simplify

$$X(z) = \frac{2z(z+1)}{z(z+3) - 1(z+3)}$$

$$\frac{X(z)}{z} = \frac{2(z+1)}{(z+3)(z-1)}$$

or we can write

$$\frac{2(z+1)}{z^2 + 2z - 3} = \frac{A}{(z+3)} + \frac{B}{(z-1)}$$

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Putting

$$2(z+1) = A(z-1) + B(z+3) \Rightarrow \text{eq (1)}$$

~~z~~ Now putting $z=1$ in eq (1)

$$2(1+1) = B(1+3)$$

$$4 = 4B$$

$$4/4 = 4/4 B$$

$$1 = B$$

$$B = 1$$

Now putting $z=-3$ in eq (1)

$$2(-3+1) = A(-3-1)$$

$$-4 = -4A$$

$$A = 1$$

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Now we will put A and B
in eq (1)

$$\frac{2(z+1)}{(z+3)(z-1)} = \frac{1}{z+3} + \frac{1}{(z-1)}$$

$$X(z) = \frac{z}{z+3} + \frac{z}{(z-1)}$$

inverse z-transform

$$X(z) = u[3] + (-1)^k$$

Q(4)

Ans:- If $x[n] = 4\delta[n] - 3\delta[n-1] + 4\delta[n-2]$

$h[n] = 2\delta[n-1] - 3\delta[n-2] + 2\delta[n-3]$

$y(z)$ $y[n] = ?$

$X(z) = 4 - 3z^{-1} + 4z^{-2}$

$h(z) = 2z^{-1} - 3z^{-2} + 2z^{-3}$

Now $H(z) \cdot X(z)$

$y(z) = (\cancel{2z^{-1}} - 3$

$y(z) = (2z^{-1} - 3z^{-2} + 2z^{-3}) \cdot (4 - 3z^{-1} + 4z^{-2})$

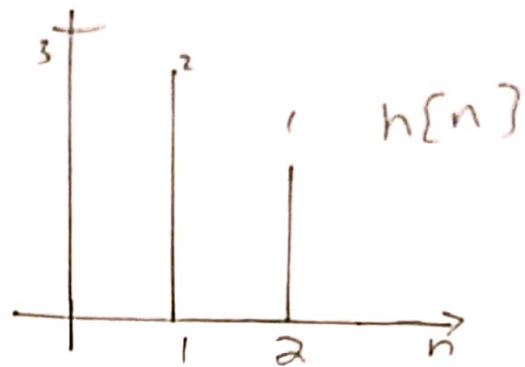
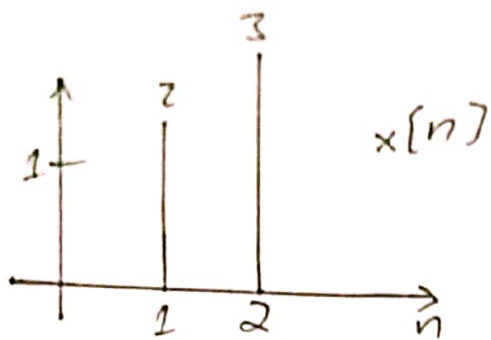
$y(z) = 8z^{-1} - 6z^{-2} + 8z^{-3} - 12z^{-2} + 9z^{-3} - 12z^{-4} + 8z^{-3} - 6z^{-4} + 8z^{-3}$

$$\begin{aligned}
 Y(z) &= 8z^{-1} - 6z^{-2} - 12z^{-2} + 8z^{-3} + 9z^{-3} \\
 &\quad + 8z^{-3} - 12z^{-4} - 6z^{-4} + 8z^{-5} \\
 &= 8z^{-1} - 18z^{-2} + 25z^{-3} - 18z^{-4} + 8z^{-5}
 \end{aligned}$$

So

$$\begin{aligned}
 y[n] &= 8\delta[n-1] - 18\delta[n-2] \\
 &\quad + 25\delta[n-3] \\
 &\quad - 18\delta[n-4] + 8\delta[n-5]
 \end{aligned}$$

Q(5) Evaluate $y[n]$ using convolution summation.



Sol:-

$$y[n] = x[n] * h[n]$$

As we know

$$x[n] = x[n] + 2x[n-1] + 3x[n-2]$$

and

$$y[n] = 3x[n] + 2x[n-1] + x[n-2]$$

$$x[n] = x[0] \delta[n] + x[1] \delta[n-1] + x[2] \delta[n-2]$$

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$$y[n] = x[0]f[n] + x[1]f[n-1] \\ + x[2]f[n-2]$$

$$x[n] = \sum_{k=0}^2 x[k]f[n-k]$$

for $y[n] = \sum_{k=0}^2 x[k]f[n-k]$
