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①

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Q 1 (a) - Estimate $\int a^4 \sqrt{1-a^2} da$

Sol: - For the integral substitute

$$u = 1 - a^2 \quad \text{and} \quad du = -2a da$$

$$= -\frac{1}{2} \int 4 \sqrt{u} du$$

The integral of $4\sqrt{u}$ is $4 \frac{u^{\frac{5}{4}}}{\frac{5}{4}}$

$$= -2u^{\frac{5}{4}} + C$$

$$\text{For } u = 1 - a^2$$

$$= -\frac{2}{5} (1 - a^2)^{\frac{5}{4}} + C$$

x _____ x

Q 1

(b)

Estimate $\int_0^1 x^3 (1+x^4)^3 dx$

Sol: $u = x^4 + 1 \rightarrow \frac{du}{dx} = 4x^3$

$$dx = \frac{1}{4x^3} du$$

②

$$= \frac{1}{4} \int u^3 du$$

Now solving

$$\int u^3 du$$

Applying power Rule

$$\int u^n du = \frac{u^{n+1}}{n+1} \quad \because n=3$$

$$= \frac{u^4}{4}$$

$$\frac{1}{4} \int u^3 du$$

$$= \frac{u^4}{16}$$

Substitution $u = x^4 + 1$

$$= \frac{(x^4 + 1)^4}{16}$$

$$\int u^3 (x^4 + 1)^3 dx$$

$$= \frac{(x^4 + 1)^4}{16} + C$$

(3)

Definite integral

$$\int_0^1 f(x) dx = \frac{15}{16}$$

Approximate

0.9375 Ans

x ————— x

x _____ ~~4~~ 4

Q 2

(a) :-

$$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0$$

Sol:-

$$(x^2 + 3x) + y^2 + z^2 - 4z + 1 = 0$$

$$\left(x^2 + 3x + \left(\frac{3}{2}\right)\right) + (y-0)^2 + \left(z^2 - 4z + \left(\frac{-4}{2}\right)^2\right)$$

⑤

$$= -1 + \left(\frac{3}{2}\right)^2 + \left(\frac{-4}{2}\right)^2$$

$$= \left(x + \frac{3}{2}\right)^2 + (y)^2 + (z - z)^2 = \frac{21}{4}$$

So,

$(x_0, y_0, z_0) = \text{centre}$

and Radius $a = \sqrt{\frac{21}{4}}$ //

x ————— x

Q2

(b) :-

$$y = \sqrt{x} \quad 0 \leq x \leq 4$$

Sol :-

Given data

$$y = \sqrt{x}$$

$$0 \leq x \leq 4 \Rightarrow a \leq x \leq b$$

$$\text{As } V = \int_a^b \pi y^2 dx$$

$$V = \int_0^4 \pi (\sqrt{x})^2 dx$$

$$V = \pi \int_0^4 2 dx = \pi \left. \frac{x^2}{2} \right|_0^4$$

⑥

$$\Rightarrow v = \frac{\pi}{2} \sqrt{(4)^2 - 0} = 8\pi //$$

$$x \text{-----} x$$

Q3 :- If $A = 2i - 4j + \sqrt{5}k$
 $B = -2i + 4j - \sqrt{5}k$

Sol:- $B \cdot A = (-2i + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$

$$B \cdot A = -4i - 16j - 5k$$

$$\boxed{B \cdot A = -25}$$

$$A \cdot A = (2i - 4j + \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$$
$$= 4 + 16 + 5$$

$$\boxed{= 25}$$

$$\text{Proj}_A B = \left(\frac{B \cdot A}{A \cdot A} \right) A$$

$$= \left(\frac{-25}{25} \right) (2i - 4j + \sqrt{5}k)$$

$$= -1 (2i - 4j + \sqrt{5}k)$$

$$\boxed{= -2i + 4j - \sqrt{5}k}$$

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Q4:- Where $y = -x^2 + 5x - 4$ $[0, 2]$

Sol:- $y = f(x) = -x^2 + 5x - 4$

$$[a, b] = [0, 2]$$

$$A = \int_a^b f(x) dx$$

$$A = \int_0^2 (-x^2 + 5x - 4) dx$$

$$A = \left(\frac{-x^3}{3} + \frac{5x^2}{2} - 4x - 0 \right) \Big|_0^2$$

$$A = \frac{(2)^3}{3} + \frac{5(2)^2}{2} - 4(2) - 0$$

$$A = \frac{-27}{3} + \frac{5(9)}{2} - 12$$

$$A = -9 + \frac{45}{2} - 12$$

$$A = -9 + 22.5 - 12$$

$$A = 13.5 - 12$$

$$A = 1.5$$

x

(8)

Q5:-

(a) $A = i - 2j - 2k$ & $B = 6i + 3j + 2k$

Sol:-

$$A = i - 2j - 2k$$

$$\sqrt{(x)^2 + (y)^2 + (z)^2}$$

$$|A| = \sqrt{1 + 4 + 4} = \sqrt{9} = 3$$

$$B = 6i + 3j + 2k$$

$$|B| = \sqrt{(6)^2 + (3)^2 + (2)^2}$$

$$|B| = \sqrt{36 + 9 + 4}$$

$$|B| = \sqrt{49} = 7$$

$$\theta = \cos^{-1} \frac{(A \cdot B)}{|A| |B|}$$

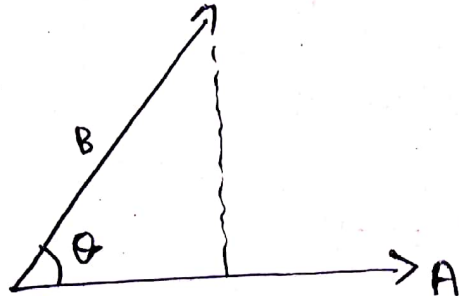
$$\theta = \frac{\cos^{-1} (ci - 2j - 2k) \cdot (6i + 3j + 2k)}{3 \times 7}$$

$$\theta = \cos^{-1} \left(\frac{(1)(6) + (-2)(3) + (-2)(2)}{21} \right)$$

21

(9)

$$\theta = \cos^{-1} \left(\frac{-4}{21} \right)$$



x ————— x

Q5 :-

$$(b) \quad x^2 + y^2 + (z-1)^2 = 1$$

Sol :- $x^2 + y^2 + (z-1)^2 = 1$

$$= (\rho \sin \phi \cos \theta)^2 + (\rho \sin \phi \sin \theta)^2 + (\rho \cos \phi - 1)^2 = 1$$

$$= \rho^2 \sin^2 \phi \cos^2 \theta + \rho^2 \sin^2 \phi \sin^2 \theta + \rho^2 \cos^2 \phi + 1 - 2\rho \cos \phi = 1$$

$$\rho \cos \phi = 1$$

$$= \rho^2 \sin^2 \phi (\cos^2 \theta + \sin^2 \theta) + \rho^2 \cos^2 \phi + 1 - 2\rho \cos \phi = 1 - 1$$

$$= \rho^2 (\sin^2 \phi + \cos^2 \phi) - 2\rho \cos \phi = 0$$

$$\rho^2 = 2\rho \cos \phi$$

$$\rho = 2 \cos \phi$$