I.D :- 7278

Name :- Hamza Ikram

Date:

Q3 :- $x^2y'' + 2xy' - 6y = 10x^2$: $y(1) = 1$, $y'(1) = -6$

Sol :-

$x^2y'' + 2xy' - 6y = 10x^2$ — (1)

Put $x = e^t$ then $\frac{dy}{dx} = y' = e^{-t}Dy$ ∴ $D = \frac{d}{dt}$

& $\frac{d^2y}{dx^2} = y'' = e^{-t}[D(D-1)]y$

using these values (1)

$e^{bt}e^{-bt} D(D-1)y + 2e^t \cdot e^0 y - 6y = 10e^{2t}$

$D^2y - Dy + 2Dy - 6y = 10e^{2t}$

$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = 10e^{2t}$

$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = 10e^{2t}$ — (2)

Assosiated homogenous eq of (2) is

$\frac{d^2y}{d} + \frac{dy}{dt} - 6y = 0$ — (3)

say d/dt m, $d^2/dt^2 = m^2$ they (3)

$m^2y + my - 6y = 0 \Rightarrow (m^2 + m - 6)y = 0$

roots are $m = -3, 2$

The complementry solution is

$y_c = c_1e^{-3t} + c_2e^{2t}$ — (4)

Now find w, w_1, w_2

$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{-3t} & e^{2t} \\ -3e^{-3t} & 2e^{2t} \end{vmatrix} = 2e^{-t} + 3e^{-t} = 5e^{-t}$

where $y_1 = e^{-3t}$, $y_2 = e^{2t}$

②

Date:

$$w_1 = \begin{vmatrix} 0 & y_2 \\ f(t) & y_2' \end{vmatrix} = \begin{vmatrix} 0 & e^{2t} \\ 10e^{2t} & 2e^{2t} \end{vmatrix} = -10e^{4t}$$

$$w_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(t) \end{vmatrix} = \begin{vmatrix} e^{-3t} & 0 \\ -3e^{-3t} & 10e^{2t} \end{vmatrix} = 10e^{-t}$$

Now

R $\quad u_1 = \int \frac{w_1}{w} dt = \int -\frac{10e^{4t}}{5e^{-t}} dt = -2\int e^{5t} dt = -\frac{2}{5} e^{5t}$

$u_2 = \int \frac{w_2}{w} dt = \int \frac{10e^{-t}}{5e^{-t}} dt = \int 2\, dt = 2t$

we have

$\Rightarrow \quad y_p = u_1 y_1 + u_2 y_2 = \frac{-2}{5} e^{5t} \cdot e^{3t} + 2t \cdot e^{2t}$

$y_p = -\frac{2}{5} e^{2t} + 2te^{2t}$

The general solution is

$y = y_c + y_p$

Q4:→ $x^2 y'' + 7xy' + 5y = x^5 \quad y(0) = 2 \;\&\; y(1) = 2$

Sol:→ $x^2y'' + 7xy' + 5y = x^5$ — ①

put $x = e^t$ thus $y' = e^{-t} Dy \quad \because D = \frac{d}{dt}$

R $\quad y'' = e^{-2t} D(D-1)y$

$\Rightarrow e^{2t} e^{-2t} D(D-1)y + 7e^{t} e^{-t} Dy + 5y = e^{5t}$

$\Rightarrow D^2 y - Dy + 7Dy + 5y = e^{5t}$

$D^2 + 6Dy + 5y = e^{5t} \rightarrow ②$

The assoicated homogenous eq of ② is

$\frac{d^2y}{dt^2} + \frac{6dy}{dt} + 5y = 0$ ③

$\frac{d}{dt} = m, \quad \frac{d^2y}{dt^2} = m^2$

$(3) \Rightarrow m^2 + 6m + 5 = 0$

The root are $m = -1, -5$

Date:

The complementry sol is

$$y_c = c_1 e^{-5t} + c_2 e^{-t}$$

Here $y_1 = e^{-5t}$, $y_2 = e^{-t}$

Now

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{-5t} & e^{-t} \\ -5e^{-5t} & -e^{-t} \end{vmatrix} = -e^{-6t} + 5e^{-6t}$$

$$W = 4e^{-6t}$$

& $$W_1 = \begin{vmatrix} 0 & y_2 \\ f(t) & y_2' \end{vmatrix} = \begin{vmatrix} 0 & e^{-t} \\ e^{5t} & -e^{-t} \end{vmatrix} = -e^{4t}$$

$$W_2 = \begin{vmatrix} y_1 & f_0 \\ y_1' & f(t) \end{vmatrix} = \begin{vmatrix} e^{-5t} & 0 \\ -e^{-5t} & e^{5t} \end{vmatrix} = 1$$

Now

$$u_1 = \int \frac{W_1}{W} dt = \int \frac{-e^{-4t}}{4e^{-6t}} dt = \frac{1}{4}\int e^{2t} dt = \frac{1}{8} e^{2t}$$

$$u_2 = \int \frac{W_2}{W} dt = \int \frac{1}{4e^{-6t}} dt \quad \frac{1}{4}\int e^{6t} dt = \frac{1}{24} e^{6t}$$

Thus P.I is

$$y_p = u_1 y_1 + u_2 y_2 = \frac{1}{8} e^{2t} \cdot e^{-5t} + \frac{1}{24} e^{6t} \cdot e^{-t}$$

$$y_p = \frac{1}{8} e^{-3t} + \frac{1}{24} e^{5t}$$

The general solution is $y = y_c + y_p$

$$y = c_1 e^{-5t} + c_2 e^{-t} + \frac{1}{8} e^{-3t} + \frac{1}{24} e^{5t}$$

* *

Date:

Q2 $x^3y''' + 2x^2y'' + 2y = 100'x + \frac{10}{x}$

Sol:-

$$x^3y''' + 2x^2y'' + 2y = 100'x + \frac{10}{x} \longrightarrow (1)$$

put $x = e^t$ then

$$\frac{dx}{dt} = e^t \Rightarrow \frac{dt}{dx} = e^{-t}$$

Now

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{dy}{dt} \cdot e^{t}$$

or

$$y' = \frac{dy}{dx} = e^{-t} Dy \quad \because \frac{d}{dt} \to D$$

Similarly

$$y'' = e^{-3t} [D(D-1)]y$$

$$y''' = e^{-3t} [D(D-1)(D-2)]y$$

using these value in eq (1)

$$e^{3t} \cdot e^{-3t} [D(D-1)(D-2)]y + 2 e^{2t} \cdot e^{-2t} [D(D-1)]y + 2y = 10e^t + 10e^{-t}$$

$$(D^3 - 3D^2 + 2D)y + (2D^2 - 2D)y + 2y = 10e^t + 10e^{-t}$$

$$D^3y - D^2y + 2y = 10e^t + 10e^{-t}$$

$$\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2} + 2y = 10e^t + 10e^{-t} \longrightarrow (2)$$

The assosited homogenous equation 2

$$\frac{d^3y}{dt^3} - \frac{d^2y}{dt^2} + 2y = 0$$

say $d/dt^2 = k^2$, $\frac{d^3}{dy^3} = k^3$

Date:

$(k^3y - k^2y + 2y) = 0$

$(k^3 - k^2 - 2)y = 0$

for non-trivial sol- $y \neq 0 \Rightarrow$

$k^3 - k^2 + 2 = 0$

Roots are

$k = -1, \; 1+i$

$y_c(t) = Ae^{-t} + (B\cos t + C\sin t)e^{t}$

when is complainmentry solution.

Q5- $(x+1)^2 y'' - 3(x+1)y' + 4y \; x^3$

Solg-

$(x+1)^2 y'' - 3(x+1)y' + 4y \; x^3 \longrightarrow (1)$

$x+1 = e^t \Rightarrow x = e^t - 1$

Diff $\log(x+1)t$

Also $(x+1)y' = Dy \; \{d/dt = D\}$

$(x+1)^2 y'' = D(D-1)y$ and $D = \frac{d}{dx}$

Thus eq(1) $\Rightarrow (D(D-1) - 3D + 4)y = e^t - 1)^2$

$(D^2 - 4D + 4)y = e^{2t} - 2e^t + 1$

eq is $D^2 - 4D + 4 = 0$

$(D-2)^2 = 0$

$D = 2, 2$

The complementry function is ~~2t~~

$C.f = (c_1 + c_2 t)e^{2t}$

Also particular integral is

$P.I = \frac{1}{(D-2)^2}(e^{2t} - 2e^t + 1)$

$= \frac{1}{(D-2)^2}e^{2t} - 2\frac{1}{(D-2)^2}et = \frac{1}{0}e^{2t} \rightarrow (2)$

6

Date:

Now

$$\cancel{\frac{1}{(D-2)}e^{2t} - 2\frac{1}{(D-2)}e^{t} + \frac{1}{(D-2)^2}} \rightarrow ②$$

$$\frac{1}{(D-2)^2}e^{2t} = \frac{1}{(2-2)^2 e^{2t}} = \frac{1}{0}e^{2t}$$

case of failure

$$\frac{1}{(D-2)^2}e^{2t} = \frac{t\,1}{2(1-2)^2}e^{t} = \frac{t^2 e^{t}}{2}$$

$$\&\quad 2\frac{1}{(D-2)^2}e^{t} = 2\frac{1}{(1-2)^2}e^{t} = 2e^{t}$$

$$\&\quad \frac{1}{(D-2)^2}(1) = \frac{1}{(D-2)^2}e^{0t} = 1/4$$

eq ② $\quad P.I = 1/2\, t^2 e^{2t} - 2e^{t} + 1/4$

Hence complete solution is ②

$$y = C = -P.I$$

$$y = (c_1 + c_2 t)e^{2t} + 1/2\, t^2 e^{2t} - 2e^{t} + \frac{1}{4}$$

respect value of e^{t}

$$y = (c_1 + c_2 \log(x+1)^2 + 1/2[(\log(x+1)^2 (x+1)^2]$$

$$2(x+1 + 1/4 \quad \text{or}$$

$$y = C_1 + C_2 \log(x+1)(x+1)^2 + 1/2(\log(x+1)^2 (x+1)^2] - 2x - 7/4$$

which is required.

Date:

Q 2:- $x^3y''' + 4x^2y'' - 5xy' - 15y = x^4$

Sol:-

$x^3y''' + 4x^2y'' - 5xy' - 15y = x^4 \rightarrow (1)$

put $x = e^t$ then $\frac{dy}{dx} = y' = e^{-t} Dy \therefore D = d/dt$

& $\frac{d^2y}{dx^2} = y'' = e^{t} \left(D(D-1) \right) y$

using these values by (1)

$e^{6t} e^{-6t} D(D-1)y + 2e^{-t} e^{t} y - 6y = 10e^{4t}$

$D^2y - Dy + 2Dy - 6y = 10e^{2t}$

$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = 10e^{2t}$

$\frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = 10e^{2t} \rightarrow (2)$

Associated homogenous eq - q (2) is

$\frac{d^2y}{d} + \frac{dy}{dt} - 6y = 0 \rightarrow (3)$

say d/dt m, $d^2/dt^2 = m^2$ then (3)

$m^2y + my - 6y = 0 \Rightarrow (m^2 + m - 6)y = 0$

roots are $m = -3, 2$

The complementary solution is

$y_c = c_1 e^{-3t} + c_2 e^{2t} \rightarrow (4)$

Now find w, w_1, w_2

$w = \begin{vmatrix} y_1 & y_2 \\ y_1 & y_2 \end{vmatrix} = \begin{vmatrix} e^{-3t} & e^{2t} \\ -3e^{3t} & 2e^{3t} \end{vmatrix} = 2e^{-t} + 3e^{-t} = 5e^{-t}$

where $w_1 = \begin{vmatrix} 0 & y_2 \\ f(t) & y_2' \end{vmatrix} = \begin{vmatrix} 0 & e^{2t} \\ 10e^{2t} & 2e^{ot} \end{vmatrix} = -10e^{4t}$

$w_2 = \begin{vmatrix} y_1 & 0 \\ y_1' & f(t) \end{vmatrix} = \begin{vmatrix} e^{-3t} & 0 \\ -3e^{-3t} & 10e^{2t} \end{vmatrix} = 10e^{-t}$

Date:

Now

$u_1 = \int \frac{w_1}{w} dt = \int -\frac{10^{4t}}{5e^{-t}} dt = -2\int e^{5t} dt = -2/5 \, e^{5t}$

$u_2 = \int \frac{w_2}{w} dt = \int \frac{10e^{-t}}{5e^{-t}} dt = \int 2 dt = 2t$

we have

$y_p = u_1 y_1 + u_2 y_2 = \frac{-2}{5} e^{5t} \cdot e^{3t} + 2t \cdot e^{2t}$

$\Rightarrow$

$y_p = -2/5 \, e^{2t} + 2te^{2t}$

The general solution is

$y = y_c + y_p$

Scanned with CamScanner