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SUBJECT	INTRO TO STRUCTURAL DYNAMICS
UNIVERSITY	IQRA NATIONAL UNIVERSITY
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1)

Q1 QUESTION No. 1

Given Data

$T_n = ?$

$I = 150 \text{ in}^2$

δ_{st} = due to 7282 Lb

Sol:

General Eq of motion:

$$k u + c\dot{u} + m\ddot{u} = p(t)$$

= As the system vibrates freely, ignoring damping effects the eq(1) becomes

$$K u + m\ddot{u} = 0$$

= As $k = 3EI/L^3 \Rightarrow \dfrac{3(29000 \text{ k/in}^2 \times 150 \text{in}^4)}{(10 \times 12 \text{ in})^3}$

$$= \frac{13050{,}000}{1728000} = k = 7.55 \text{ k/in}$$

$$= 7.55 \text{ k/in} \times 1000/12 = 90600 \text{ lb/ft}$$

$$m = \frac{7282 \text{ Lb sec}^2}{32.2 \text{ ft}} \qquad m = 226.14 \text{ slug}$$

= $\omega_n = \sqrt{k/m} = \sqrt{\frac{90600}{226.14}}$

= $\omega_n = 20.015$ rad/sec

Time Period.

$T_n = 2\pi/\omega_n = \frac{2(3.14)}{20.01}$

$= 0.3138$ sec

= By substituting values in (1)

$90600\,u + 226.14\,\ddot{u} = 0$

= General sol. of Eq of motion for undamped free vibration

$u(t) = u(0)\cos(\omega_n t) + \dot{u}(0)/\omega_n \sin(\omega_n t)$

$u(0) = 1/2''$

$= 1/24$ ft & $\dot{u}(0) = 0$

$u(t) = (1/24)\cos(20.015t) + 0$

$u(t) = \frac{1}{24}\cos(20.015t)$

Q1

Equivalent Static force at "t"

$f_s(t) = k\,u(t) = \frac{90600 \times \cos(20.05t)}{24}$

$f_s(t) = 3775(\cos(20.05t))$

Now for u_0 = Amplitude ⇒ Amplitude of equivalent Static force

$ku_0 = 90600 \times 1/24$

$ku_0 = 3775$ lb

$u_0 = \sqrt{(u(0))^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2}$

$= \sqrt{(1/24)^2 + 0}$

$= \sqrt{(1/24)^2}$

$u_0 = 1/24$

Question: Question number (Two) (02)

Q2: (2) (b) (Two) Eq. of motion for damped free vibration

$ku + c\dot{u} + m\ddot{u} = 0$ ∴ $k = 90600$ lb/ft

$m = 226.14$ lb/sec²

$c = \zeta \times 2m\omega_n$ (The damping ratio = 0.025)

$c = (0.025) \times 2(226.14)(20.015)$

$c = 226.30$ lb/sec/ft

Q2:

Putting the values in eq

$$90600\,u + 226.3\,\dot{u} + 31.06\,\ddot{u} = 0$$

Solution of Eq of motion for damped free vibration.

$$u(t) = e^{-\zeta\omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega}\left[\dot{u}(0) + u(0)\,\zeta\omega_n\right]\right]$$

As $\omega_m = 20.01$ rad/sec

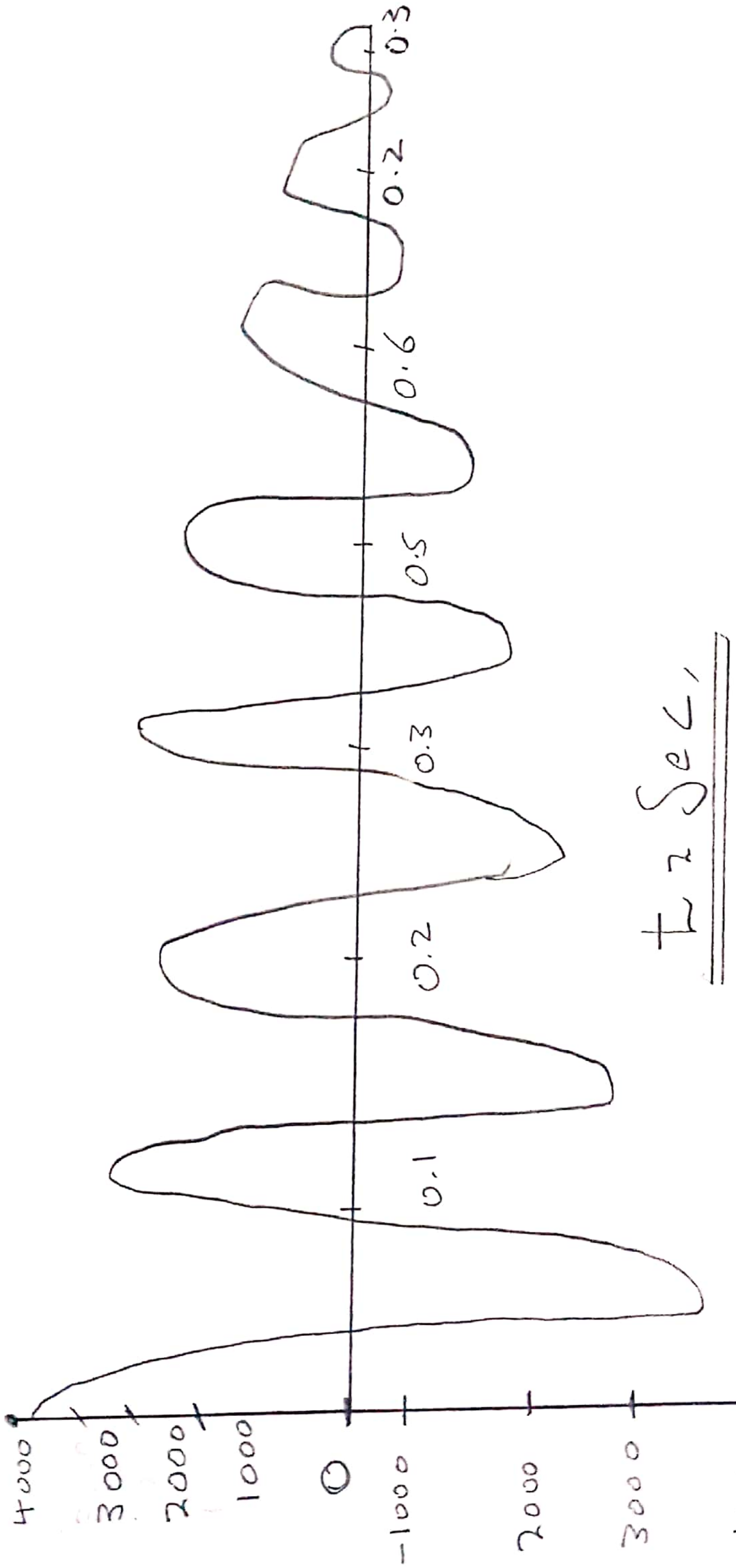
$$u(t) = e^{-0.025 \times 20.01}\left[u\ \frac{1}{24}\cos(20.01t) + \frac{1}{20.01}\left[0 + \frac{1}{24}\,s\,(0.025)(20.01t)\right] \times \sin(20.01t)\right.$$

$$u(t) = e^{-0.5}\left(0.041\cos(20.01t) + 0.001025\sin(20.01t)\right)$$

$$f_{st} = k.u(t) \Rightarrow 90600\,u(t)$$

$$f_{st} = e^{-0.5}\left[3714.6\cos(20.01t) + 92.86\sin(20.01t)\right]$$

Q2:


4000
3000
2000
1000
0
-1000
2000
3000
4000
0.1
0.2
0.3
0.5
0.6
0.2
0.3
T = Sec

Q2:

$$u(1) = e^{-0.5}(0.03852 + 0.0003507)$$

$$= e^{-0.5}(0.0388)$$

$$= 0.01196 \approx 0.012$$

QUESTION No. 3 (Three)

Q3: → 7.282 inch

Cycles = 7

T = 3.57 sec

Amplitude of displacement = 0.9"

a) Damping ratios

$u_1 = 7282$ inch

$$\zeta = \frac{1}{2\pi j} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

Q3

$$= \frac{1}{2(3.14)(\zeta)} \ln\left(\frac{7.28}{0.9}\right)$$

$$7(6.28)\zeta = \ln(8.088)$$

$$43.96\zeta = 2.09$$

$$\zeta = 2.09/43.06$$

$$\zeta = 4.75\%$$

(b) $T_n = ?$

if cycles of vibrations are completed in 3.57 sec

Time required to complete One cycle $\frac{3.57}{7} = T_D$

$$T_D = 0.51 \text{ sec}$$

Now

$$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$$

$$\frac{2\pi}{\omega_D} = 2\pi / \omega_n \sqrt{(1-\zeta^2)}$$

Q3

$$T_D = \frac{T_n}{\sqrt{(1-\zeta^2)}}$$

$$T_n = T_D\sqrt{(1-\zeta^2)}$$

$$= 0.51\sqrt{1-(0.0475)^2}$$

$$= 0.51\sqrt{0.9977}$$

$$T_n = 0.5094 \text{ sec}$$

(c) $K = ?$

$$K = \frac{60 \cos 60^\circ}{7.28}$$

$$= 4.12087 \text{ k/in}$$

$$= 49440 \text{ lb/ft}$$

$$\omega_n = \sqrt{k/m} = \sqrt{k/(w/g)}$$

$$= \sqrt{\frac{kg}{w}}$$

$$w = Mg/\omega_n^2$$

Q3

$\omega_n = 2\pi / T_n$

$W = K g \frac{T_n^2}{4\pi^2}$

$W = \frac{49440 \text{ lb}}{ft} \times \frac{32.2 \text{ ft}}{sec^2} \frac{(0.5094)}{4\pi^2}$

$W = 1591968 (0.00658)$

$= 10476.72 \text{ lb}$

$W = 10.47 \text{ kl}$

(e) $c = ?$

$\zeta = \frac{c}{2m\omega_n}$

$c = \zeta \, 2m \, \omega_n$

$= \zeta \, 2m \, (2\pi / T_n)$

$c = \frac{(0.0475) 4 (3.14) \left(\frac{10476.7}{32.2} \right)}{0.5}$

$c = 381.33 \text{ lb sec/ft}$

Q3

(f) No. of cycles to reduce displacement Amplitude from

7.28 to 0.5in $j=?$

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$j = \frac{1}{2\pi(0.0475)} \ln\left(\frac{7.22}{0.5}\right)$$

$$= \frac{1}{0.2983}(2.67)$$

$j = 2.95$ or 9 cycles.