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1.

Q1(a) Ans:

1st we find 'p'

$p = \sqrt{x^2 + y^2}$

$x = -2$, $y = 6$

putting values:

$p = \sqrt{(-2)^2 + (6)^2}$

$p = \sqrt{4 + 36}$

$\boxed{p = \sqrt{40}}$

Now we find 'φ'

As $\tan\varphi = (y/x)$

$\varphi = \tan^{-1}(y/x)$

putting values:

$\varphi = \tan^{-1}(-6/2)$

$\boxed{\varphi = -71.57}$

And $\boxed{z = 3}$

②

Q1 Ans: (part b).

Given point (3,4,5) in cartesian coordinates

Sherical cordinates = ?

For spherical cordinates we find (r, θ, ϕ)

As $r = \sqrt{x^2+y^2+z^2}$

$x = 3, y = 4, z = 5$

putting values:

$r = \sqrt{(3)^2+(4)^2+(5)^2} = \sqrt{9+16+25}$

$\boxed{r = \sqrt{40}}$ or $\boxed{r = 7.07}$

Now $\cos\theta = (z/r)$.

$\theta = \cos^{-1}(z/r)$

putting values:

$\theta = \cos^{-1}\left(5/\sqrt{40}\right)$.

$\because \sqrt{40} \Rightarrow 5\sqrt{2}$

(3)

So:

$$\theta = \cos^{-1}\left(5/5\sqrt{2}\right)$$

$$\boxed{\theta = 45^\circ}$$

$$\tan\phi = (y/x)$$

$$\phi = \tan^{-1}(y/x)$$

putting values:

$$\phi = \tan^{-1}(4/3)$$

$$\boxed{\phi = 53^\circ}$$

(4)

Q1: Ans: part (c)

Given $A = (2, 3, -1)$

Spherical coordinates = ?

For spherical coordinate we have to find (r, θ, ϕ)

$r = \sqrt{x^2 + y^2 + z^2}$

$x = 2,\ y = 3,\ z = -1$

putting values:

$r = \sqrt{(2)^2 + (3)^2 + (-1)^2}$

$r = \sqrt{4 + 9 + 1}$

$\boxed{r = \sqrt{14}}$ or $\boxed{r = 3.7}$

Now: $\cos\theta = (z/r)$

$\theta = \cos^{-1}(z/r)$

$\theta = \cos^{-1}(-1/3.7)$

$\boxed{\theta = 105.5^\circ}$

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$\tan\phi = (y/x)$

$\phi = \tan^{-1}(y/x)$

putting value:

$\phi = \tan^{-1}(3/2)$

$\boxed{\phi = 56.31^\circ}$

(6)

Q(d) Ans:

Given (4, 25, 120)

Find cartesian coordinate

For this we have to

find (x, y, z)

$x = r \sin\theta \cos\phi \rightarrow (i)$

~~sin = 25°~~ , ~~cos~~

$\theta = 25°$, $\phi = 120$, $r = 4$

putting values in (i).

$x = 4 \sin 25° \cos 120°$

$\boxed{x = -0.84}$

$y = r \sin\theta \sin\phi$

putting values:

$y = 4 \sin 25° \sin 120°$

$\boxed{y = 1.46}$

(7)

$Z = r\cos\theta$

putting values:

$Z = 4\cos 25^{\circ}$

$$\boxed{Z = 3.62}$$

(8).

Part (7) Ans:

As we know:

$$F = \frac{q_1 q_2}{(4\pi\varepsilon_0 r^2)}$$

putting values:

$$F = -2 \times 9 / (10^{-9} \times 12).$$

$$\boxed{F = -18 \times 10^9}$$

so:

$$E = F/q$$

$$F = (-18 \times 10^9)$$

$$q = 2$$

putting values:

$$\varepsilon = \frac{18 \times 10^{+9}}{2} \Rightarrow \boxed{\varepsilon = 9 \times 10^9}$$

(9)

part (g) Ans:

As we know:

$$E = Q / 4\pi\varepsilon_0 r^2$$

$E = 40 V/cm \Rightarrow \boxed{E = 4000 V/m}$

$r = 30 cm \Rightarrow \boxed{r = (0.3)^2 m}$

putting values:

$$Q = (4000 \times (0.3)^2) / (9 \times 10^9)$$

$$\boxed{Q = 4 \times 10^{-8} C}$$

part (h) Ans:

Given:

$q_1 = 2\times10^{-7}$, $q_2 = 4.5\times10^{-7}$

$F = 0.1N$

$r = ?$

As: Force is:

$F = q_1 q_2 / 4\pi \epsilon_0 r^2$.

$r^2 = q_1 q_2 / 4\pi \epsilon_0 F$

$$r^2 = \frac{(2\times10^{-7})(4.5\times10^{-7})}{4(3.17)(8.8\times10^{-11})(0.1)}$$

$$r^2 = \sqrt{\frac{(2\times10^{-7})(4.5\times10^{-7})}{4(3.17)(8.8\times10^{-12})(0.1)}}$$

By taking squre root b/s

$\boxed{r = 0.09m}$

Q # 2 (a)

$$\vec{A} = \sqrt{3}\,\hat{i} + \hat{j}$$

$$B = 2\hat{i}$$

$$\vec{A} \cdot \vec{B} = 2\sqrt{3}$$

As:

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

$$\theta = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{AB}$$

$$A = \sqrt{(\sqrt{3})^2 + (1)^2} = \sqrt{4} \Rightarrow 2$$

$$B = 2$$

$$\theta = \cos^{-1}\left(\frac{2\sqrt{3}}{2 \times 2}\right)$$

$$\theta = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\boxed{\theta = 30^\circ}$$

(b) (i)

$$f = ax^2 + by^3z$$

$$\Delta f = \nabla(ax^2 + by^3z).$$

$$= a\frac{\partial x^2}{\partial x}\hat{i} + \left[b\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}k\right)y^3z\right]$$

$$\nabla f = a\,\partial x\hat{i} + b\left[z\frac{\partial}{\partial y}y^3\hat{j} + y^3\frac{\partial}{\partial z}zk\right]$$

$$= 2ax\hat{i} + b\left[3zy^2\hat{j} + y^3\hat{k}\right]$$

$$\boxed{\nabla f = 2ax\hat{i} + 3bzy^2\hat{j} + by^3\hat{k}}$$

(B)

b (ii)

$f = ar^2 \sin\varphi + brz\cos 2\varphi$

Solution:

$$\nabla f = \frac{\partial f}{\partial r} i_r + \frac{1}{r}\frac{\partial f}{\partial \varphi} i_\varphi + \frac{\partial f}{\partial z} i_z$$

$$= 2(ar\sin\varphi + bz\cos 2\varphi) i_r$$
$$+ (ar\cos\varphi - 2bz\sin\varphi) i_\varphi$$
$$+ \quad br\cos 2\varphi\, i_z$$

Q1 (e) Ans:

Given:
charges = $q_1 = 2nc$
= $q_2 = -1nc$

distance = r = 4cm.

Solution:
Before the charge are brought constant:

$F = (q_1 q_2) / 4\pi\epsilon_0 r^2$.

putting values:

$$F = \frac{(2nc)(-1nc)}{4(3.14)(\quad)(4)^2}$$

$$\boxed{F = 11.23\,\mu N}$$

After charges are brought into constant and separated charge on each sphere is:

$= (q_1 + q_2) / 2$

$= (2nc + (-1nc)) / 2 = 0.5nc$

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$q_1 \neq q_2 = 0.5\,mC$

Force 'F' due to

$q_1 = q_2$

$$F = \frac{0.5}{4(3.14)(\quad)(4)^2}$$

$$\boxed{F = 1.404\,\mu N.}$$

Q: Three point charge are placed on the y-axis as show. Find the electric field at point P on the x-axis.

Sol

The distance between charge 2Q and point "P" is

$r^2 = b^2 + a^2$

So

$r = \sqrt{b^2 + a^2}$

let as sume that charge 2Q make angle (α) and ($-\alpha$) with x-axis

magnitude of $|\vec{E_1}| = |\vec{E_2}| = \frac{kq}{r^2}$

$= \frac{k(2Q)}{r^2}$

$= \frac{k(2Q)}{b^2 + a^2}$

So resultant of $\vec{E_1}$ and $\vec{E_2}$ is

$\vec{E_{2+2}} = \vec{E_1} + \vec{E_2} = \vec{E_{1x}} + \vec{E_{2x}}$

(y-component will be cancel)

$= \frac{k(2Q)}{b^2+a^2}(\cos(\alpha) + \cos(-\alpha))$

$= \frac{k(2Q)}{b^2+a^2}(2\cos(\alpha)$ ∴ $\cos(\alpha) = \cos$

$$\vec{E} = \frac{4kQ\cos(\alpha)}{b^2+a^2} \longrightarrow (9)$$

→ Now electric field at point 'p' due to charge "-Q".

→ As charge is negative Electric field at point will be to directed towards charge "-Q"

$$\boxed{-Q}\ O \xleftarrow{\vec{E_A}} \cdot P$$

$$\vec{E_A} = \frac{-k(Q)}{b^2}$$

Net electric field at point "p" will be

$$\vec{E}_{net} = \vec{E_A} + (\vec{E_1} + \vec{E_2})$$

$$= \frac{-k(Q)}{b^2} + \frac{4kQ\cos\alpha}{b^2+a^2}$$

$$= \frac{-kQ(a^2+b^2) + 4kQb^2\cos\alpha}{b^2(a^2+b^2)}$$

$$= \frac{kQ}{b^2(a^2+b^2)}\left[4b^2\cos\alpha - (a^2+b^2)\right]$$

Where $k = 9\times10^9\ Nm^2/C^2$

$$\vec{E}_{net} = \frac{9\times10^9 Q}{b^2(a^2+b^2)}\left[4b^2\cos\alpha - (a^2+b^2)\right]$$

$= \cos($
$\alpha)$

$$\alpha = \tan^{-1}\left(\frac{a}{b}\right)$$

So

$$\vec{E}_{net} = \frac{9 \times 10^9 Q}{b^2(a^2+b^2)}\left(4b^2 \cos\left(\tan^{-1}\left(\frac{a}{b}\right) + (a^2 - b^2)\right)\right)$$