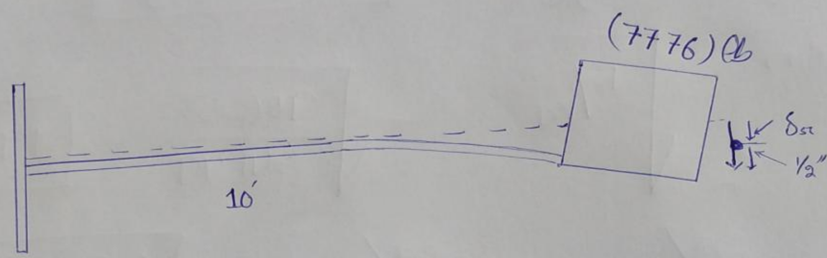
IQRA NATIONAL UNIVERSITY PESHAWAR

DEPARTMENT OF CIVIL ENGINEERING

SUBJECT:	STRUCTURAL DYNAMICS AND EARTHQUAKE	NAME: ABDUL BASIT
SEMESTER:	8TH	ID: 7776 SECTION: C

Q.NO (01) **ANSWER:**

QNO # 01



Sol

The general EOM for SDOF system is

$$ku + c\dot{u} + m\ddot{u} = P(t)$$

In our case system is undamped ($c=0$) undergoing free vibration ($P(t)=0$)

Hence EOM become $ku + m\ddot{u} = 0$ ——— (1)

$K = 3EI/L^3$

$$= \frac{3 \times 29000 \frac{K}{in^2} \times 150\ in^4}{(10 \times 12 in)^3} = \boxed{7.55\ K/in}$$

In order to eliminate the chances of mistake during calculation, it is more appropriate to use fundamental units like lb, ft sec or kg m, sec.

$K = 7.55\ k/in = 90625\ lb/ft.$

$m = \dfrac{7776\ lb\,sec^2}{32.2\,ft} = \boxed{241.49\ slug}$

$\omega_n = \sqrt{k/m} = \sqrt{\dfrac{90625}{241.49}} = \boxed{19.37\ rad/sec}$

$T_n = 2\pi/\omega_n = 2\pi/19.37 = \boxed{0.324\ Sec}$

Substituting the corresponding values in eq (1)

$90625u + 241.49\ddot{u} = 0.$

where "k" is in lb/ft and "m" is in lbsec/ft²

General Solution to EOM for undamped free vibration is

$u(t) = u(0)\cos(\omega_n t) + \dot{u}(0)/\omega_n \sin(\omega_n t)$

$u(0) = 1/2'' = 1/24\,ft$ and $\dot{u}(0) = 0.$

$u(t) = 1/24 \times \cos(19.37t) + 0 = 1/24 \times \cos(19.37t)$

Equivalent static force at any time "t" is

$f_s(t) = K \cdot u(t) = \dfrac{90625\cos(19.37t)}{24}$

$f_s(t) = 3776.04\cos(19.37t)$

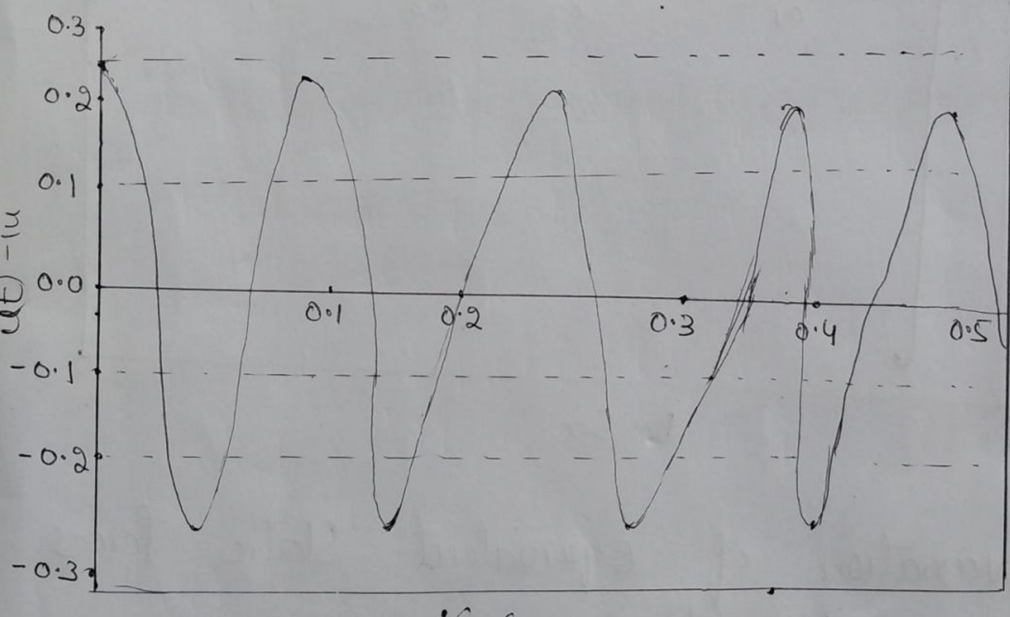
Amplitude of dynamic displacement, u_0 for

$$u_0 = \sqrt{\left[(u(0))^2 + \left(\dot{u}(0)/\omega_n\right)^2\right]} = \sqrt{\left(\left(1/24\right)^2 + 0\right)} = 1/24 \text{ ft}$$

Amplitude of equivalent static force f_{s0}

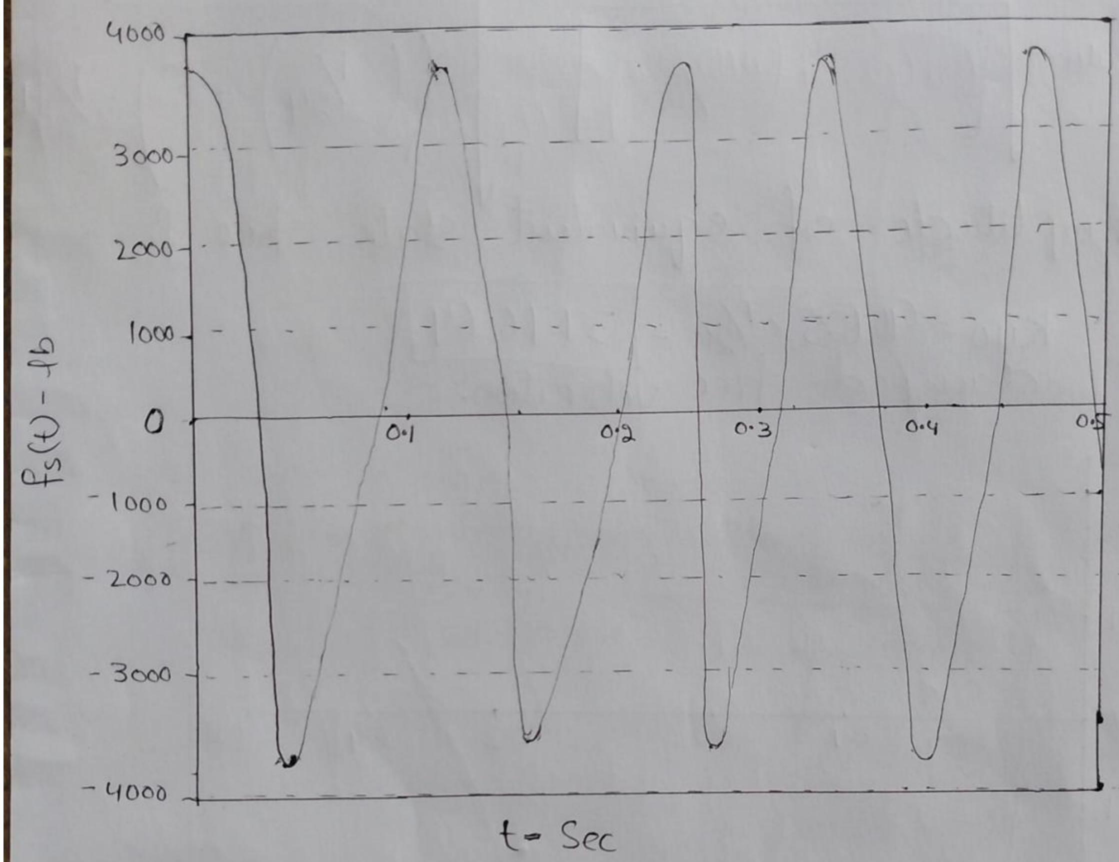
$$Ku_0 = 90625 \times 1/24 = \boxed{3776.64}$$

⇒ Undamped free vibration.



Variation of displacement with time.

Undamped Free vibration.



variation of Equivalent Static forces with time.

Q.NO (03) ANSWER:

Q No #3

Given data :-

- ⇒ Force = 60 Kips.
- ⇒ $u_1 = 7776/1000 = 7.776$ in

* After ; J = 7 (cycles).

⇒ completed = 3.57 sec.

⇒ $u_{J+1} = 2.286$ cm $= 0.9$ in

⇒ Ignore the verticle vibrating.

⇒ Required

(a) Damping ratios.
(b) Natural period of undamped vibration.
(c) Stifness of structures.
(d) weight of tank.
(e) Damping co-efficent.
(f) Number of cycles to reduce the displacement emplitude to 0.5".

②

Solution :-

(a) ζ = Damping ratio = ?

As,

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

By Putting values.

$$7 = \frac{1}{2(3.14)\zeta} \ln\left[\frac{7.776}{0.9}\right]$$

$$\zeta(7 \times 3 \times 3.14) = 2.15$$

$$\zeta = \frac{2.15}{43.96}$$

$$\boxed{\zeta = 0.0489}$$

$$\boxed{\zeta = 4.89\%}$$

(3)

(b) $T_n = ?$

As "seven" cycles are completed in "3.57" sec

Thus time required to complete one cycle.

$= 7/3.57 = 1.96 sec$

$T_D = 1.96\ sec$

Now $\omega_D = \omega_n \sqrt{(1-\zeta^2)}$

$\Rightarrow \frac{2\pi}{\omega_D} = \frac{2\pi}{\omega_n} \left(\sqrt{(1-\zeta^2)}\right)$

As

$T_D = T_n / \sqrt{1-\zeta^2}$

$\Rightarrow T_n = T_D \left(\sqrt{1-\zeta^2}\right)$

$\Rightarrow 1.96\left(\sqrt{1-(0.0489)^2}\right)$

$T_n = 1.957 sec$

Natural period of undamped vibration.

(c) Stiffness of Structure,

$K = ?$

As

$$K = \frac{F \cdot \cos\theta}{2} \quad \begin{pmatrix} F = 60k \\ \theta = 60^\circ \end{pmatrix}$$

$$K = \frac{60 \cdot \cos(60^\circ)}{2}$$

$$\boxed{K = 15 \ K/in}$$

(d) weight of Tank "w" = ?

As

$$w_n = \sqrt{K/m} = \sqrt{\frac{K}{(w/g)}} = \sqrt{K \cdot g/w}$$

$$\Rightarrow w_n^2 = K \cdot g/w \Rightarrow \left(w = K \cdot g/w_n^2\right)$$

By Putting values of $w_n = 2\pi/T_n$

$$w = K \cdot g / (4\pi^2/T_n^2) = K \cdot g \left(\frac{T_n^2}{4\pi^2}\right)$$

$$w = \frac{18000 \ lb}{ft} \cdot \frac{32.2 \ ft}{sec^2} \left(\frac{(1.96)^2}{4(3.14)^2}\right)$$

$$w = 56284.75 \ lb = 56.284 \ k/lb$$

(e)

Damping co-efficent ; "c" = ?

it is know that ; $\zeta = \frac{c}{2m\omega_n}$

$\Rightarrow c = \zeta(2m\omega_n) = \zeta(2m(2\pi/T_n))$

By Putting values.

$$c = \frac{0.0489\left(2\left(\frac{56284}{32.2}\right)2(3.14)\right)}{1.957}$$

$c = 518.286$ lb sec/ft

("F") No of cycles to reduce displacement altitude from "6.872" in to "0.5in"

$J = ?$

$$J = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$= \frac{1}{2(3.14)(0.0489)} \ln\left[\frac{7.776}{0.9}\right]$$

$= 7.022$ OR 7

J = 7 cycles

Q.NO (02) **ANSWER:**

QNO 02 ①

Given data.

$E = 29000$ ksi

$I = 150$ in^4

$\delta_{st} = 7776$

Take $\zeta = 2.5\%$

Sol

E.O.M for damped free vibration

$Ku + C\dot{u} + m\ddot{u} = 0$ ——— ①

it is known from problem NO 1 That

$K = 90625$ lb/ft.

$m = 241.49$ lb.sec^2/ft.

$C = \zeta \times 2 m \omega_n$

$C = 0.025 \times 2 (241.49)(19.39)$.

$C = 234.12$ lb.sec/ft

②

By substituting values of K, c and m in eq ①

$Ku + c\dot{u} + m\ddot{u} = 0$

$90625 + 234.12\dot{u} + ~~240~~\ 241.49\ddot{u} = 0$

Solution to The E.O.m for damped free vibration is

$u(t) = e^{-\zeta\omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{u}(0) + u(0)(\omega_n)\right]\sin(\omega_D t)\right]$

tion

1

$\omega_D = 19.39$ rad/sec

$u(t) = e^{-0.025 \times 19.39t}\left[\frac{1}{24} \times \cos(19.39t) + \frac{1}{19.39} \times 0 + \frac{1}{24} \times 0.025 \times 19.39 \times \sin(19.39t)\right]$

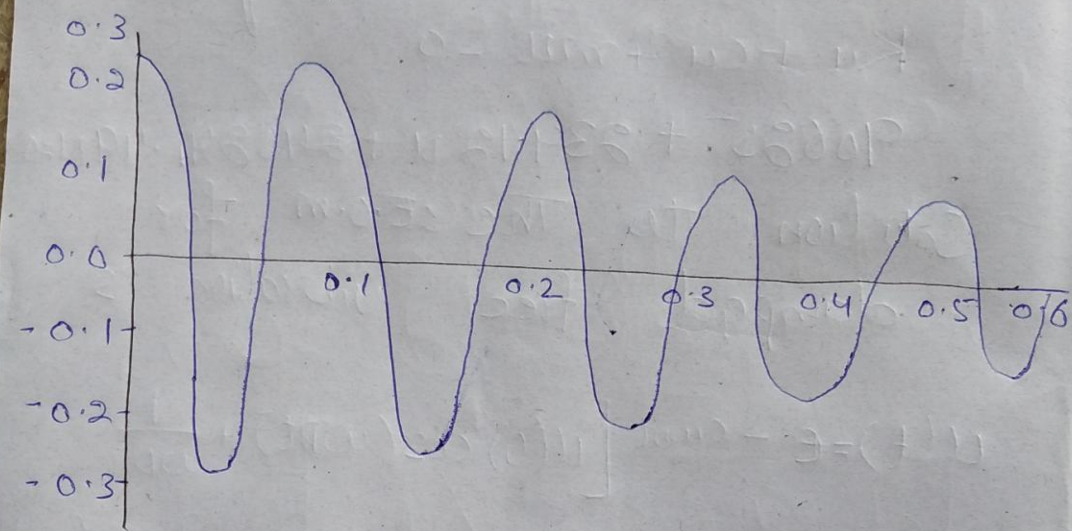
$u(t) = e^{-0.4875}\left[0.04167 \times \cos(19.39t) ~~+0.015~~ + 0.0515 \times 0.0203 \sin 19.39t\right]$

$u(t) = e^{-0.48475}\left[0.04167 \times \cos(19.39t) + 0.001045 \sin 19.39t\right]$

$f_s(t) = Ku(t) = 90625 \times u(t)$

$\Rightarrow f_s(t) = e^{-0.48675}\left[3776 \cos(19.39) + 94.34 \sin(19.39t)\right]$

Damped free vibration



t-sec

Damped free vibration