NAME # "ZARAK REHMAN"

I.D # "7666"

SUBJECT # "NUMERICAL ANALYSIS"

SUBMITED TO # "MAM SHUMAILA MAZHER"

"FINAL PAPER"

Que :-

Apply both ..... three deimal places?

By Euler's Method :-

Given data

$$y*(0) = 1 \quad , h = 0.1 \quad , x_0 = 0$$

By formula

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h(2x_n)$$

1st Interation :-

$n = 0$

$$y_1 = y_0 + h(2x_0)$$

$$y_1 = 1 + 0.1(2(0))$$

$y_1 = 1 + 0.1$

$$\boxed{y_1 = 1.1}$$

$x_{n+1} = x_n + h$

$x_1 = x_0 + h$

$x_1 = 0 + 0.1$

$$\boxed{x_1 = 0.1}$$

2nd Interation:-

$n = 1$

$y_2 = y_1 + h(2x_1)$

$y_2 = 1.1 + 0.1\,(2(0.1))$

$$\boxed{y_2 = 1.12}$$

$x_{n+1} = x_n + h$

$x_2 = x_1 + h$

$x_2 = 0.1 + 0.1$

$$\boxed{x_2 = 0.2}$$

3rd Interation :-

$$y_3 = y_2 + h(2x_2) \qquad n = 2$$

$$y_3 = 1.12 + 0.1 \quad (2(0.2)$$

$$\boxed{y_3 = 1.16}$$

$$x_{n+1} = x_n + h$$

$$x_3 = x_2 + 0.1$$

$$x_3 = 0.2 + 0.1$$

$$\boxed{x_3 = 0.3}$$

(b) By Modified Euler method :-

$$\frac{dy}{dx} = 2x$$

Given data :-

$$y_0 = 1 \;,\; x_0 = 0 \;,\; h = 0.1$$

formula

$$y_{n+1} = y_n + h[f(x_n)]$$

$$y_{n+1} = y_n + h(2x_n) \quad \text{——(1)}$$

$$y_{n+1} = y_n + h/2\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1})\right]$$

$$y_n + h/2\ [2x_n + 2x_n]$$

$$y_n + h/2\ [4x_n]$$

1st Iteration:-

$$x_{n+1} = x_n + h$$

$$x_1 = x_0 + h$$

$$x_1 = 0 + 0.1$$

$$x_1 = 0.1$$

$$y_1 = y_0 + h/2(4x_0)$$

$$y_1 = 1 + \frac{0.1}{2}(4(0))$$

$y_1 = 1$

2nd Iteration:-

$n = 1$

$x_2 = x_1 + h$

$x_2 = 0.1 + 0.1$

$x_2 = 0.2$

$y_2 = y_1 + h/2 \ (4x_1)$

$y_2 = 1 + \frac{0.1}{2} \ (4(0.1))$

$y_2 = 1.02$

3rd Iteration :-

$n = 2$

$x_3 = x_2 + h$

$x_3 = 0.2 + 0.1$

$x_3 = 0.3$

$y_3 = y_2 + h/2 (4x_2)$

$= 1.02 + \frac{0.1}{2} (4(0.2))$

$y_3 = 1.06$

Que :- 2.

Use the fourth ..... places:

Given data :-

$$y = 0 \quad , \quad x = 0 \quad , \quad h = 0.2$$

$$0 \leq x \leq 0.6$$

$$y_{n+1} = y_n + K$$

Ist Iteration :-

$$n = 0$$

$$y_1 = y_0 + K \quad , \quad K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

$$K_1 = hf(x_n, y_n)$$

$$K_1 = h(x_0^2 - x_0 y_0)$$

$$K_1 = 0.2(0^2 - 0 - 0)$$

$$\boxed{K_1 = 0}$$

$$K_2 = hf\left(x_n + h/2 \, , \, y_n + h/2\right)$$

$$= 0.2 f\left(x_0 + h/2 \, , \, y_0 + h/2\right)$$

$$= 0.2 f\left(0 + 0.2/2 \, , \, 0 + \frac{0.2}{2}\right)$$

$= 0.2\, f(0.1, 0.1)$

$\Rightarrow 0.2\,(0.1^2 + 0.1 - 0.1)$

$\boxed{K_2 = 0.0020}$

$K_3 = hf\left(x_n + h/2,\ y_n + \frac{K_2}{2}\right)$

$= 0.2\, f\left(0 + \frac{0.2}{2},\ 0 + \frac{0.002}{2}\right)$

$= 0.2f\,(0.1, 0.001)$

$\Rightarrow 0.2\,(0.1^2 + 0.1 - 0.001)$

$\boxed{K_3 = 0.0218}$

$K_4 = hf\,(x_n + h,\ y_n + K_3)$

$= 0.2f\,(0 + 0.2,\ 0 + 0.0218)$

$\Rightarrow 0.2f\,(0.2, 0.0218)$

$\Rightarrow 0.2\,(0.2^2 + 0.2 - 0.0218)$

$\boxed{K_4 = 0.0436}$

Pg (9)

$k = 1/6 \ (0 + 2(0.0029) + 2(0.0218) + 0.0436)$

$k = 0.0152$

$y_1 = 0 + 0.0152$

$\boxed{y_1 = 0.0152}$

Pg(10)

Que: 3

A rocket is .... 10 seconds?

Given data:

$a = 0$, $b = 10$, $n = 10$

$$h = \frac{b-a}{n}$$

$$= \frac{10-0}{10}$$

$h \Rightarrow 1$

Sol:-

x	0	1	2	3	4	5	6	7	8	9	10
f(x)	10.1	17.2	24.4	29.2	34.6	41.8	50.9	57.8	60.3	61.2	62.1

Using formula

$$\int f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2(f(x_1) + f(x_2) + f(x_3) \cdots - f(x_9) + f(x_{10})\right]$$

$$= \frac{1}{2}\left[10.1 + 2(17.2 + 24.4 + 29.2 + 34.6 + 41.2 + 50.9 + 57.8 +) + 62.1 + 60.3 + 61.2 -\right]$$

$$= 412.9$$

$$\boxed{\int f(x)\,dx = 412.9}$$

Que :4

$$\int_2^3 \ln(x^3+1)\,dx$$

Use 10 strips

Solution:-

$n = 10$

$h = \frac{3-2}{10}$

$= 0.1$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
f(x)	0.693	0.846	1.003	1.162	1.320	1.476	1.628	1.777	1.922	2.069

Now using formula

Pg(13)

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4\left(f(x_1) + f(x_3) \ldots\right] + 2\left[f(x_2) \ldots + f(x_n)\right]\right.$$

$$\frac{0.1}{3}\left[0.693 + 4(0.846 + 1.162 + 1.476 + 1.777) + 2(1.003 + 1.320 + 1.628 + 1.922) + 2.062\right.$$

So

$$= 1.184$$

$$\int_a^b f(x)\,dx = 1.184$$