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subject = calculus.

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Question No # 1

Soli:

$\int \phi \sqrt{1-\theta^2}\, d\theta$

Applying u-subsitude.

$u = 1-\theta^2$

$= \int -\frac{u \sqrt{u}}{2}\, du.$

Take constant out

$= -\frac{1}{2} \int u \sqrt{u}\, du.$

Applying Radical rule.

$= -\frac{1}{2} \cdot \int u^{1/4}\, du.$

$= -\frac{1}{2} \frac{u^{1/4+1}}{1/4+1}$

$= -\frac{1}{2} \left(\frac{1-\theta^2}{1/4+1}\right)^{1/4+1}$

$= -\frac{2}{5}\left(-\theta^2+1\right)^{5/4}$

$= -\frac{2}{5}\left(-\theta^2+1\right)^{5/4} + c$

Ans.

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Question No # 1b.

Sol:→

$$\int_0^1 x^3 (1+x^4)^3 \, dx$$

$$x^3 (1+x^4)^3 ; x^3 + 3x^7 + 3x^{11} + x^{15}$$

$$\Rightarrow \int_0^1 x^3 + dx + \int_0^1 3x^7 dx + \int_0^1 3x^{11} dx + \int_0^1 x^{15} dx$$

$$= \left[\frac{x^{3+1}}{3+1}\right]_0^1 + 3\left[\frac{x^{7+1}}{7+1}\right]_0^1 + 3\left[\frac{x^{11+1}}{11+1}\right]_0^1 + \left[\frac{x^{15+1}}{15+1}\right]_0^1$$

$$\Rightarrow \left[\frac{x^4}{4}\right]_0^1 + 3\left[\frac{x^3}{8}\right]_0^1 + 3\left[\frac{x^{12}}{12}\right]_0^1 + \left[\frac{x^{16}}{16}\right]_0^1$$

$$\Rightarrow \frac{1}{4} + 3\left(\frac{1}{8}\right) + 3\left(\frac{1}{12}\right) + \left(\frac{1}{16}\right)$$

$$= \frac{1}{4} + \frac{3}{8} + \frac{1}{4} + \frac{1}{16}$$

$$\Rightarrow \frac{1}{4} + \frac{3}{8} + \frac{1}{4} + \frac{1}{16}.$$

$$\Rightarrow \frac{4+6+4+1}{16}$$

$$\boxed{\Rightarrow \frac{15}{16}}$$ Ans

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Question No #2.

Sol:→

Given that.

$$y=\sqrt{x}$$

$$0 \le x \le 4 \Rightarrow a \le x \le b$$

As

$$V=\int_a^b \pi y^2 \, dx$$

$$V=\int_0^4 \pi(\sqrt{x})^2 \, dx.$$

$$V=\pi\int_0^4 x \, dx = \pi \cdot \frac{x^2}{2}\Big|_0^4$$

$$V=\frac{\pi}{2}\left[(4)^2-0\right]$$

$$\boxed{V=8\pi} \text{ Ans}$$

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Question No#3

Sol:→

$$B\cdot A = (-2i + 4j - \sqrt{5}k)\cdot(2i - 4j + \sqrt{5}k)$$

$$B\cdot A = -4i - 16j - 5k.$$

$$\boxed{B\cdot A = -25}$$

$$A\cdot A = (2i - 4j + 5k)\cdot(2i - 4j + \sqrt{5}k)$$

$$= 4 + 16 + 5$$

$$\boxed{= 25}$$

$$Proj_A B = \left(\frac{B\cdot A}{A\cdot A}\right)A.$$

$$= \left(\frac{-25}{25}\right)(2i - 4j + \sqrt{5}k)$$

$$= -1\,(2i - 4j + \sqrt{5}k)$$

$$\boxed{= -2i + 4j - \sqrt{5}k}$$ Ans.

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Question No # 4.

Fubine theorm

$$= \iint_R f(x,y)\,dA.$$

$$= \int_0^2 \int_{-1}^1 f(x,y)\,dA.$$

$$f(x,y) = 1 - 6x^2y$$

$$= \int_0^2 \int_{-1}^1 (1 - 6x^2y)\,dy\,dx$$

$$= \int_0^2 \left(\int_{-1}^1 dy - 6x^2 \int_{-1}^1 y\,dy \right) dx$$

$$= \int_0^2 \left(y \Big|_{-1}^1 - \frac{6x^2y^2}{2} \Big|_{-1}^1 \right) dx$$

$$= \int_0^2 \left(1 + 1 - 3x^2 - 3x^2 \right) dx.$$

$$= \int_0^2 \left(2 - 6x^2 \right) dx$$

$$= 2\int_0^2 dx - 6 \int_0^2 x^2\,dx$$

$$= 2x \Big|_0^2 - \overset{2}{\cancel{6}} \frac{x^2}{3} \Big|_0^2$$

$$= 4 - 2(8)$$

$$= 4 - 16$$

$$\boxed{= -12}$$ Ans

Question No # 5A.

$y = -2x^3 + 6x^2 - 3$

First find $f'(x)$ & $f''(x)$

$f'(x) = dy/dx = \frac{d}{dx}\left[-2x^3 + 6x^2 - 3\right]$

$= -2\frac{d}{dx}x^3 + 6\frac{d}{dx}x^2 - \frac{d}{dx}3$

$= -2(3x^2) + 6(2x) - 0$

$= -6x^2 + 12x - 0$

$f(x) = -6x^2 + 12x.$

Now

$f''(x) = \frac{d}{dx}(-6x^2 + 12x)$

$f''(x) = -12x + 12.$

for max. & min.

$f'(x) = 0$

$-6x^2 + 12x = 0$

$6\left[-x^2 + 2x\right] = 0$

$-x^2 + 2x = 0$

$x\left[-x + 2\right] = 0$

$-x + 2 = 0$

$\boxed{x = 2}$

Now

put $x = 2$ in $f''(x)$

$$f''(x) = (-12x + 12)$$

put $x = 2$

$$f''(2) = (-12(2) + 12)$$

$$= -24 + 12$$

$$f''(2) = -12$$

So $f''(x) < 0$

$y = -2x^3 + 6x^2 - \frac{80}{3}$ function is minum at $x = 2$.

Question No#15(B)

Sol:→

$$x^2+y^2+(z-1)^2=1$$

$$(\rho \sin\phi \cos\phi)^2+(\rho \sin\phi \sin\phi)^2$$
$$+(\rho \cos\phi-1)^2=1$$

$$\rho^2 \sin^2\phi \cos^2\phi+\rho^2 \sin^2\phi \sin^2\phi+\rho^2\cos^2\phi+1-2\rho\cos\phi=1$$

$$\rho^2 \sin^2\phi (\cos^2\phi+\sin^2\phi)+\rho^2\cos^2\phi+1-2\rho\cos\phi=1$$

$$\rho^2(\sin^2\phi)+\rho^2\cos^2\phi+2\rho\cos\phi=1-1$$

$$\rho^2(\sin^2\phi+\cos^2\phi)-2\rho\cos\phi=0$$

$$\rho^2=2\rho\cos\phi=2\cos\phi.$$

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