

ID : 14943

NAME : AQIB ALI

Section : B

Class : SE

Semester : 2<sup>nd</sup>

Paper : Operation Research



①

# Question No # 1

There are total of 5 machines and five employments are to be segregated and the related Cost network is as following. Locate the best possible task.

Solutions:-

		Machines				
J		A	B	C	D	E
	1	6	12	3	11	15
O	2	4	2	7	1	10
	3	8	11	10	7	11
B	4	16	19	122	23	21
	5	9	5	7	6	10
S						

## Step # 1

Row minimization

		Machines					
J		A	B	C	D	E	
O	1	6	12	3	11	15	3
B	2	4	2	7	1	10	1
S	3	8	11	10	7	11	7
	4	16	19	122	23	21	16
	5	9	5	7	6	10	5



(2)

Step # 2

Row Subtraction

		Machines				
		A	B	C	D	E
J	1	3	9	0	8	12
O	2	3	1	6	0	9
B	3	1	4	3	0	4
S	4	0	3	106	6	5
	5	4	0	2	1	5

Step # 3

Column Minimization

		Machines				
		A	B	C	D	E
J	1	3	9	0	8	12
O	2	3	1	6	0	9
B	3	1	4	3	0	4
S	4	0	3	106	6	5
	5	4	0	2	1	5



③

Step # 4

Column Subtraction

		Machines				
		A	B	C	D	E
J	1	3	9	0	8	8
O	2	3	1	6	0	5
B	3	1	4	3	0	0
S	4	0	3	106	6	1
	5	4	0	2	1	1

Step # 5

Draw horizontal and vertical lines through circles

		Machines				
		A	B	C	D	E
J	1 ← 3	3	9	0	8	8
O	2 ← 3	3	1	6	0	5
B	3 ←	1	4	3	0	0
S	4 ←	0	3	106	6	1
	5 ←	4	0	2	1	1

Hence  $\bar{J} = 5$   
Optimal Solution.



4

# Step # 6

## Timing

		Machines				
		A	B	C	D	E
J	1	6	12	3	11	15
O	2	4	2	7	1	10
B	3	8	11	10	7	11
S	4	16	19	122	23	21
	5	9	5	7	6	10

Jobs	Operators	Time
1	C	3
2	D	1
3	E	11
4	A	16
5	B	5

Total Processing time = 36 cr/hr





(5)

## Question No # 2

$$\begin{aligned} \text{Minimization } Z &= 2x_1 + 3x_2 \\ \text{Subj } &= \frac{1}{2}x_1 + \frac{1}{4}x_2 \leq 4 \\ & x_1 + 3x_2 \geq 20 \\ & x_1 + x_2 = 10 \\ & x_1 + x_2 \geq 0 \end{aligned}$$

Solution :-

$$\begin{aligned} \frac{1}{2}x_1 + \frac{1}{4}x_2 + S_1 &= 4 \\ x_1 + 3x_2 - S_2 + a_1 &= 20 \\ x_1 + x_2 + a_2 &= 10 \end{aligned}$$

Step # 2

Set the Objective function equal to zero.

$$\begin{aligned} Z &= -2x_1 - 3x_2 \\ Z &= -2x_1 - 3x_2 - mA_1 = mA_2 \\ Z + 2x_1 + 3x_2 + mA_1 + mA_2 &= 0 \\ 2x_1 + 3x_2 + mA_1 + mA_2 + Z &= 0 \end{aligned}$$



## Step # 3

Create a Simple table

	$x_1$	$x_2$	$S_1$	$S_2$	$a_1$	$a_2$	$b_1$	$b_2$
$\rightarrow R_1$	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	0	4
$\rightarrow R_2$	1	3	0	-1	1	0	0	20
$\rightarrow R_3$	1	1	0	0	0	1	0	10
$\rightarrow R_4$	2	3	0	0	$m$	$m$	1	0

$$R_4 + (-mR_3)$$

$$\begin{array}{r}
 2 \quad 3 \quad 0 \quad 0 \quad m \quad m \quad 1 \quad 0 \\
 -m \quad -m \quad 0 \quad 0 \quad 0 \quad -m \quad 0 \quad -10m \\
 \hline
 2-m \quad 3-m \quad 0 \quad 0 \quad m \quad 0 \quad 1 \quad -10m
 \end{array}$$

	$x_1$	$x_2$	$S_1$	$S_2$	$a_1$	$a_2$	$b_1$	$b_2$
	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	0	4
	1	3	0	-1	1	0	0	20
	1	1	0	0	0	1	0	10
	$2m$	$3-m$	0	0	$m$	0	1	$-10m$

$$R_4 + (-mR_2)$$

$$\begin{array}{r}
 2-m \quad 3-m \quad 0 \quad 0 \quad m \quad 0 \quad 1 \quad -10m \\
 -m \quad -3m \quad 0 \quad m \quad -m \quad 0 \quad 0 \quad -20m \\
 \hline
 2-2m \quad 3-4 \quad 0 \quad m \quad 0 \quad 0 \quad 1 \quad -30m
 \end{array}$$



	$x_1$	$x_2$	$S_1$	$S_2$	$a_1$	$a_2$	$b_i$	
$S_1$	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	0	4
$a_1$	1	3	0	-1	1	0	0	20
$a_2$	1	1	0	0	0	1	0	10
$Z$	$2-2m$	$3-4m$	0	$m$	0	0	1	$-30m$

Step # 4

Select the pivot Column and Row

	$x_1$	$x_2$	$S_1$	$S_2$	$a_1$	$a_2$	$b_i$	
$S_1$	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	0	4
$a_1$	1	3	0	-1	1	0	0	20
$a_2$	1	1	0	0	0	1	0	10
$Z$	$2-2m$	$3-4m$	0	$m$	0	0	1	$-30m$

Step # 5

Select the Pivot Row and Column

	$x_1$	$x_2$	$S_1$	$S_2$	$a_1$	$a_2$	$b_i$	
$S_1$	$\frac{1}{2}$	$\frac{1}{4}$	1	0	0	0	0	4 16
$a_1$	1	3	0	-1	1	0	0	20 66 4
$a_2$	1	1	0	0	0	1	0	10 10
$Z$	$2-2m$	$3-4m$	0	$m$	0	0	1	$-30m$



# Step #6

Select the Pivot which is the entry in the pivot Column and pivot Row

	$x_1$	$x_2$	$S_1$	$S_2$	$a_1$	$a_2$	$b$	
$S_1$	$\frac{1}{2}$	$\frac{1}{4}$	-1	0	0	0	0	4
$a_1$	1	3	0	-1	1	0	0	20
$a_2$	1	1	0	0	0	1	0	10
2	$2-2m$	$3-4m$	0	$m$	0	0	1	$-30m$

# Step #7

Perform Row Operations to make pivot equal to 1 and the remaining element in pivot Column equal to zero

Multiply  $R_2$  by  $\frac{1}{3}$

	$x_1$	$x_2$	$S_1$	$S_2$	$a_1$	$a_2$	$b$	
$S_1$	2	1	4	0	0	0	0	16
$a_1$	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	$20/3$
$a_2$	1	1	0	0	0	1	0	10
2	$2-2m$	$3-4m$	0	$m$	0	0	1	$-30m$



(9)

$$R_1 + (-1R_2)$$

$$\begin{array}{cccccccc}
 2 & 1 & 4 & 0 & 0 & 0 & 0 & 16 \\
 -\frac{1}{3} & -1 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 & 0 & -\frac{20}{3} \\
 \hline
 \frac{5}{3} & 0 & 4 & \frac{1}{3} & -\frac{1}{3} & 0 & 0 & \frac{28}{3}
 \end{array}$$

	$x_1$	$x_2$	$S_1$	$S_2$	$a_1$	$a_2$	$z$	
$S_1$	$\frac{5}{3}$	0	4	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	$\frac{28}{3}$
$a_1$	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	$\frac{20}{3}$
$a_2$	1	1	0	0	0	1	0	-10
2	$2-2m$	$3-4m$	0	$m$	0	0	1	$-30m$

$$R_3 + (-1R_2)$$

$$\begin{array}{cccccccc}
 1 & 1 & 0 & 0 & 0 & 1 & 0 & 10 \\
 -\frac{1}{3} & -1 & 0 & \frac{1}{3} & -\frac{1}{3} & 0 & 0 & -\frac{20}{3} \\
 \hline
 \frac{2}{3} & 0 & 0 & \frac{1}{3} & -\frac{1}{3} & 1 & 0 & \frac{10}{3}
 \end{array}$$

	$x_1$	$x_2$	$S_1$	$S_2$	$a_1$	$a_2$	$z$	
$S_1$	$\frac{5}{3}$	0	4	$\frac{1}{3}$	$-\frac{1}{3}$	0	0	$\frac{28}{3}$
$a_1$	$\frac{1}{3}$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	0	$\frac{20}{3}$
$a_2$	$\frac{2}{3}$	0	0	$\frac{1}{3}$	$-\frac{1}{3}$	1	0	$\frac{10}{3}$
2	$2-2m$	$3-4m$	0	$m$	0	0	1	$-30m$



$$R_4 + (- (3-4m)R_2)$$

$$\begin{array}{cccccccc} 2-2m & 3-4m & 0 & m & 0 & 0 & 1 & -30m \\ -1+4m & -3+4m & 0 & -4m & 1+4m & 0 & 0 & -10+8m \\ \hline 1+2m & 0 & 0 & 1-3m & 1+4m & 0 & 1 & -10+50m \end{array}$$

Step # 8

	$a_1$	$a_2$	$S_1$	$S_2$	$a_1$	$a_2$	$a_3$	
$a_2$	$5/3$	0	4	$1/3$	$-1/3$	0	0	$28/3$
$S_1$	$1/3$	1	0	$-1/3$	$1/3$	0	0	$20/3$
$a_2$	$2/3$	0	0	$1/3$	$-1/3$	1	0	$10/3$
2	$1+2m$	0	0	$1-3m$	$1+4m$	0	1	$-10+50m$

Step # 8

Repeat the Identifying Process by the most.

$$a_2 = 28/3$$

$$S_1 = 20/3$$

$$a_2 = 10/3$$

$$2 = -10 + 50m$$



# Question #3

Use Vogel's approximation Method to obtain the initial feasible Solution.

Origin	1	2	3	4	Supply
1	20	22	17	4	120
2	24	37	9	7	70
3	32	37	20	15	50
Demand	60	40	30	110	240

## Solution

	1	2	3	4						
1	x 20	22	x 17	4	120	13	(13)	-	-	-
2	10	x 24	30	9	70	2	2	2	(17)	24 (24)
3	50	x	x	x	50	5	5	5	17	(32)
	32	37	20	15	240					
	60	40	30	110						
	10	0	0	30						
	0									
4	15	8	3							
4	-	8	3							
8	-	11	8							
8	-	-	8							
8	(24)	-	-							
8	(24)	-	-							



(2)

$$\begin{aligned} \text{total Cost} &= 40 \times 22 + 80 \times 4 + \\ & 30 \times 9 + 30 \times 7 + 50 \times 32 \\ &= 880 + 320 + 240 \\ & + 270 + 210 + 1600 \\ &= 3520 \end{aligned}$$

$$\text{total Cost} = 3520 \text{ Ans.}$$