Name	Asim Ali (senior)
ID	7763
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Submitted to	Mam : Shomaila Mazhar
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* INU Peshawer *

Q.No 1 Find the Fourier Series of representation of $f(t) = 1+t, -\pi \le t \le \pi$

Solution:

Here we use the formula

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + \sum_{n=1}^{\infty} b_n \sin nt \rightarrow (i)$$

Here

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\,dt$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} (1+t)\,dt$$

$$= \frac{1}{2\pi}\left\{\int_{-\pi}^{\pi} dt + \int_{-\pi}^{\pi} t\,dt\right\} = \frac{1}{2\pi}\left\{ t\Big|_{-\pi}^{\pi} + \frac{t^2}{2}\Big|_{-\pi}^{\pi}\right\}$$

$$= \frac{1}{2\pi}\left\{(\pi-(-\pi)) + \left(\frac{\pi^2}{2} - \frac{(-\pi)^2}{2}\right)\right\}$$

$$= \frac{1}{2\pi}\left\{(\pi+\pi) + \left(\frac{\pi^2}{2} - \frac{\pi^2}{2}\right)\right\}$$

$$= \frac{1}{2\pi}\left\{2\pi + 0\right\}$$

$$= \frac{1}{\cancel{2\pi}}(\cancel{2\pi})$$

$$\boxed{a_0 = 1}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt\,dt$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\cos nt\,dt$$

$$= \frac{1}{\pi}\int_{-\pi}^{\pi}\cos nt\,dt + \frac{1}{\pi}\int_{-\pi}^{\pi} t\cos nt\,dt$$

$$= \frac{1}{\pi}\left\{\frac{\sin nt}{n}\Big|_{-\pi}^{\pi}\right\} + \frac{1}{\pi}\left[\left\{t\frac{\sin nt}{n}\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi}\frac{d}{dt}(t)\,\frac{\sin nt}{n}\,dt\right\}\right]$$

$$= \frac{1}{n\pi}\left\{\sin nt\Big|_{-\pi}^{\pi}\right\} + \frac{1}{\pi}\left\{\frac{t\sin nt}{n}\Big|_{-\pi}^{\pi} - \int_{-\pi}^{\pi} 1\cdot\frac{\sin nt}{n}\,dt\right\}$$

$$= \frac{1}{n\pi}\left\{\sin n\pi - \sin n(-\pi)\right\} + \frac{1}{\pi}\left\{\left(\frac{\pi\sin n\pi}{n} - \frac{\pi\sin n(-\pi)}{n}\right) - \frac{1}{n}\int_{-\pi}^{\pi}\sin nt\,dt\right\}$$

$$= \frac{1}{n\pi}\left\{\sin n\pi + \sin n\pi\right\} + \frac{1}{\pi}\left\{\frac{\pi\sin n\pi}{n} + \frac{\pi\sin n\pi}{n}\right\} - \frac{1}{n\pi}\int_{-\pi}^{\pi}\sin nt\,dt$$

$$= \frac{1}{n\pi}(0+0) + \frac{1}{\pi}\left(\frac{0}{n}+\frac{0}{n}\right) - \frac{1}{n\pi}\left(-\frac{\cos nt}{n}\right)\Big|_{-\pi}^{\pi}$$

$$= 0 + 0 + \frac{1}{n^2\pi}\cos nt\Big|_{-\pi}^{\pi}$$

$$= \frac{1}{n^2\pi}\left[\cos n\pi - \cos n(-\pi)\right]$$

$$= \frac{1}{n^2\pi}\left[-1 - (-1)\right]$$

$$= \frac{1}{n^2\pi}(-1+1)$$

$$= \frac{1}{n^2\pi}(0)$$

$$= 0$$

and

$$bn = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\ \sin nt\ dt$$

$$bn = \frac{1}{\pi}\int_{-\pi}^{\pi} (1+t)\ \sin nt\ dt$$

$$bn = \frac{1}{\pi}\left\{\int_{-\pi}^{\pi} \sin nt\ dt + \int_{-\pi}^{\pi} t\ \sin nt\ dt\right\}$$

$$bn = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin nt\ dt + \frac{1}{\pi}\int_{-\pi}^{\pi} t\ \sin nt\ dt$$

$$bn = \frac{-1}{\pi}\left\{\frac{\cos nt}{n}\Big|_{-\pi}^{\pi}\right\}.$$

$$+ \frac{1}{\pi}\left[\frac{-t\cos nt}{n}\Big|_{-\pi}^{\pi}\right] + \frac{1}{\pi}\int_{-\pi}^{\pi}\frac{\cos nt}{n}\ dt.$$

$$bn = -\frac{1}{n\pi}\left\{\cos n\pi - \cos n(-\pi)\right\} - \frac{1}{n\pi}\left(t\cos nt \Big|_{-\pi}^{\pi}\right)$$

$$+ \frac{1}{n\pi}\int_{-\pi}^{\pi} \cos nt\, dt$$

$$\cancel{bn = -\frac{1}{n\pi}\left[-1-(-1)\right] - \frac{1}{n\pi}\left\{\pi\cos n\pi - (-\pi)\cos\right.}$$

$$bn = -\frac{1}{n\pi}\left[-1-(-1)\right] - \frac{1}{n\pi}\left\{\pi\cos n\pi - (-\pi)\cos n(-\pi)\right\}$$

$$+ \frac{1}{n\pi}\left\{\frac{\sin nt}{n}\Big|_{-\pi}^{\pi}\right\}$$

$$bn = -\frac{1}{n\pi}(-1+1) - \frac{1}{n\pi}(\pi\cos n\pi + \pi\cos n\pi)$$

$$+ \frac{1}{n^2\pi}\left\{\sin nt \Big|_{-\pi}^{\pi}\right\}$$

$$bn = -\frac{1}{n\pi}(0) - \frac{1}{n\pi}(2\pi\cos n\pi)$$

$$+ \frac{1}{n^2\pi}\left\{\sin n(\pi) + \sin n\pi\right\}$$

$$bn = 0 - \frac{1}{n\pi}(2\pi\cos n\pi) + \frac{1}{n^2\pi}(0+0)$$

$$bn = -\frac{1}{n\cancel{\pi}}\, 2\cancel{\pi}\cos n\pi + 0$$

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$$b_n = -\frac{1}{n} 2\cos n\pi$$

$$b_n = \frac{2}{n}(-1)^{n+1}$$

Putting the values of a_0, a_n and b_n in equation 1

$$f(t) = 1 + 0 + \sum_{n=1}^{\infty} \frac{2}{n}(-1)^{n+1} \sin nt$$

$$f(t) = 1 + \sum_{n=1}^{\infty} \frac{2}{n}(-1)^{n+1} \sin nt$$

Q: 2

Calculate the characteristic equation the Eigen values of the System where A is given by

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Solution:

As we know that

$$(A - \lambda I)x = 0$$

A = Given Matrix

The charasteristic equation is given

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

using the formula

$$\lambda^3 - \left|\text{sum of Diagonal elem}\right| \lambda^2 + \left|\text{Sum of Diagonal minor}\right| \lambda - |A| = 0$$

Sum of Diagonal elements $= 1+1+2 = 4$

Sum of Diagonal minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= (-6) + (2) + (1)$

$= -6 + 2 + 1$

$= -3$

By putting values in eq

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$= 1(2-8) - 0 + 1(6-0)$

$= 6-6$

$= 0$

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By putting Values in eq

$$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$$

$$\lambda^3 - 4\lambda^2 - 3\lambda = 0$$

$$\lambda(\lambda^2 - 4\lambda - 3) = 0$$

$$\lambda = 0$$

$$\lambda^2 - 4\lambda - 3 = 0$$

Using Quadratic formula :-

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$
$b = -4$
$c = -3$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4 + \sqrt{28}}{2}, \quad \lambda = \frac{4 - \sqrt{28}}{2}$$

$$\lambda = \frac{4 + \sqrt{4 \times 7}}{2}, \quad \lambda = \frac{4 - \sqrt{4 \times 7}}{2}$$

$$\lambda = \frac{4 + 2\sqrt{7}}{2}, \quad \lambda = \frac{4 - 2\sqrt{7}}{2}$$

$\lambda = \frac{\cancel{2}(2+\sqrt{7})}{\cancel{2}}$ $\lambda = \frac{2(2-\sqrt{7})}{2}$

$\lambda = 2+\sqrt{7}$, $\lambda = 2-\sqrt{7}$

$\lambda = (0,\ 2+\sqrt{7},\ 2-\sqrt{7})$

is the required Answer.

Q: No 3 Solve the following system of linear equations

$$5x + 4z + 2m = 3$$
$$x - y + 2z + m = 1$$
$$4x + y + 2z = 1$$
$$x + y + z + m = 0$$

Solution:

Writing in matrix form

$$\begin{bmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & +1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ m \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

The Augmented Matrix "A_b" is

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{array}\right]$$

We Solve the given System by Gauss Elimination method

$$\underset{R_1 \leftrightarrow R_4}{\sim} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right]$$

$$R_2 - R_1 \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right]$$

$$R_3 - 4R_1 \quad \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 5 & 0 & 4 & 2 & 3 \end{array}\right]$$

$$R_4 - 5R_1 \quad \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & -2 & 1 & 0 & 1 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right]$$

$$R_2 \div (-2) \quad \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1/2 & 0 & -1/2 \\ 0 & -3 & -2 & -4 & 1 \\ 0 & -5 & -1 & -3 & 3 \end{array}\right]$$

$$R_3 + 3R_2 \quad \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 0 & -\frac{7}{2} & -4 & -\frac{1}{2} \\ 0 & -5 & -1 & -3 & 3 \end{array}\right]$$

$$R_4 + 5R_2 \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1/2 & 0 & -1/2 \\ 0 & 0 & -7/2 & -4 & -1/2 \\ 0 & 0 & -7/2 & -3 & 1/2 \end{array}\right]$$

$$R_3 \times -\frac{2}{7} \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1/2 & 0 & -1/2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & -7/2 & -3 & 1/2 \end{array}\right]$$

$$R_4 + \frac{7}{2} R_3 \left[\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1/2 & 0 & -1/2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

Using backward substitutions

$$0.x + 0.y + 0.z + 1.m = 1$$

$$\Rightarrow \boxed{m = 1}$$

$$0.x + 0.y + 1.z + \frac{8}{7} m = \frac{1}{7}$$

$$z + \frac{8}{7}(1) = \frac{1}{7}$$

$$z = \frac{1}{7} - \frac{8}{7}$$

$$z = \frac{1-8}{7}$$

$$z = \frac{-7}{7}$$

$$\boxed{z = -1}$$

$$0.x + 1.y - \frac{1}{2}z + 0.m = -\frac{1}{2}$$

$$y - \frac{1}{2}z = -\frac{1}{2}$$

$$y - \frac{1}{2}(-1) = -\frac{1}{2}$$

$$y + \frac{1}{2} = -\frac{1}{2}$$

$$y = -\frac{1}{2} - \frac{1}{2}$$

$$y = \frac{-1-1}{2}$$

$$y = \frac{-2}{2}$$

$$\boxed{y = -1}$$

$$1.x + 1.y + 1.z + 1.m = 0$$

$$x + (-1) + (-1) + 1 = 0$$

$$x - 1 - \cancel{1} + \cancel{1} = 0$$

$$\boxed{x = 1}$$

So

$x = 1$, $y = -1$, $z = -1$ and $m = 1$

is the Solution.

Verification:

Now putting the above values in any one of the equation.

eg

$$5x + 4z + 2m = 3$$

$$5(1) + 4(-1) + 2(1) = 3$$

$$5 - 4 + 2 = 3$$

$$1 + 2 = 3$$

$$3 = 3$$

So the equation is satisfied.

Q. No (4) Verify that (16)

$u(x,t) = \sin(x+2t)$

is a Solution of the one-dimensional equation.

Solution:

Given That

$u(x,t) = \sin(x+2t)$

Differentiat w.r.t x partialy

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\sin(x+2t)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)\frac{\partial}{\partial x}(x+2t)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)\ (1+0)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\cos(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)\cdot\frac{\partial}{\partial x}(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)(1+0)$$

$$\boxed{\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)}$$

and $u(x,t) = \sin(x+2t)$

Differentiate w.r.t "t"

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\sin(x+2t)$$

$$\frac{\partial u}{\partial t} = \cos(x+2t) \quad (0+2)$$

$$\frac{\partial u}{\partial t} = 2\cos(x+2t)$$

$$\frac{\partial^2 u}{\partial t^2} = (2) - \sin(x+2t) \quad (0+2)$$

$$\boxed{\frac{\partial^2 u}{dt^2} = -4\sin(x+2t)}$$

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As we know that one-dimensional wave equation is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$-4\sin(x+2t) = c^2[-\sin(x+2t)]$$

$$-4\sin(x+2t) = -c^2\sin(x+2t)$$

$$-4\sin(x+2t) + c^2\sin(x+2t) = 0$$

For the orbitrary constant $c = \pm 2$

$$-4\sin(x+2t) + (\pm 2)^2\sin(x+2t) = 0$$

$$-4\sin(x+2t) + 4\sin(x+2t) = 0$$

$$0 = 0$$

Then it will be verified for the orbitrary constant

$c = 2$.