Paper: Differential equation.

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Q.No.1 Objective type Questions:

i):- The order of matrix A is $m \times p$ and the order of B is $p \times n$.
Then the order of matrix AB is?

Solution:

$$A = [a_{ij}]_{m \times p} \quad \text{and} \quad B = [b_{ij}]_{p \times n}.$$

Then the order product AB is defined to be the matrix $[c_{ij}]_{m \times n}$.
where.

$$c_{ij} = [a_{i1}, a_{i2}, \dots a_{in}] \cdot \begin{bmatrix} b_{ij} \\ b_{2j} \\ b_{nj} \end{bmatrix} = a_1 b_{ij} +$$

$$a_2 b_{2j} + a_{in} b_{nj}.$$

$$\text{i.e} = AB = [c_{ij}]_{m \times n}.$$

ii):- The number of non-zero rows in an Echelon form?

Sol:- The number of non-zero row in an Echelon form of a matrix determine the rank of the matrix.

$$\begin{bmatrix} 2 & 3 & -4 & 1 \\ 0 & 1 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & 2 & 3 \\ 0 & 0 & -5 \\ 0 & 0 & 0 \end{bmatrix}.$$

Q1 iii):- $B = \begin{bmatrix} 1 & 4 \\ 2 & 0 \end{bmatrix}$

Sol:-

A matrix B is said to be singular if $|B| = 0$.

$$|B| = \begin{vmatrix} 1 & 4 \\ 2 & 0 \end{vmatrix}$$

$$0 = (1 \times a - 4 \times 2).$$

$$0 = a - 8$$

$$\boxed{a = 8}$$

iv): If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Sol:-

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$|A| = (ab - bc).$$

$$= (2i)(-i) - (i)(i).$$

$$= -2i^2 - i^2$$

$$= -2(-1) - (-1) \qquad \therefore i^2 = -1$$

$$= 2 + 1$$

$$\boxed{|A| = 3}$$

v):- The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

Sol:-

$A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is a scalar matrix.

vi):- Solution of $\frac{dy}{dx} + 2xy = y$?

Sol:-

$$\frac{dy}{dx} + 2xy = y.$$

$$\frac{dy}{dx} = y - 2xy.$$

$$\frac{dy}{dx} = y(1-2x).$$

$$\frac{1}{y}\,dy = (1-2x)\,dx.$$

$$\int \frac{1}{y}\,dy = \int (1-2x)\,dx$$

$$\ln y = \int 1\,dx - \int 2x\,dx$$

$$\ln y = x - 2\int x\,dx$$

$$\ln y = x - 2\left(\frac{x^{1+1}}{1+1}\right) + C$$

$$\ln y = x - \cancel{x}\left(\frac{x^2}{\cancel{x}}\right) + c$$

$$\ln y = x - x^2 + c$$

Taking expanential "e"

$$e^{\ln y} = e^{x - x^2 + c}$$

$$y = e^x \cdot e^{-x^2} \cdot e^c$$

Let $e^c = c$.

$$y = e^{x - x^2} c.$$

$$\boxed{y = ce^{x - x^2}}$$

vii):- The order and degree of differential equation.

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \text{ is ?}$$

Sol:- The order of differential equation is 1.

The degree of differential equation is 3.

viii):- $\dfrac{d^2y}{dx^2} - 4xy = \sin \dfrac{d^2y}{dx}$ is?

Sol:-

The order of differential equation is 2.

The degree of differential equation is 1.

ix):- The differential equation

$$2\frac{dy}{dx} + x^2y = 2x + 3, \ y(0) = 5 \text{ is ?}$$

Sol:-

$2\dfrac{dy}{dx} + x^2y = 2x+3, \ y(0) = 5$ is impossible.

x):- $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$

Sol:-

$$\sim \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} \qquad \begin{matrix} R_2 - R_1 \\ R_3 - R_1 \end{matrix}$$

$$(b-a)(c-a)\begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b^2-a^2 \\ 0 & 1 & c^2-a^2 \end{vmatrix}$$

$= (b-a)(c-a)(c+\cancel{a}-b-\cancel{a})$.

$= (b-a)(c-a)(c-b)$.

$= (a-b)(b-c)(c-a)$.

Q2. Express the determination.

(i) $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

Sol:-

$$a \cdot b \cdot c \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$C_2 - C_1$

$C_3 - C_1$

$$a \cdot b \cdot c \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix}$$

$$a \cdot b \cdot c \cdot (b-a)(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & c+a \end{vmatrix}$$

$= abc \cdot (b-a)(c-a)(c+\cancel{a}-b-\cancel{a})$

$= abc \quad (b-a)(c-a)(c-b).$

$= abc\,(a-b)(b-c)(c-a).$

Q2 Find the Eigen value.

ii):-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol:- For ~~find~~ eigen values consider.

$$A - \lambda I = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}$$

For eigen values consider:

$\Rightarrow |A - \lambda I| = 0.$

$$\Rightarrow \left| \begin{bmatrix} -2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix} \right| = 0$$

$-1-3+\lambda.$
$3-\lambda+1.$
$4-\lambda.$

Using $R_3 - R_2$.

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ 0 & -4+\lambda & 4-\lambda & 0 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix}$$

Expand by column first.

$$(2-\lambda)\begin{vmatrix} 3-\lambda & -1 & -1 \\ -4+\lambda & 4-\lambda & 0 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 & 0 \\ -4+\lambda & 4-\lambda & 0 \\ -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by colum_3 Expand by C_3.

$$(2-\lambda)\left\{-1\begin{vmatrix} -4+\lambda & 4-\lambda \\ -1 & -1 \end{vmatrix} + (2-\lambda)\begin{vmatrix} 3-\lambda & -1 \\ -4+\lambda & 4-\lambda \end{vmatrix}\right\}$$

$$+1\left\{(2-\lambda)\begin{vmatrix} 1 & -1 \\ -4+\lambda & 4-\lambda \end{vmatrix}\right\} = 0$$

$$\Rightarrow -(2-\lambda)(4-\lambda+4-\lambda) + (2-\lambda)^2\Big((3-\lambda)(4-\lambda)-4+\lambda\Big)$$

$$+1\Big((2-\lambda)(\lambda-4-4+\lambda)\Big) = 0.$$

$$\Rightarrow (\lambda-2)(8-2\lambda) + (\lambda-2)^2(12-7\lambda+\lambda^2-4+\lambda)$$

$$+(2-\lambda)(2\lambda-8)\Big) = 0.$$

$$\Rightarrow (\lambda-2)(8-2\lambda) + (\lambda-2)^2(\lambda^2-6\lambda+8) + (\lambda-2)(8-2\lambda) = 0$$

$$\Rightarrow (\lambda-2)\{8-2\lambda+(\lambda-2)(\lambda^2-6\lambda+8)+8-2\lambda\} = 0$$

$$\Rightarrow (\lambda-2)(\cancel{16}-4\lambda+\lambda^3-6\lambda^2+8\lambda-2\lambda^2+12\lambda-\cancel{16}) = 0$$

$$\Rightarrow (\lambda-2)=0 \qquad \lambda^3-8\lambda^2+16\lambda = 0$$

$$\Rightarrow \lambda-2=0. \qquad \lambda(\lambda^2-8\lambda+16)=0$$

$$\Rightarrow \boxed{\lambda=2} \qquad \Rightarrow \boxed{\lambda=0}. \quad \lambda^2-8\lambda+16.$$

$$\Rightarrow \lambda^2-4\lambda-4\lambda+16=0$$

$$\Rightarrow \lambda(\lambda-4)-4(\lambda-4)=0$$

$$\Rightarrow (\lambda-4)(\lambda-4)=0$$

$$\Rightarrow \boxed{\lambda = 4, 4}.$$

Q3:- The rate of change in the form of differential equation is given by

$(x^2 + 3y^2)dx - 2xydy = 0.$

find the general solution at $x = 2$ and $y = 6$.

Sol:-

$$2xy\,dy = (x^2 + 3y^2)\,dx.$$

$$2xy\frac{dy}{dx} = x^2 + 3y^2.$$

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy} \quad \text{---} (i).$$

Put the value of $y = Vx$.

$$\frac{dy}{dx} = V + x\frac{dv}{dx}.$$

$$V + x\frac{dv}{dx} = \frac{x^2 + 3V^2x^2}{2xVx}.$$

$$V + x\frac{dv}{dx} = \frac{x^2(1 + 3V^2)}{2x^2V}$$

$$V + x\frac{dv}{dx} = \frac{1 + 3V^2}{2V}$$

eq (A). $\Rightarrow$ $$\frac{x^2 + y^2}{x^3} = 5.$$

$$x^2 + y^2 = 5x^3$$

$$y^2 = 5x^3 - x^2$$

$$\boxed{y = \sqrt{5x^3 - x^2}}$$