

Q1:
ANS:

Solution:- Sample space for two fair dice.

$$SS = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$$

Let

A = the sum is 7

B = Sum is odd

C = Sum is greater than 8

D = two dice had same outcome.

$$A = \{ (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) \}$$

$$B = \{ (1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (3,6), (4,1), (4,3), \\ (4,5), (5,2), (5,4), (5,6), (6,1), (6,3), (6,5) \}$$

$$C = \{ (3,6), (4,5), (4,6), (5,4), (5,5), (5,6) \}$$

$$D = \{ (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) \}$$

$$(A \cap B) = \{ (1,6), (2,5), (3,4), (4,3), (5,2), (6,1) \}$$

$$A \cap C = \emptyset$$

$$A \cap D = \emptyset$$

$$i): P(A/B) = ?$$

$$P(A) = \frac{6}{36}, P(B) = \frac{18}{36}, P(C) = \frac{10}{36}, P(D) = \frac{6}{36}$$

$$P(A \cap B) = \frac{6}{36}, P(A \cap C) = 0, P(A \cap D) = 0$$

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$$\text{Now } P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{6/36}{18/36} = \frac{6}{18} = \frac{1}{3}$$

$$P(A/B) = 1/3$$

$$\text{ii): } P(A/C) = \frac{P(A \cap C)}{P(C)} = \frac{0}{10/36} = 0$$

$$P(A/C) = 0$$

$$\text{iii): } P(A/D) = \frac{P(A \cap D)}{P(D)} = \frac{0}{6/36} = 0$$

$$P(A/D) = 0$$

x ————— x

Q2:
Ans: =>

- i) A = more than 7?
- ii) B = Less than 7?
- iii) C = Exactly 7?

$$A = \{(2,6) (3,5) (3,6) (4,4) (4,5) (4,6) (5,3) (5,4) (5,5) (5,6) (6,2) (6,3) (6,4) (6,5) (6,6)\}$$

$$B = \{(1,1) (1,2) (1,3) (1,4) (1,5) (2,1) (2,2) (2,3) (2,4) (3,1) (3,2) (3,3) (4,1) (4,2) (5,1)\}$$

$$C = \{(1,6) (2,5) (3,4) (4,3) (5,2) (6,1)\}$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{15}{36}$$

$$P(A) = \frac{15}{36}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{15}{36}$$

Hence proved

$$P(A) = P(B) = \frac{15}{36}$$

Now

P(c) exactly 7

$$P(c) = \frac{n(c)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

$$P(c) = \frac{1}{6}$$

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Q # 2

Assumption:- By throwing a single of two dice there are 36 possibilities. It can give us 7, more than 7, & also less than 7 so we pick from the combination series for only & exactly 7, more than 7 & also less than 7 & then find these possibilities through which we find that probability for more than 7 & less than 7 are equal which are $\frac{15}{36}$ & probability for exactly 7 are $\frac{6}{36}$ or $\frac{1}{6}$.



Q 3: ⇒
ANS: ⇒

By Binomial distribution

$$P(X=x) = \binom{n}{x} p^x q^{n-x}$$

$$P(A) = \frac{2}{3}$$

$$p = \frac{2}{3}, q = \frac{1}{3}, n = 8$$

i) exactly four games $x=4$

$$P(X=4) = \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^{8-4}$$

$$= 70 (0.1775) (0.01234)$$

$$P(X=4) = 0.1706$$

ii) At least 4 games.

$$P(X \geq 4) = \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4 + \binom{8}{5} \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^3 + \binom{8}{6} \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^2 + \binom{8}{7} \left(\frac{2}{3}\right)^7 \left(\frac{1}{3}\right) + \binom{8}{8} \left(\frac{2}{3}\right)^8 \left(\frac{1}{3}\right)^0$$

$$= 0.1706 + 0.2731 + 0.2731 + 0.156 + 0.039.$$

$$P(X \geq 4) = 0.9118$$

$$\text{iii): } P(3 \leq X \leq 6) = \binom{8}{3} \left(\frac{2}{3}\right)^3 \left(\frac{1}{3}\right)^5 + \binom{8}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^4 + \binom{8}{5} \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right)^3$$

$$+ \binom{8}{6} \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^2$$

$$= 0.06828 + 0.1706 + 0.2731 + 0.2731$$

$$P(3 \leq X \leq 6) = 0.78508 \rightarrow \text{Completed.}$$

Q5:

Ans:-

Binomial Distribution:->

The no. of successes "x" in "n" trials of a binomial experiment is called binomial random variable. The probability distribution of the binomial random variable is called binomial dist.

Definition:- if X is a binomial random variable then its probab. function is.

$$f(x) = \binom{n}{x} p^x q^{n-x} \quad x=0,1,2,\dots,n$$

other wise

Where n & p are two parameters of the binomial distⁿ.

$$\begin{cases} p+q=1 \end{cases}$$

$$\Rightarrow q=1-p.$$

M.Ean:- by def:

$$\Rightarrow E(X) = \sum_{x=0}^n x f(x) = \sum_{x=0}^n x \binom{n}{x} p^x q^{n-x}$$

$$\Rightarrow E(X) = 0 \cdot \binom{n}{0} p^0 q^{n-0} + 1 \cdot \binom{n}{1} p^1 q^{n-1} + 2 \cdot \binom{n}{2} p^2 q^{n-2} + \dots + n \binom{n}{n} p^n q^{n-n}$$

$$\Rightarrow E(X) = 0 \cdot 1 \cdot 1 \cdot q^n + 1 \cdot n \cdot p q^{n-1} + 2 \cdot \frac{n(n-1)}{2} p^2 q^{n-2} + \dots + n \cdot 1 \cdot p^n q^0$$

$$\Rightarrow E(X) = 0 + n p q^{n-1} + 1 \cdot n(n-1) p^2 q^{n-2} + \dots + n p^{n-1}$$

$$\Rightarrow E(X) = n p q^{n-1} + 1 \cdot n(n-1) p^2 q^{n-2} + \dots + n p^{n-1}$$

$$\Rightarrow E(X) = n p q^{n-1} + 1 \cdot n(n-1) p^2 q^{n-2} + \dots + n p^{n-1} \cdot p^1$$

$$\Rightarrow E(X) = n p [q^{n-1} + 1 \cdot (n-1) p q^{n-2} + \dots + p^{n-1}]$$

$$\Rightarrow E(X) = n p [q+p]^{n-1} \quad \text{where } q+p=1$$

$$\Rightarrow E(X) = n p (1)^{n-1}$$

$$\Rightarrow E(X) = n p.$$

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ii) Variance :- by def

$$V(x) = E(x^2) - (E(x))^2 \quad \text{--- (1)}$$

$$\Rightarrow E(x^2) = E[x + x(x-1)]$$

$$\Rightarrow E(x^2) = E(x) + E(x(x-1)) \quad \text{--- (a)}$$

$$\Rightarrow E(x^2) = np + \sum_{x=0}^n x(x-1) C_n^x p^x q^{n-x}$$

$$\Rightarrow E(x^2) = np + \left[0 \cdot (0-1) C_n^0 p^0 q^{n-0} + 1 \cdot (1-1) C_n^1 p^1 q^{n-1} + \right.$$

$$\left. 2 \cdot (2-1) C_n^2 p^2 q^{n-2} + \dots + n(n-1) C_n^n p^n \right]$$

$$\Rightarrow E(x^2) = np + \left[0 + 0 + 2 C_n^2 p^2 q^{n-2} + 3(3-1) C_n^3 p^3 q^{n-3} + \dots + n(n-1) p^n \right]$$

$$\Rightarrow E(x^2) = np + \left[\frac{2 \cdot n(n-1)}{2} p^2 q^{n-2} + \frac{3 \cdot n(n-1)(n-2)}{3} p^3 q^{n-3} + \dots + n(n-1) p^n \right]$$

$$\Rightarrow E(x^2) = np + \left[n(n-1) p^2 q^{n-2} + n(n-1)(n-2) p^3 q^{n-3} + n(n-1)(n-2) p^3 q^{n-3} + \dots + n(n-1) p^{n-2+2} \right]$$

NOTE :- In discrete case x^2 cannot find directly therefore

$$x^2 = x(x-1) + x$$

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Q6:

ANS:-

If $p=q=1/2$ Then distⁿ is Symmetrical

If $p > q$ then distⁿ is -vely Skewed

If $p < q$ then distⁿ is +vely Skewed.

Binomial frequency distⁿ:-

If a binomial experiment is repeated "N" times then it is called binomial frequency distⁿ. It is denoted by

$$\Rightarrow \text{bi, freq; dist}^n = Nf(x)$$

$$\Rightarrow \text{bi, freq; dist}^n = N C_x^n p^x q^{n-x} \quad x = 0, 1, 2, 3 \dots n$$

Practical topic:- fitting of binomial distⁿ.

For the fitting of binomial distⁿ.

1) To find the parameters of binomial distⁿ.
i.e "p" & "n"

2) Mean $= \bar{x} = np$

3) Find the expected frequency by the formula

$$Ef = Nf(x)$$

$$\Rightarrow Ef = N C_x^n p^x q^{n-x} \quad x = 0, 1, 2, 3 \dots n$$

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Q7:
Ans:

Measure	A	B	C	D
Mean	45	60	50	25
S.D	3	11	5	15
Sample Size Size	1500	3,200	500	2,700
C.V	$= \frac{3}{45} \times 100$ $= 6.6$ $\approx 7\%$	$\frac{11}{60} \times 100$ $= 18.3$ $\approx 18\%$	$\frac{5}{50} \times 100$ $= 10$ $\approx 10\%$	$\frac{15}{25} \times 100$ $= 60$ $\approx 60\%$

Group "A" Coefficient variation is just 7% which is low for group "B, C & D" which ~~is~~ ~~is~~ mean group "A" is best from B, C & D.

Q4
Ans:

Sol: As we know that two events A & B in the same sample space are defined to be independent if the probability that one event occurs not effected by whether the other event.

$$\text{As } P(A/B) = P(A) \text{ ?}$$

$$\{ P(B/A) = P(B) \}$$

Then it will be independent only if $P(A \cap B) = P(A) \cdot P(B)$

As it given then

$$C \in \{1, 2, \dots, M\}$$

\{ B is independent of all C.

$$A \cap B = \{C_1, C_2, C_3\}$$

$$P(A) = \frac{3}{5}, P(B) = \frac{3}{5}, P(A \cap B) = \frac{3}{5} = 1$$

Let A = red die show an even
B = green " " 5 & 6.

A \cap B = red die show an even number & give show a 6.

$$A = 18 \text{ out comes}$$

$$B = 12 \text{ "}$$

$$A \cap B = 6$$

$$P(A) = \frac{18}{36} = \frac{1}{2}$$

$$P(B) = \frac{12}{36} = \frac{1}{3}$$

$$P(A \cap B) = \frac{6}{36} = \frac{1}{6}$$

$$P(A \cap B) = P(A) \cdot P(B) = \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6} = \frac{1}{6}$$

$$\frac{1}{6} = \frac{1}{2} \cdot \frac{1}{3}$$

Hence proved A & B is

independent.

The end