EXAM:

MID TERM

PAPER:

DIFFERENTIAL EQUATION

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SUBMITTED TO:

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QNO 1:

SOLVE THE FOLLOWING OBJECTIVE TYPE QUESTIONS.

1:- SOLUTION:

The order of matrix AB is "~~4×4~~" . "m×n"

2:- SOLUTION:

The number of non zero rows in a Echelon form is ~~one~~ Known as "Rank of Matrix"

3:- SOLUTION:

$$\begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix}$$

$$a1 - 8 = 0$$

$$a = 8$$

4:- SOLUTION:

$$|A| = 2i(i) - i(i)$$

$$= 2i^2 - i^2$$

$$i^2 = (-1)$$

$$= -2(-1)^2 - (-1)^2$$

$$= 2 + 1$$

$$= 3$$

5: SOLUTION:

SCALAR MATRIX.

6:- SOLUTION:

$$\frac{dy}{dx} + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy$$

$$\frac{dy}{dx} = y(1-2x)$$

$$\frac{dy}{y} = (1-2x)\,dx$$

Integrating both.

$$\int \frac{1}{y}\,dy = \int (1-2x)\,dx$$

$$\ln y = \int 1\,dx - 2\int x\,dx$$

$$\ln y = x - \frac{2x^2}{2} + c$$

$$\ln y = (x - x^2) + c$$

$$\ln y = (x - x^2) + c$$

Taking anti log.

$$y = e^{(x-x^2)+c}$$

$$y = e^{x-x^2} \cdot e^{c}$$

$$y = e^{x-x^2} \cdot c$$

$$y = e^{x \cdot x^2} \cdot c$$

7 : SOLUTION:

Order = 1 Degree = 3

8 : SOLUTION:

Order = 2 Degree = 1

9 : SOLUTION:

$$2y + \frac{x^3}{3} = x^2 + 3x + 10$$

10 : SOLUTION:

$$S = (b-a)(c-a)(c-b)$$

Q No 2:

PART A:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the Product of factors which are linear in a, b, c.

SOLUTION:

let $S = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$

Expanding the Matrix S by R_1

$$S = \left[a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & c^3 \end{vmatrix}\right]$$

$$S = a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2c^3 - a^3b^2)$$

$$S = ab^2c^3 - ab^3c^2 - ba^2c^3 + ba^3c^2 + ca^2c^3 - ca^3b^2 \quad (1)$$

Now Taking a, b, c common from eq (1)

$= abc\,(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$

$= abc\,[bc(c-b) - ac(c+a) + ab(b-a)]$

RESULT:

$= abc\,(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$

OR

$\Rightarrow abc\,[bc(c-b) - ac(c+a) + ab(b-a)]$

PART (ii)

Find the Eigen value.

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

SOLUTION:

we have

$$S = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

we have $|B - \lambda I| = 0 \rightarrow (1)$

I = Identity Matrix

$$\Rightarrow \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 & 0 \\ 0 & \lambda & 0 & 0 \\ 0 & 0 & \lambda & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix}$$

$$S = \begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix}$$

Expanding the Matrix by R_1,

$$2-\lambda \begin{bmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{bmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} -$$

$$(-1) \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} - 0 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & 3-\lambda \\ 0 & -1 & -1 \end{vmatrix} = 0$$

Again Expanding the first Matrix by R_1

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$ Expanding by R_1

$$= 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda) \left[\left((3-\lambda)(2-\lambda) - (-1)(-1) \right] + 1 \left((-1)(2-\lambda) - (-1)(-1) \right] - 1 \left((-1)(-1) - (-1)(3-\lambda) \right) \right) \right]$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-2+\lambda-1) - (1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5) + (-3+\lambda) - (4-\lambda)$$

$$= 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$$

$$= 8\lambda^2 + 8 - \lambda^3 - 18\lambda$$

Re odering it.

$$\Rightarrow -\lambda^3 + 8\lambda^2 - 18\lambda + 8 \longrightarrow (1)$$

Now taking the 2nd Matrix.

$$= 1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expanding it by coloumn 1 (C_1)

$$\Rightarrow -1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1) + 1(-2+\lambda-1)$$

$$= 5\lambda-5-3-\lambda^2-\lambda$$

$$= -\lambda^2+6\lambda-8 \rightarrow (2)$$

Now taking the 3rd Matrix.

$$-1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expanding it by coloumn 1 (C1)

$$= -\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1)\begin{vmatrix} 3-\lambda & 1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$= -(-1(-2+\lambda-1) + 1(6-3\lambda-2\lambda+\lambda^2-1)]$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= -\lambda^2+6\lambda-8 \rightarrow (3)$$

Putting the values of eq (1) (2) and (3)

$= (2-\lambda)(-\lambda^3 + 8\lambda^2 - 18\lambda + 8) - \lambda^2 + 6\lambda - 8 - \lambda^2 + 6\lambda - 8$

$= -2\lambda^3 + 16\lambda^2 - 36\lambda + 16 + \lambda^4 - 8\lambda^3 + 18\lambda^2 - 8\lambda - \lambda^2 + 6\lambda - 8 - \lambda^2 + 16\lambda - 8$

$= \lambda^4 - 2\lambda^3 - 8\lambda^3 + 16\lambda^2 + 16\lambda^2 - \lambda^2 - \lambda^2 - 36\lambda - 8\lambda + 6\lambda + 6\lambda + 16 - 6$

$= \lambda^4 - 10\lambda^3 + 32\lambda^2 - 32\lambda = 0 \quad \rightarrow (5)$

Solving eq (5) By Synthetic division we get:

$\lambda(\lambda-2)(\lambda^2 - 8\lambda + 16) = 0 \quad (\lambda = 0)$

$\lambda - 2 = 0 \rightarrow \lambda = 2$

$\lambda^2 - 8\lambda + 16 = 0$

Solving it by factorization:

$\lambda^2 - 8\lambda + 16 = 0$

$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$

$\lambda(\lambda - 4) - 4(\lambda - 4)$

$(\lambda - 4) \quad (\lambda - 4)$

$\lambda - 4 = 0 \quad \lambda - 4 = 0$

$\kappa - 4 = 0$ $\qquad$ $\kappa - 4 = 0$

$\kappa = 4$ $\qquad$ $\kappa = 4$

RESULT:

$\kappa_1 = 0$

$\kappa_2 = 2$

$\kappa_3 = 4$

$\kappa_4 = 4$

Q no 3:

The rate of change in the form of differential equation is given by

$(x^2 + 3y^2)\,dx - 2xy\,dy = 0$

Find the general solution at $x = 2$ and $y = 6$

SOLUTION:

$$(x^2+3y^2)\,dx - 2xy\,dy = 0$$

$$(x^2+3y^2)\,dx = 2xy\,dy$$

$$= \frac{x^2+3y^2}{2xy} = \frac{dy}{dx}$$

$$= \frac{x^2}{2xy} + \frac{3y^2}{2xy} = \frac{dy}{dx}$$

$$= \frac{x}{2y} + \frac{3y}{2x} = \frac{dy}{dx}$$

So we have

$$\frac{dy}{dx} = \frac{x}{2y} + \frac{3y}{2x}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \longrightarrow (1)$$

Now compariy eq (1) with $\frac{dy}{dx} = g(y/x)$

So from this it is cleab that eq (1) is homogeneous eq of degree (1)

Put $\frac{y}{x} = V$ $\quad y = Vx$

$$\frac{dy}{dx} = V\frac{dx}{dx} + x\frac{dV}{dx}$$

$$\frac{dy}{dx} = V + x\frac{dV}{dx}$$

Putting in eq.

$$V + x\frac{dV}{dx} = \frac{1}{2}\left[\frac{1}{V} + 3V\right]$$

xliy 2 to b.s

$$2V + 2x\frac{dV}{dx} = \frac{1}{2}\left[\frac{1}{V} + 3V\right] \times 2$$

$$2V + 2x\frac{dV}{dx} = \frac{1}{V} + 3V$$

$$2x\frac{dV}{dx} = \frac{1}{V} + 3V - 2V$$

$$2x\frac{dV}{dx} = \frac{1}{V} + V$$

$$2x\frac{dV}{dx} = \frac{V^2 + 1}{V}$$

$$\frac{2v}{v^2+1}\,dv = \frac{1}{2x}\,dx \rightarrow (5)$$

Now integrating eq (5)

$$\int \frac{2v}{v^2+1}\,dv = \frac{1}{2}\int \frac{1}{x}\,dx$$

$$\ln(v^2+1) = \ln x + c$$

$$\frac{v^2+1}{x} = c$$

$$v^2+1 = xc$$

Now Putting the value of "v"

$$\left(\frac{y}{x}\right)^2 + 1 = xc$$

$$\frac{y^2}{x^2} + 1 = xc$$

$$\frac{y^2+x^2}{x^2} = xc$$

$$y^2+x^2 = x^2 \cdot xc$$

$$y^2+x^2 = x^3 c \rightarrow (B)$$

Now Putting the value of "x" and "y"

$x = 2 \qquad y = 6$

$y^2 + x^2 = x^3 c$

$(6)^2 + (2)^2 = (2)^3 c$

$36 + 4 = 8c$

$8c = 40$

$\div$ing b/s by 8

$$\frac{8c}{8} = \frac{40}{8}$$

$c = 5$

Putting the value of "c" in eq" B"

$y^2 + x^2 = 5x^3$

$y^2 = 5x^3 - x^2$

Taking square root b.s

$\sqrt{y^2} = \sqrt{5x^3 - x^2}$

$y = \pm\sqrt{5x^3 - x^2}$

RESULT:

$y = \pm\sqrt{x^2(5x-1)}$

$y = \pm x\sqrt{5x-1}$