Iqra National University, Peshawar
Department of Electrical Engineering

Mid – Term Examinationsummer2020
Date:20/8/2020

Course Code:	MTH 101	**Course Title:**	Linear Algebra
Prerequisite:	NA	**Instructor:**	HIMAYTULLAH
Module:	1 **Program:** BEE	**Total Marks:** 30	**Time Allowed:**

Note: Attempt all questions.PLO: program learning outcome C:Cognitive

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Instructor :- sir Himayatullah
Course :- Linear algebra

Q 1.	(a)	. Let A = $\begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$. **Identify** the (3,2) entry of AB.	Marks 5 / PLO 1 C1

Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$

Identify (3,2) entry of AB

$$A \cdot B = \begin{bmatrix} 0 & 1 & -2 \end{bmatrix} \cdot \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$$

$$= (0 + (-1) + (-4))$$

$$= 0 - 1 - 4$$

$$= -5$$

(b)	**Label** the quadratic polynomial that interpolate the points (1,3), (2,4), (3,4)	Marks 5
		PLO 1 C1

b) Find the quadratic Polynomial that interpolate the Point (1,3), (2,4), (3,7)

$$a_2 x_1^2 + a_1 x_1 + a_0 = y_1$$

$$a_2 x_2^2 + a_1 x_2 + a_0 = y_2$$

$$a_2 x_3^2 + a_1 x_3 + a_0 = y_3$$

$(x_1, y_1) = (1,3)$ $(x_2, y_2) = (2,4)$

$(x_3, y_3) = (3,7)$

Sol:-

$$a_2(1)^2 + a_1(1) + a_0 = 3$$

$$a_2(2)^2 + a_1(2) + a_0 = 4$$

$$a_2(3)^2 + a_1(3) + a_0 = 7$$

$$= \quad a_2 + a_1 + a_0 = 3$$

$$4a_2 + 2a_1 + a_0 = 4$$

$$9a_2 + 3a_1 + a_0 = 7$$

$$a:b = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 4 & 2 & 1 & 4 \\ 9 & 3 & 1 & 7 \end{array}\right]$$

$$\tilde{R} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & -6 & -8 & -20 \end{array}\right] \begin{array}{l} \\ R_2 - 4R_1 \\ R_3 - 9R_1 \end{array}$$

$$\tilde{R} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 3 \\ 0 & -2 & -3 & -8 \\ 0 & 0 & 1 & 4 \end{array}\right] R_3 - 3R_2$$

$a_2 + a_1 + a_0 = 3$ — (i)

$-2a_1 - 3a_0 = -8$ — (ii)

$a_0 = 4$ — (iii)

Now Put in 2

$-2a_1 - 3(4) = -8$

$-2a_1 - 12 = -8$

$-2a_1 = 4$

$a_1 = -2$

Put in (i)

$$a_2 + a_1 + a_0 = 3$$

$$a_2 = -2 + 4 = 3$$

$$a_2 = 3 - 2$$

$$a_2 = 1$$

Q 2	(a)	. If A and B are $n \times n$ matrices where $\lvert A \rvert = 2$ and $\lvert B \rvert = -3$,calculate $\lvert A^{-1}B^{T} \rvert$.	Marks 5 PLO 2 C2

Q2:- If A & B are nxn matrices where $|A|=2$ and $|B|=-3$ calculate $|A^{-1}B^{t}|$

Sol:- Since $|A^{-1}B^{t}| = |A^{-1}||B^{t}|$

$\Rightarrow \frac{1}{|A|}|B|$ become $|B^{t}| = |B|$

so $|A^{-1}B^{t}| = \frac{1}{|A|}|B|$

$= \frac{1}{2} \cdot 3 = \frac{3}{2}$

b) $x + y + 2z = 1$

(b)	**Estimate** the linear system of equation $x + y + 2z = 1$ $x - 2y + z = -5$ $3x + y + z = 3$	Marks 5 PLO2 C2

$x - 2y + z = -5$

$3x + y + z = 3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & -3 & -1 & -6 \\ 0 & -2 & -5 & 0 \end{array}\right] \begin{array}{l} R_2 - R_1 \\ R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1/3 & 2 \\ 0 & -2 & -5 & 0 \end{array}\right] \frac{R_2}{-3}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1/3 & 2 \\ 0 & 0 & -\frac{13}{2} & 4 \end{array}\right] R_3 + 2R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1/3 & 2 \\ 0 & 0 & 1 & -\frac{8}{13} \end{array}\right] R_3 \times \frac{2}{-13}$$

$$x + y + 2z = 1 \quad \text{—(i)}$$

$$y + \tfrac{1}{3}z = 2 \quad \text{—(ii)}$$

$$z = \frac{-8}{13} \quad \text{—(iii)}$$

Now Put eq (iii) – in (ii)

$$y + \tfrac{1}{3} \times \frac{-8}{13} = 2$$

$$y - \frac{8}{39} = 2$$

$$y = 2 + \frac{8}{39}$$

$$y = \frac{78+8}{39} = \frac{86}{39}$$

Now Put value of y in (i)

$$x + \frac{86}{39} + 2\left(\frac{-8}{13}\right) = 1$$

$$x + \frac{86}{39} - \frac{16}{13} = 1$$

$$x + \frac{38}{39} = 1$$

$$x = 1 - \frac{38}{39}$$

$$x = \frac{1}{39}$$

Q 3		Find A^{-1} where $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$.	Marks 10
			PLO 2 C2

Q3:- Find A^{-1} where $A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$

Sol:-

$$|A| = \begin{vmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{vmatrix}$$

$$= 3\begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} + 2\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1\begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$= 3(-18-0) + 2(-15-2) + 1(0-6)$$

$$= 3(-18) + 2(-17) + 1(-6)$$

$$= -54 - 34 - 6$$

$$= -94$$

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} = -18$$

$$= (-1)^2(-18-0)$$

$$= -18$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = 17$$

$$(-1)^3 (-15 - 2)$$

$$-1(-17) = 17$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6$$

$$= (-1)^4 (0 - 6)$$

$$= 1(-6) = -6$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = -6$$

$$(-1)^3 (6 - 0)$$

$$(-1)(6) = -6$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -10$$

$$= (-1)^4 (-9 - 1)$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix}$$

$$= (-1)^5 (0+2)$$

$$= -1(2) = -2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} = -10$$

$$= (-1)^4 (-4-6)$$

$$= 1(-10) = -10$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = -1$$

$$= (-1)^5 (6-5)$$

$$= -1(1) = -1$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -2 \\ 5 & 6 \end{vmatrix}$$

$$= (-1)^6 (18+10)$$

$\Rightarrow 1(28) = 28$

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^t$$

$$= \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{adj } A$$

$$A^{-1} = \frac{1}{-94} \begin{bmatrix} 18 & 6 & 10 \\ -17 & 10 & 1 \\ 6 & 2 & -28 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{18}{-94} & \frac{6}{-94} & \frac{10}{-94} \\ \frac{-17}{-94} & \frac{10}{-94} & \frac{1}{-9} \\ \frac{6}{-94} & \frac{2}{-94} & \frac{-28}{-94} \end{bmatrix} = \begin{bmatrix} \frac{-9}{47} & \frac{-3}{47} & \frac{-5}{47} \\ \frac{17}{94} & \frac{-5}{47} & \frac{1}{-9} \\ \frac{-3}{47} & \frac{-1}{47} & \frac{14}{47} \end{bmatrix}$$