



Iqra National University, Peshawar
Department of Electrical Engineering



Assignment
Date:20/4/2020

Course Code: MTH 102 Course Title: Calculus and analytic geometry
Prerequisite: _____ Instructor: HIMAYATULLAH
Module: 3 Program: BEE Total Marks: 30

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Q1.	(a)	. Identify $\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$	Marks 5
			CLO1 C1
	(b)	Find the first order derivatives of the function $y = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x} + 1\right)$	Marks 5
			CLO1 C1
Q2	(a)	. A dynamite blast blows up a heavy rock with launch velocity of 160m/sec reaches a height of $s = 160t - 16t^2$ ft after t sec, (i) How high does the rock go (ii) Find the velocity and speed of the rock when it is 256 ft above the ground on the way up and down (iii) find the acceleration of the rock at time 5sec	Marks 10
			CLO2 C2
Q3	(a)	Does the curve $y = x^4 - 2x^2 + 2$ have nay horizontal tangent if so where ?	Marks 10
			CLO1 C1

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(1)

Identify.

b)

$$\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

So

Sol

$$\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

$$= \frac{\sqrt{2+0} - \sqrt{2}}{0} = \frac{\sqrt{2} - \sqrt{2}}{0}$$

$$= \frac{0}{0} \Rightarrow \text{o/o form}$$

So then

$$\lim_{h \rightarrow 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$$

ring and ring by $\sqrt{2+h} + \sqrt{2}$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{2+h} - \sqrt{2}) \times (\sqrt{2+h} + \sqrt{2})}{h(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{2+h} - \sqrt{2}) (\sqrt{2+h} + \sqrt{2})}{h(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h \rightarrow 0} \frac{(\sqrt{2+h})^2 - (\sqrt{2})^2}{h(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h \rightarrow 0} \frac{2+h - 2}{h(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{2+h} + \sqrt{2})}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{2+h} + \sqrt{2}}$$

(2)

$$\lim_{h \rightarrow 0} \frac{1}{\sqrt{2+h} + \sqrt{2}}$$

$$= \frac{1}{\sqrt{2+0} + \sqrt{2}}$$

$$= \frac{1}{\sqrt{2} + \sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}}$$

ⓑ $y = \left(n + \frac{1}{n}\right) \left(n - \frac{1}{n} + 1\right)$

Sol
 $y = \left(n + \frac{1}{n}\right) \left(n - \frac{1}{n} + 1\right)$

$$\frac{dy}{dn} = \frac{d}{dn} \left(n + \frac{1}{n}\right) \left(n - \frac{1}{n} + 1\right)$$

$$= \left(n + n^{-1}\right) \frac{d}{dn} \left(n - n^{-1} + 1\right) + \left(n - n^{-1} + 1\right) \frac{d}{dn} \left(n + n^{-1}\right)$$

$$= \left(n + n^{-1}\right) \left(1 + n^{-2}\right) + \left(n - n^{-1} + 1\right) \left(1 - n^{-2}\right)$$

$$= \left(n + \frac{1}{n}\right) \left(1 + \frac{1}{n^2}\right) + \left(n - \frac{1}{n}\right) \left(1 - \frac{1}{n^2}\right)$$

(3)

$$= n + \cancel{n} \cancel{x} + \cancel{x} + \frac{1}{n^3} + \cancel{n} - \cancel{n} \cancel{x} - \frac{1}{n^2} + \frac{1}{n^2} + \frac{1}{n^2}$$

$$= 2n + 1 - \frac{1}{n^2} + \frac{1}{n^3} \quad \underline{\text{Ans}}$$

Q2(a)

Given. $S = 160t - 16t^2$ ft

① $S = 160t - 16t^2$ ft

Velocity is,

$$V = \frac{ds}{dt} = \frac{d}{dt} (160t - 16t^2)$$

$$= \frac{d}{dt} 160t - \frac{d}{dt} 16t^2$$

$$V = 160 - 32t$$

Maximum height,

$$V = 0$$

So

$$160 - 32t = 0$$

$$\frac{160}{32} = \frac{32t}{32}$$

$$t = 5 \text{ sec}$$

(4)

$$S_{\max} = S(s) = 160(s) - 16(s)^2$$

$$S_{\max} = 400 \text{ ft}$$

(b) Given that

$$S = 256 \text{ ft}$$

then

$$160t - 16t^2 = 256$$

$$16t^2 - 160t + 256 = 0$$

$$\frac{16}{16} (t^2 - 10t + 16) = \frac{0}{16}$$

$$t^2 - 10t + 16 = 0$$

$$t^2 - 8t - 2t + 16 = 0$$

$$t(t - 8) - 2(t - 8) = 0$$

$$(t - 8)(t - 2) = 0$$

$$t - 8 = 0, \quad t - 2 = 0$$

$$t = 8, \quad t = 2$$

$$t_1 = 8 \text{ sec}, \quad t_2 = 2 \text{ sec}$$

Since

$$V = 160 - 32t$$

(5)

$$t_1 = 2s$$
$$V(2) = 160 - 32(2)$$

$$= 160 - 64$$

$$V(2) = 96 \text{ m/s} \Rightarrow \text{Velocity upward}$$

$$t_2 = 8s$$

$$V_8 = 160 - 32(8)$$

$$= 160 - 256$$

$$= -96 \text{ m/s Velocity downward}$$

(c) Since

$$v = 160 - 32t$$

$$\text{Acceleration, } a = \frac{dv}{dt} = \frac{d}{dt}(160 - 32t)$$

$$a = 0 - 32 \text{ m/s}^2$$

$$a = -32 \text{ m/s}^2$$

|| ——— || ——— || ——— || ——— || ——— ||

Q 3 (a)

$$y = m^4 - 2m^2 + 2$$

h

Sol

$$y = m^4 - 2m^2 + 2$$

$$\frac{dy}{dm} = \frac{d}{dm}(m^4 - 2m^2 + 2)$$

(B)

$$= \frac{d}{dn} n^4 - \frac{d}{dn} 2n^2 + \frac{d}{dn} 2$$

$$= 4n^3 \frac{dn}{dn} - 2 \times 2n \frac{dn}{dn} + 0$$

$$= 4n^3 - 4n + 0$$

$$\frac{dy}{dn} = 4n^3 - 4n$$

If the tangent is horizontal
-then

$$\frac{dy}{dn} = 0$$

So,

$$4n^3 - 4n = 0$$

$$4n(n^2 - 1) = 0$$

$$4n = 0, \quad n^2 - 1 = 0$$

$$\frac{4n}{4} = \frac{0}{4}$$

$$n = 0$$

$$n^2 - 1 = 0$$

$$\frac{n^2}{n^2} = \frac{1}{1}$$

$$n = 0, \quad ,$$

$$n = \pm 1$$

So

$$n = 0$$

Their corresponding point is

$$y = n^4 - 2n^2 + 2$$

(9)

for,

$$x = 0$$

$$y = x^4 - 2x^2 + 2$$

$$= (0)^4 - 2(0)^2 + 2$$

$$= 0 - 0 + 2$$

$$y = 2$$

$$x = 1$$

$$y = x^4 - 2x^2 + 2$$

$$= (1)^4 - 2(1)^2 + 2$$

$$= 1 - 2 + 2$$

$$x = -1$$

$$y = x^4 - 2x^2 + 2$$

$$= (-1)^4 - 2(-1)^2 + 2$$

$$= 1 - 2 + 2$$

$$= 1$$

hence, $(0, 2)$, $(1, 1)$, $(-1, 1)$

