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Subject:→ Differential Equation

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Quiz

Topic:→ Matrices

Question

A yarn merchant sells brands A, B, C of yarn each of which is a blend of Pakistani, Egyptian and American cotton in the ratio 1:2:1, 2:1:1, 2:0:2 respectively. If one Kilogram of A, B, C costs 40, 50 and 60 rupees respectively, find the cost of a Kilogram of cotton of each country.

Solution

$x = A$, $y = B$, $Z = x$

Let x, y and Z be the cost/Kg of Pakistani, Egyptian and American cotton respectively

P.T.O

According to the condition

$\frac{1}{4}x + \frac{2}{4}y + \frac{1}{4}z = \boxed{40} \rightarrow ①$

Sum of Ratio = 4

$\frac{2}{4}x + \frac{1}{4}y + \frac{1}{4}z = \boxed{50} \rightarrow ②$

$\frac{2}{4}x + \frac{2}{4}z = \boxed{60} \rightarrow ③$

Xing "4" both Sides on equation ①, ② and ③, we get

$① \Rightarrow \quad x + 2y + z = \boxed{160}$

$② \Rightarrow \quad 2x + y + z = \boxed{200}$

$③ \Rightarrow \quad 2x + 0y + z = \boxed{120}$

Now, we use these equations in matrix form

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

So,

$$A_1 = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B_1 = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

$\Rightarrow \quad A_1 \times B_1$

Now, using Cramer's Rule

P.T.O

$$A_1 x = \begin{bmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{bmatrix}$$

we just Replace B_1 in 1st Column of A_1

$$|A_1 x| = 160 \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} - 2 \begin{bmatrix} 200 & 1 \\ 120 & 1 \end{bmatrix} + \begin{bmatrix} 200 & 1 \\ 120 & 1 \end{bmatrix}$$

$$= 160\,(1-0) - 2\,(200-120) + 1\,(0-120)$$

$$= 160 - 2(80) - 120 = 160 - 160 = \boxed{120}$$

$$A_1 x| = \boxed{-120}$$

Now,

$$x \quad \frac{|A_1 x|}{|A_1|} \rightarrow (4)$$

Now find $|A_1|$

$$|A_1| = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = 1(1-0) - 2(2-1) + 1(0-1)$$

$$= 1 - 2 - 1 = \boxed{-2}$$

Now

$$(4) \Rightarrow x \quad \frac{|A_1 x|}{|A_1|} = \frac{-120}{-2} = \boxed{60}$$

$$x = A = \boxed{60}$$

Also,

$$y = \frac{|A_1 y|}{|A_1|} \rightarrow (5)$$

P.T.O

A_1y $\begin{bmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{bmatrix}$ just Replace B_1 in 2nd Column of A_1

$$|A_1y| = 1(200-120) - 160(2-1) + 1(240-200)$$

$$= 80 - 160 + 40$$

$$|A_1y| = \boxed{-40}$$

$$(5) \Rightarrow y = \frac{|A_1y|}{|A_1|} = \frac{-40}{-2} = \boxed{20}$$

$$y = B = \boxed{20}$$

Again $z = \dfrac{|A_1z|}{|A_1|} \rightarrow (6)$

$$A_1z = \begin{bmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{bmatrix}$$

$$|A_1z| = 1(120-0) - 2(240-200) + 160(0-1)$$

$$= 120 - 80 - 160$$

$$= \boxed{120}$$

$$(6) \Rightarrow z = \frac{|A_1z|}{|A_1|} = \frac{-120}{-2} = \boxed{60}$$

P.T.O

$\Rightarrow$ $Z = C = \boxed{60}$

Hence

$$(x, y, z) = (60, 20, 60)$$

OR

$$(A, B, C) = (60, 20, 60)$$

it means that

$\Rightarrow$ Pakistani blend Cost/Kg of Cotton = $\boxed{60}$

$\Rightarrow$ Egyptian blend Cost/Kg of Cotton = $\boxed{20}$

$\Rightarrow$ American blend Cost/Kg of Cotton = $\boxed{60}$