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Subject = Calculus.

Q1:-

find PQ where P is the point in Three diamesional space with coordinate (4, 1, 3) & the point Q with coordinate (1, 2, 4). Find the distance b/w P & Q, Find the position vactor of the point dividing PQ in the ratio 1:3.

Solution:

Coordinate of $P = (4, 1, 3)$

$\vec{OP} = 4i + 1j + 3k$

or $OQ = \vec{OQ} - \vec{OP}$

$= (i + 2j + 4k) - (4i + 1j + 3k)$

$= -3i + 1j + 1k$ —(1)

Now distance between $P \& Q = |PQ|$

$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$

$= \sqrt{9+1+1}$

$= \sqrt{11}$ → (2)

let M be the point which divided PQ in ratio 1:3, Then by ratio Therem Position vactor of $M = \vec{OM}$

$$= \frac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1+3}$$

$$= \frac{12i + 3j + 9k + i + 2j + 4k}{4}$$

$$= \frac{13i + 5j + 13k}{4} \quad \text{--- (3)}$$

ence eq (1), (2) & (3) are the
required sol.

Q2

Evaluate $\int \frac{4x^3+10x+4}{2x^2+x}\,dx$.

Solution:

$$\begin{array}{r|l} & 2x-1 \\ \hline 2x^2+x & 4x^3+10x+4 \\ & \pm 4x^3 \quad \pm 2x^2 \\ \hline & -2x^2+10x+4 \\ & \mp 2x^2 \mp x \\ \hline & 11x+4 \end{array}$$

So $\quad 2x-1+\frac{11x+4}{2x^2+x} = \frac{4x^3+10x+4}{2x^2+x}$

$$\Rightarrow \int \frac{4x^3+10x+4}{2x^2+x} = \int 2x-1+\int\frac{11x+4}{2x^2+x} \rightarrow (1)$$

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx \quad \text{——} (2)$$

Now find.

$$\int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$\frac{11x+4}{x(2x+1)} = A/x + B/2x+1 \quad \text{——} (A)$$

$$\cancel{\frac{11x+4}{x(2x+1)}} = \cancel{A/x + \frac{B}{(2x+1)}}$$

$$\frac{11x+4}{x(2x+1)} = \frac{A\cdot 2(x+1)+Bx}{x(2x+1)}$$

$$11x+4 = A(2x+1)+Bx \quad \text{--- (3)}$$

put $x=0$ in eq

$$\boxed{4=A}$$

Now put $x = -1/2$ in eq (3)

$$11(-1/2)+4 = B(-1/2)$$

$$-11/2+4 = -B/2$$

$$\frac{-11+8}{\cancel{2}} = B/\cancel{2}$$

$$-3 = -B$$

$$\Rightarrow \boxed{B=3}$$

putting value of A & B in A

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$ Taking integral

$$\int \frac{11x+4}{x(2x+1)}\,dx = \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$= 4\ln|x| + \tfrac{3}{2}\ln|2x+1|$$

putting these value in 2

$$= x^2 - x + 4\ln|x| + \tfrac{3}{2}\ln|2x+1|$$

put value in 1

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx =$$

$$x^2 - x + 4\ln|x| + \tfrac{3}{2}\ln|2x+1| + C$$

Ans.

$\frac{\quad}{2x^2} \quad \frac{\quad}{2x^2} \quad \frac{\quad}{2x^2}$

Q NO3 (1)

a) $\int_0^2 x^2 e^x \, dx$

Now first find Integration.

$\int x^2 e^x dx$

$= x^2 \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x^2 \right) dx$

$= x^2 e^x - \int e^x (2x) dx$

$= x^2 e^x - 2 \int x e^x dx$

$= x^2 e^x - 2 \left[x \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x \right) dx \right]$

$= x^2 e^x - 2 \left[x e^x - \int e^x dx \right]$

$= x^2 e^x - 2x e^x + 2e^x$

Now put limits.

$\left| x^2 e^x - 2x e^x + 2e^x \right|_0^2$

$= \left(2^2 e^2 - 2(2) e^2 + 2e^2 - (0 - 0 + 2e^0) \right)$

$= \left(4e^2 - 4e^2 + 2e^2 - 2 \right)$

$= 2e^2 - 2$ Ans.

2

B part

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$$

First Find Integration

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = ? \quad \text{—— (1)}$$

let $y = \sqrt{x}$

$$dy/dx = \frac{1}{2\sqrt{x}}$$

$$\boxed{2\,dy = \frac{1}{\sqrt{x}}\,dx} \text{ putin—(1)}$$

$$\int \sin(y)(2dy) = 2\int \sin(y)\,dy$$

$$= \cancel{2(-\cos y)}$$

$$= 2(-\cos y)$$

$$= -2\cos y$$

put $y = \sqrt{x}$

$$= -2\cos\sqrt{x}$$

Put Limits

$$= -2 \left| \cos\sqrt{x} \right|_1^2 = -2 \left(\cos\sqrt{2} - \cos 1 \right)$$

$$= -2\cos\sqrt{2} + 2\cos(1) = \text{Ans.}$$

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Q No 7:-

The laplace's equation 3·d is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \rightarrow (A)$$

So $u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$

$$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$= -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x(-3/\cancel{2})(x^2+y^2+z^2)^{-5/2}(\cancel{2}x) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-3/2}$$

Now $\frac{\partial u}{\partial y} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$

$$\frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[y(-3/\cancel{2})(x^2+y^2+z^2)^{-5/2}(\cancel{2}y) + (x^2+y^2+z^2)^{-3/2}\right.$$

$$\boxed{\frac{\partial^2 u}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{-5/2} + (x^2+y^2+z^2)^{-3/2}} \to 2$$

$$\frac{\partial u}{\partial z} = -\tfrac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}z)$$

$$\frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}$$

$$\boxed{\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}} \to 3$$

putting 1 2 & 3 in A

$$3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2} -$$

$$(x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$= (x^2+y^2+z^2)^{-5/2}\left[3x^2-(x^2+y^2+z^2)+3y^2-(x^2+y^2+z^2) + 3z^2-(x^2+y^2+z^2)\right]$$

$$= (x^2+y^2+z^2)^{-5/2}\left[\cancel{3x^2}-\cancel{x^2}-\cancel{y^2}-z^2+3y^2-\cancel{x^2}-\cancel{y^2}-z^2+ 3z^2-\cancel{x^2}-\cancel{y^2}-z^2\right]$$

$$(x^2+y^2+z^2)^{-5/2}(0) = 0$$

So the given $u(x, y, z)$ is solution of laplace equation.