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Section - B

Depart - (S.E)

Semester - 2nd

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Q1:- Compute adjoint of:-

(i)

$$A = \begin{bmatrix} 1 & 2 & 2\text{nd ID} \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

Find its adjoint or adj(A)

$$A = \begin{bmatrix} 1 & 2 & 6 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

~~$A_{11} = (-1)^{1+1}((3\times 2)$~~

$A_{11} = (-1)^{1+1}(3\times 2 - 1\times 1) = (-1)^2 6 - 1 = 50$

$(-1)^2(5) = (1)(5) = 5$

(2)

$A_{12} = (-1)^{1+2}(2\times2 - 1\times3) = (-1)^3(4-3) = (-1)(1) = -1$

$A_{13} = (-1)^{1+3}(2\times1 - 3\times3) = (-1)^4(2-9) = (1)(-7) = -7$

$A_{21} = (-1)^{2+1}(2\times2 - 6\times1) = (-1)^3(4-6) = (-1)(-2) = 2$

$A_{22} = (-1)^{2+2}(2\times2 - 6\times3) = (-1)^4(2-18) = (1)(-16) = -16$

$A_{23} = (-1)^{2+3}(1\times1 - 2\times3) = (-1)^5(1-6) = (-1)(-5) = 5$

$A_{31} = (-1)^{3+1}(2\times1 - 6\times3) = (-1)^4(2-18) = (1)(-16) = -16$

$A_{32} = (-1)^{3+2}(1\times1 - 6\times2) = (-1)^5(1-12) = (-1)(-11) = 11$

$A_{33} = (-1)^{3+3}(1\times3 - 2\times2) = (-1)^6(3-4) = (1)(-1) = -1$

$$adj(A) = \begin{bmatrix} 5 & -1 & -7 \\ 2 & -16 & 5 \\ -16 & 11 & -1 \end{bmatrix}^T$$ let we have transpose of adj(A)

$$\begin{bmatrix} 5 & 2 & -16 \\ -1 & -16 & 11 \\ -7 & 5 & -1 \end{bmatrix}$$ Ans

(ii) $$B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$$

Find its adjoint or adj(B)

$A_{11} = (-1)^{1+1}(-1\times8 - (-2\times8)) = (-1)^2(-8-(-16))$

$(-1)^2(-8+16) = (1)(8) = 8$

$A_{12} = (-1)^{1+2}(2\times8 - 8\times5) = (-1)^3(16-40) =$
$(+1)(-24) = -24$

$A_{13} = (-1)^{1+3}(2\times-2-(-1\times5) = (-1)^4(-4-(-5)$
$(-1)^4(-4+5) = (1)(1) = 1$

$A_{21} = (-1)^{2+1}(4\times8 - 5\times-2) = (-1)^3(32-(-10)$
$(-1)(42) = -42$

$A_{22} = (-1)^{2+2}(3\times8 - 5\times5) = (-1)^4(24-25)$
$(+1)(-1) = -1$

$A_{23} = (-1)^{2+3}(3\times-2-4\times5) = (-1)^5(-6-20)$
$(-1)(+26) = 26$

$A_{31} = (-1)^{3+1}(4\times8 - (5\times-1) = (-1)^4(32-(-5)$
$(1)(32+5) = (1)(37) = 37$

$A_{32} = (-1)^{3+2}(3\times8 - 5\times2) = (-1)^5(24-10)$
$(-1)^5(14) = -14$

$A_{33} = (-1)^{3+3}(3\times-1 - 4\times2) = (-1)^6(-3-8)$
$(-1)^6(-3-8) = (1)(-11) = -11$

$\text{adj}(B)\begin{bmatrix} 8 & -24 & 1 \\ -42 & -1 & 26 \\ 37 & -14 & -11 \end{bmatrix}^T$ Let we have transpose of adj(B)

$$\begin{bmatrix} 8 & -42 & 37 \\ -24 & -1 & -14 \\ 1 & 26 & -11 \end{bmatrix}$$ Ans

Q2:- Find the cofactors of A_{21}, A_{31}, A_{33} if

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

A_{21}, A_{31}, A_{33}

$$A = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

$$A_{21} = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

$$A_{21} = \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix} = -4 - (-9) = -4 + 9 = 5$$

$$\boxed{A_{21} = 5}$$

$$A_{31} = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

$$A_{31} = \begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix} = -2 - 9 = -11$$

$$\boxed{A_{31} = -11}$$

$$A_{33} = \begin{bmatrix} 1 & -2 & 3 \\ -2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

$$A_{33} = \begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix} = 3 - (-(-4)) = 3 - 4 = -1$$

$$\boxed{A_{33} = -1}$$

Q3:- Find Eigen values and Eigen Vectors of

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$A - \lambda I = 0$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & 1 & 1 \\ 1 & 3-\lambda & 2 \\ -1 & 1 & 2-\lambda \end{bmatrix}$$

$2-\lambda((3-\lambda)(2-\lambda)-2) - 1((1)(2-\lambda)+2) + 1(1-(3-\lambda)(-1))$

$2-\lambda(6-3\lambda-2\lambda+\lambda^2-2) - (2-\lambda+2) + (1+3+\lambda)$

~~$2-\lambda(6-5\lambda$~~

$2-\lambda(4-5\lambda+\lambda^2) - (4-\lambda) + (4-\lambda)$

$2-\lambda(4-5\lambda+\lambda^2)$

$2-\lambda = 0 \qquad 4-5\lambda+\lambda^2 = 0$

$2 = \lambda$

$\boxed{\lambda = 2}$

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$4 - 4\lambda - \lambda + \lambda^2 = 0$

$\lambda^2 - 4\lambda - \lambda + 4 = 0$

$\lambda(\lambda - 4) - 1(\lambda - 4) = 0$

$(\lambda - 4)$ $(\lambda - 1)$

$\lambda - 4 = 0$ $\lambda - 1 = 0$

$\boxed{\lambda = 4}$ $\boxed{\lambda = 1}$