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Page # 01

Q1: Find the general solution of

$y' + 3.5y = 2.8$

Ans:

Solution:

$$y' + P(x)y = r(x)$$

$$P(x) = 3.5, \quad r(x) = 2.8$$

First Find "h"

$$h = \int p(x)\,dx = \int 3.5\,dx$$

$$h = 3.5x$$

$$y = e^{-h}\left[\int e^{h} r(x)\,dx + c\right]$$

$$= e^{-3.5x}\left[\int e^{3.5x}(2.8)\,dx + c\right]$$

$$= e^{-3.5x}\left[2.8\,\frac{e^{3.5x}}{3.5}\right] + c$$

$$y = \frac{2.8}{3.5}(1) + ce^{-3.5x}$$

Q2: Find the orthogonal trajectories of $y = Cx$

Ans.

Solution:-

① First we construct the differential equation for the family of straight lines $y = Cx$. By differentiating the left equation with respect to $x$, we get:

$$y' = C = \text{const.}$$

Eliminate the constant $C$ from the system of equations:

$$\begin{cases} y = Cx \\ y' = C \end{cases}, \Rightarrow y' = \frac{y}{x}.$$

we obtain the differential equation of the initial set of straight lines.

② Replace $y'$ with $\left(-\frac{1}{y'}\right)$. This gives the differential equation of the orthogonal trajectories:

$$-\frac{1}{y'} = \frac{y}{x}, \Rightarrow y' = -\frac{x}{y},$$

$$\Rightarrow y\,dy = -x\,dx,$$

$$\Rightarrow \int y\,dy = -\int x\,dx$$

$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C,$$

$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} = C,$$

$$\Rightarrow x^2 + y^2 = 2C.$$

By replacing $2C$ with $R^2$ we see that the orthogonal trajectories for the family of straight lines are concentric circles (Figure 1):

$$x^2 + y^2 = R^2.$$

**Q3:**

(a) Estimate the general solution of

$$4y'' - 20y' + 25y = 0$$

**Ans:**

**Solution:**

$$4y'' - 20y' + 25y = 0$$

$$4\lambda^2 - 20\lambda + 25 = 0$$

$$4\lambda^2 - 10\lambda - 10\lambda + 25 = 0$$

$$2\lambda(2\lambda - 5) - 5(2\lambda - 5) = 0$$

$$(2\lambda - 5)(2\lambda - 5) = 0$$

$$2\lambda - 5 = 0 \qquad 2\lambda - 5 = 0$$

$$\lambda_1 = \frac{5}{2} \qquad \lambda_2 = \frac{5}{2}$$

$$y = (C + Cx)\, e^{\frac{5}{2}x}$$

Q 3
(B) Identify an ODE $y'' + ay' + by = 0$ for the basis $e^{2x}, e^{x}$.

Ans Solution:-

Since,

$y_1 = e^{2x}$ , $y_2 = e^{x}$

$y = C_1 e^{2x} + C_2 e^{x}$

$\lambda_1 = 2$ , $\lambda_2 = 1$

$\lambda_1 - 2$ , $\lambda_2 - 1 = 0$

$(\lambda - 2)(\lambda - 1) = 0$

$\lambda^2 - 2\lambda - \lambda + 2 = 0$

$\lambda^2 + 3\lambda + 2 = 0$

$y'' - 3y' + 2y = 0$

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Q4: Calculate the general solution of

$y'' + 2y + y = 0.$

Ans: Solution:

$$y'' + 2y' + y = 0$$

$$\lambda^2 + 2\lambda + 1 = 0$$

$$(\lambda + 1)^2 = 0$$

$$\lambda_1 = \lambda_2 = -1$$

$$y = (C_1 + C_2 x)e^{-x}$$

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Q 5

Analyze the general solution of

$x^2y'' + 3xy' + y = 0.$

Ans

Solution:

$$x^2y'' + 3xy' + y = 0$$

$$m^2 + (a-1)m + b = 0$$

$$a = 3 \qquad b = 1$$

$$m^2 + (3-1)m + 1 = 0$$

$$m^2 + 2m + 1 = 0$$

$$(m+1)^2 = 0$$

$$m = m_1 = m_2 = -1$$

So,

$$y = (c_1 + c_2 \ln x)$$