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Subject - Applied Maths II

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Q 1:

Find the general solution of
 $y' + 3.5y = 2.8$

Ans:

Solution:

$$y' + P(x)y = r(x)$$

$$P(x) = 3.5, \quad r(x) = 2.8$$

First find "h"

$$h = \int P(x) dx = \int 3.5 dx$$

$$h = 3.5x$$

$$y = e^{-h} \left[\int e^h r(x) dx + C \right]$$

$$= e^{-3.5x} \left[\int e^{3.5x} (2.8) dx + C \right]$$

$$= e^{-3.5x} \left[2.8 \frac{e^{3.5x}}{3.5} \right] + C$$

$$y = \frac{2.8}{3.5} (1) + C e^{-3.5x}$$



Q2:

Find the orthogonal trajectories of
 $y = cx$.

Ans:

Solution:

① First we construct the differential equation for the family of straight lines $y = cx$. By differentiating the last equation with respect to x , we get:

$$y' = c = \text{const.}$$

Eliminate the constant c from the system of equations:

$$\begin{cases} y = cx \\ y' = c \end{cases}, \Rightarrow y' = \frac{y}{x}.$$

we obtain the differential equation of the initial set of straight lines.

② Replace y' with $(-\frac{1}{y'})$. This gives the

Differential equation of the orthogonal trajectories:

$$-\frac{1}{y} = \frac{y}{x}, \Rightarrow y' = -\frac{x}{y},$$

$$\Rightarrow y \, dy = -x \, dx,$$

$$\Rightarrow \int y \, dy = - \int x \, dx$$

$$\Rightarrow \frac{y^2}{2} = -\frac{x^2}{2} + C,$$

$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} = C,$$

$$\Rightarrow x^2 + y^2 = 2C.$$

By replacing $2C$ with R^2 we see that the orthogonal trajectories for the family of straight lines are concentric circles (Figure 1):

$$x^2 + y^2 = R^2.$$



Q3:

(a)

Estimate the general solution of

$$4y'' - 20y' + 25y = 0$$

Ans:

Solution:

$$4y'' - 20y' + 25y = 0$$

~~$4\lambda^2 - 20\lambda + 25 = 0$~~

$$4\lambda^2 - 20\lambda + 25 = 0$$

$$4\lambda^2 - 10\lambda - 10\lambda + 25 = 0$$

$$2\lambda(2\lambda - 5) - 5(2\lambda - 5) = 0$$

$$(2\lambda - 5)(2\lambda - 5) = 0$$

$$2\lambda - 5 = 0 \quad 2\lambda - 5 = 0$$

$$\lambda_1 = \frac{5}{2}$$

$$\lambda_2 = \frac{5}{2}$$

$$y = (C + Cx)e^{\frac{5}{2}x}$$



Q 3
(B)Identify an ODE $y'' + ay' + by = 0$ for the basis e^{2x}, e^x .AnsSolution:-

Since,

$$y_1 = e^{2x}, \quad y_2 = e^x$$

$$y = C_1 e^{2x} + C_2 e^x$$

$$\lambda_1 = 2, \quad \lambda_2 = 1$$

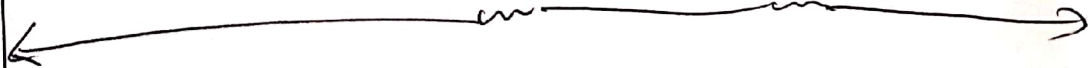
$$\lambda_1 - 2, \quad \lambda_2 - 1 = 0$$

$$(\lambda - 2)(\lambda - 1) = 0$$

$$\lambda^2 - 2\lambda - \lambda + 2 = 0$$

$$\lambda^2 - 3\lambda + 2 = 0$$

$$y'' - 3y' + 2y = 0$$



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Q4:

Calculate the general solution of

$$y'' + 2y' + y = 0.$$

Ans:

Solution:

$$y'' + 2y' + y = 0$$

$$\lambda^2 + 2\lambda + 1 = 0$$

$$(\lambda + 1)^2 = 0$$

$$\lambda_1 = \lambda_2 = -1$$

$$y = (C_1 + C_2x)e^{-x}$$

Q5

Analyze the general solution of
 $x^2 y'' + 3xy' + y = 0.$

Ans

Solution:

$$x^2 y'' + 3xy' + y = 0$$

$$m^2 + (a-1)m + b = 0$$

$$a = 3 \quad b = 1$$

$$m^2 + (3-1)m + 1 = 0$$

$$m^2 + 2m + 1 = 0$$

$$(m+1)^2 = 0$$

$$m = m_1 = m_2 = -1$$

So,

$$y = (C_1 + C_2 \ln x)$$