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- **Paper = Discrete Structure**
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- **Examination: Final Paper**


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**Q.1**

a) Explain the concept of Biconditional statement.
b) Let p, q, and r represent the following statements:

P: Sam had pizza last night.
Q: Chris finished her homework.
R: Pat watched the news this morning

Give a formula (using appropriate symbols) for each of these statements.

i. Sam had pizza last night if and only if Chris finished her homework.
ii. Pat watched the news this morning iff Sam did not have pizza last night.
iii. Pat watched the news this morning if and only if Chris finished her homework and Sam did not have pizza last night.

In order for Pat to watch the news this morning, it is necessary and sufficient that Sam had pizza last night and Chris finished her homework.

## Answer(A): Biconditional statement:

## Definition:

A biconditional statement is a combination of a conditional statement and its converse written in the if and only if form.

## Explanation:

- A biconditional is true if and only if both the conditionals are true.
- Two line segments are congruent *if and only if* they are of equal length.
- It is a combination of two conditional statements, “if two line segments are congruent then they are of equal length” and “if two line segments are of equal length then they are congruent”.
- Bi-conditionals are represented by the symbol ↔↔ or ⇔⇔.
- p↔qp↔q means that p→qp→q and q→pq→p .

- That is, $p\leftrightarrow q = (p\rightarrow q) \land (q\rightarrow p) p\leftrightarrow q = (p\rightarrow q) \land (q\rightarrow p)$.

**Example:** Write the two conditional statements associated with the bi-conditional statement below.

A rectangle is a square if and only if the adjacent sides are congruent.

The associated conditional statements are:

a) If the adjacent sides of a rectangle are congruent then it is a square.

b) If a rectangle is a square then the adjacent sides are congruent.

- **Conditional:** If the polygon has only four sides, then the polygon is a quadrilateral. (true)
- **Converse:** If the polygon is a quadrilateral, then the polygon has only four sides. (true)
- **Conditional:** If the quadrilateral has four congruent sides and angles, then the quadrilateral is a square. (true)
- **Converse:** If the quadrilateral is a square, then the quadrilateral has four congruent sides and angles. (true)

**Part B:** Let p, q, and r represent the following statements:

P: Sam had pizza last night.
Q: Chris finished her homework.
R: Pat watched the news this morning

Give a formula (using appropriate symbols) for each of these statements.

## Statements:

i. Sam had pizza last night if and only if Chris finished her homework.
ii. Pat watched the news this morning iff Sam did not have pizza last night.
iii. Pat watched the news this morning if and only if Chris finished her homework and Sam did not have pizza last night.

In order for Pat to watch the news this morning, it is necessary and sufficient that Sam had pizza last night and Chris finished her homework.

**Answer:B.**

i. $p \leftrightarrow q$
ii. $r \leftrightarrow \sim p$
iii. $(r \leftrightarrow q) \wedge (\sim p)$
iv. $r \wedge p \wedge q$

**Q.2**

a) Let's p, q, r represent the following statements:
P: it is hot today.
Q: it is sunny
R: it is raining
Express in words the statements using Bicondtional statement represented by the following formulas:

i. $q \leftrightarrow p$
ii. $p \leftrightarrow ( q \wedge r )$
iii. $p \leftrightarrow ( q \vee r)$
iv. $r \leftrightarrow ( p \vee q)$

**Answer:**

i. It is Sunny if and only if it is hot today.
ii. It is hot today iff it is sunny and it is raining.
iii. It is hot today iff it is sunny and or it is raining.
iv. It is raining iff it is hot today or it is sunny.

**Q.3**

Explain Argument with proper examples. Differentiate Valid and Invalid argument through proper examples, also construct a truth table showing valid and invalid arguments. (Note: Examples and truth table should not belongs to your book or slides)

**Answer:** **Argument:**

## Definition:

An argument is a work of persuasion. You use it to convince others to agree with your claim or viewpoint when they have doubts or disagree. While we sometimes think of arguments as hostile and bitter, they don't need to be that way – in fact, a good argument is quite calm, reasonable, and fair-minded.

OR

An **argument** is a set of statements called **premises** together with a **conclusion**. An argument consisting of two premises and a conclusion is called **a syllogism**.

**Examples:** 1. either a leopard is a cat or a bird. (First premise)

It is not the case that a leopard is a bird. (Second premise)

Therefore a leopard is a cat. (Conclusion)

2. Either a leopard is a fish or a bird. (First premise)

It is not the case that a leopard is a bird. (Second premise)

Therefore a leopard is a fish. (Conclusion)

3. If the Moon is a planet then the Sun is an asteroid.

The Moon is a planet.

So the Sun is an asteroid.

4. If the Earth is a planet then it is larger than a protoplanet.

The Earth is larger than a protoplanet.

Therefore, a leopard is a cat.

5. If the Earth is a planet then it is larger than a protoplanet.

The Earth is larger than a protoplanet.

Therefore, the Earth is a planet.

Arguments can be discredited if any of the premises are false (or their truth is uncertain). This is, however, not the only way an argument can be discredited. Argument 4 has true premises and a true conclusion but, nevertheless, is a poor argument. Argument 5 is an inadequate argument as well but it is harder to spot. This is because this form allows for true premises to lead to a false conclusion. Here's an example of an argument with the same form as argument 5 that we recognize is a bad argument:

## Differentiate Valid and Invalid argument through proper examples, also construct a truth table showing valid and invalid arguments.

## Valid and Invalid argument:

### Definition:

So, we will construct a **truth table** that covers every possible **truth** value arrangement, and then we'll look to see if there is a row in which all the premises are true and the conclusion is false. If we find such a row, the **argument** is **invalid**; and if there is not such a row, then the **argument** is **valid**.

OR

The logical form of an argument is the second method for evaluating arguments. An argument is **valid** if and when all the premises are true. If this is the case then the conclusion must be true (i.e. if you accept the truth of the premises it forces you to accept the truth of the conclusion). We can check the validity of an argument with a truth table. When you are given a valid argument and you know the premises are true, the argument proves the conclusion to be true.

## Example:

Determine if the following arguments are valid just by looking at their form. Note that some are a combination of two or more argument types.

a. If one is a wuzzle then one is a woozle

If one is a woozle then one is a finkle

Therefore if one is a wuzzle then one is a finkle.

b. If p is a prime number larger than 2 then p is odd.

p is odd.

Therefore p is a prime number.

c. You either like Coke or you like Pepsi.

You like Coke.

So you don’t like Pepsi.

d. Lizzlestipes and quadrinons.

If Lizzlestipes then fizbots.

If quadrinons then apoplexis.

Therefore fizbots and apoplexies.

## **Truth table showing valid and invalid arguments:**

Consider the first argument arguing a leopard is a cat. It has the form:

p or q.

Not q.

Therefore p.

Is it a valid argument? Well, it’s valid if whenever the premises (p or q) and (not q) are true, so the conclusion p must be true.

Here is the truth table with "p or q" and "not q" illustrated.

| p | q | Premise1: $p \vee q$ | Premise1: $\neg q$ | Conclusion: p |
|---|---|---|---|---|
| T | T | T | F | T |
| T | T | T | T | T |
| F | T | T | F | T |
| F | F | F | T | F |

The second row of truth values is the only row where both premises are true. When both premises were true, the conclusion was true as well. This means the argument is valid. Note that we have not determined if the actual premises are correct – we've just noted that, if the premises are true it must be the case the conclusion is true as well.

The second argument is also valid because it has exactly the same form as the first (and hence the same truth table). Because its first premise is false, though, this argument is not logically sound.

Argument 4 has the form:

If p then q.

Q

Therefore r.

Here is the truth table that illustrates the premises and the conclusion's possible truth values all at once. Since there are 3 simple statements (p, q and r) involved it is bigger (twice as big!) than the previous truth tables.

| p | q | r | Premise1: if p then q. | Premise2:q | Conclusion:r |
|---|---|---|---|---|---|
| T | T | T | T | T | T |
| T | T | F | T | T | F |
| T | F | T | F | F | T |
| T | F | F | F | F | F |
| F | T | T | T | T | T |
| F | T | F | T | T | F |
| F | F | T | T | F | T |
| F | F | F | T | F | F |

The first, second, fifth and sixth rows of truth values all satisfy the condition that both premises are true. But row 2 and row 6 have the premises true yet the conclusion false. This is an invalid argument. Note that as soon as we see the second row of truth values we can conclude that the argument was invalid without reading through the rest of the truth table.

**Q.4**

a) Explain the concept of Union, also explain membership table for union by giving proper example of truth table.

b) Explain the concept of Intersection, also explain membership table for Intersection by using proper example of truth table. (Note: Examples and truth table should not belongs to your book or slides)

# Answer part A: Union:

## Definition:

The set made by combining the elements of two sets.

OR

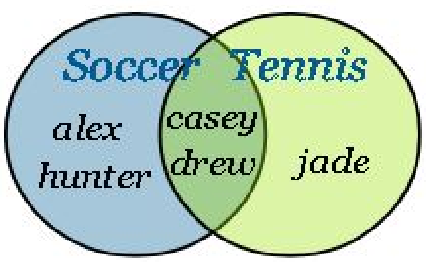
So the union of sets A and B is the set of elements in A, or B, or both.

The symbol is a special "U" like this: ∪ .

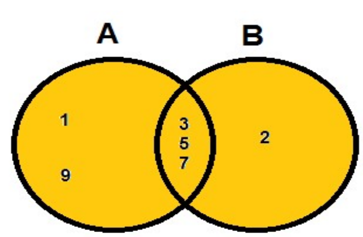
**Example:**

Soccer = {alex, hunter, casey, drew}
Tenniss = {casey, drew, jade}
Soccer ∪ Tennis = {alex, hunter, casey, drew, jade}
In words: the union of the "Soccer" and "Tennis" sets is alex, hunter, casey, drew and jade

## Diagrams:




### Another Example of Union:




A={1,3,5,7,9} and B={2,3,5,7,}, what are A∪ B:

A∪ B= {1, 2, 3, 5, 7, 9}

The Union is any region including either A or B.

## Membership Tables:

We combine sets in much the same way that we combined propositions. Asking if an element xx is in the resulting set is like asking if a proposition is true. Note that xx could be in any of the original sets.

What does the set A∪ (B∩C)A∪ (B∩C) look like? We use 11 to denote the presence of some element xx and 00 to denote its absence. Type equation here.

| $A$ | $B$ | $C$ | $A \cap B$ | $\overline{A \cap B}$ | $\overline{A}$ | $\overline{B}$ | $\overline{A} \cup \overline{B}$ |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |

| $A$ | $B$ | $C$ | $B \cap C$ | $A \cup (B \cap C)$ |
|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |

## Part B:

### Intersection:

#### Definition:

**The intersection of two sets has only the elements common to both sets.**

- If an element is in just one set it is not part of the intersection.
- The symbol is an upside down U like this: ∩

**Example:** The intersection of the "Soccer" and "Tennis" sets is just Casey and drew (only Casey and drew are in both sets), which can be written:

Soccer ∩ Tennis = {Casey, drew}

## Membership Table For Intersection:

The Membership table for intersection of sets A and B is given below

• The truth table for conjunction of two statements P and Q is given below

• In the membership table of Intersection, replace 1 by T and 0 by F then the table is same as of conjunction

• So membership table for Intersection is similar to the truth table for conjunction ( $\wedge$ ).

| A | B | $A \cap B$ |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 0 |
| 0 | 1 | 0 |
| 0 | 0 | 0 |

| P | Q | $P \wedge Q$ |
|---|---|---|
| T | F | T |
| T | F | T |
| F | T | T |
| F | F | T |

# QNo5:

a) Explain the concept of Venn diagram with examples.

b) Given the set $P$ is the set of even numbers between 15 and 25. Draw and label a Venn diagram to represent the set $P$ and indicate all the elements of set $P$ in the Venn diagram.

c) Draw and label a Venn diagram to represent the set

$R$ = {Monday, Tuesday, Wednesday}.

d) Given the set $Q = \{x : 2x - 3 < 11, x$ is a positive integer $\}$. Draw and label a Venn diagram to represent the set $Q$.

## Answer Part A:

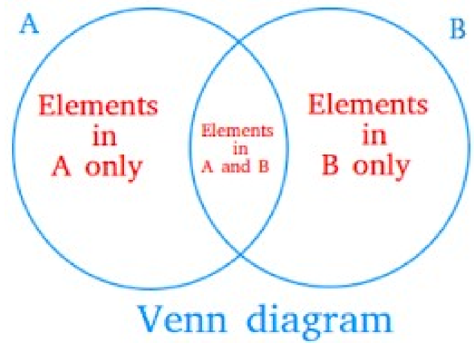
## Venn diagram:

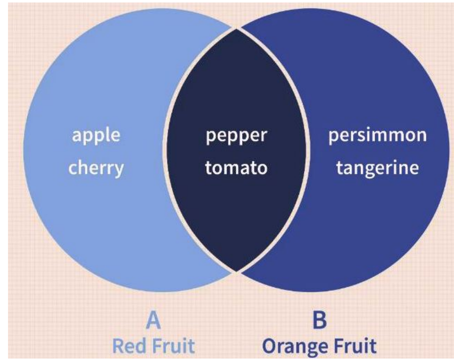
A Venn diagram is an illustration that uses circles to show the relationships among things or finite groups of things.

OR

- In a Venn diagram, sets are represented by shapes; usually circles or ovals. The elements of a set are labeled within the circle.

- Examples of Venn Diagrams:
- A Venn diagram could be drawn to illustrate fruits that come in red or orange colors. Below, we can see that there are orange fruits (circle B) such as persimmons and tangerines while apples and cherries (circle A) come in red colors. Peppers and tomatoes come in both red and orange colors, as represented by the overlapping area of the two circles.




## Explanation:

A Venn diagram is an illustration that uses circles to show the relationships among things or finite groups of things. Circles that overlap have a commonality while circles that do not overlap do not share those traits. Venn diagrams help to visually represent the similarities and differences between two concepts.

**Application of Venn diagram:**

**Possible Classroom Examples:**

A survey of 300 workers yielded the following information: 231 belonged to a union, and 195 were Democrats. If 172 of the union members were Democrats, how many workers were in the following situations?

a. Belonged to a union or were Democrats.
b. Belonged to a union but were not Democrats.
c. Were Democrats but did not belong to a union.
d. Neither belonged to a union nor were Democrats.

A recent transportation survey of w urban commuters (that is $n(U) = w$) yielded the following information: x rode neither trains nor busses, y rode trains, and z rode only trains. How many people rode the following?

a. Trains and buses.
b. Only buses.
c. Buses.
d. Trains or buses.

Given the sets.

U = { 0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

A = {0, 2, 4, 5, 9}

B = {1, 2, 7, 8, 9}

Use DeMorgan's laws to find:

a. $(A \cap B')'$
b. $(A \cup B')'$

A nonprofit organization's board of directors, composed of four women (Angela, Betty, Carmine, and Delores) and three men (Ed, Frank, and Grant), holds frequent meetings. A meeting can be held at Betty's house, at Delores's house, or at Frank's house.

- Delores cannot attend any meetings at Betty's house.
- Carmine cannot attend any meetings on Tuesday or on Friday.
- Angela cannot attend any meetings at Delores's house.
- Ed can attend only those meetings that Grant also attends.
- Frank can attend only those meetings that both Angela and Carmine attend.

If the meeting is held on Tuesday at Betty's, which of the following pairs can be among the board members who attend?

1. Angela and Frank

2. Ed and Betty

3. Carmine and Ed

4. Frank and Delores

5. Carmine and Angela

**b)**

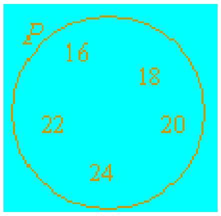
Given the set P is the set of even numbers between 15 and 25. Draw and label a Venn diagram to represent the set P and indicate all the elements of set P in the Venn diagram.

**Solution:**

List out the elements of *P*.

P = {16, 18, 20, 22, 24} ← 'between' does not include 15 and 25

Draw a circle or oval. Label it P . Put the elements in P.

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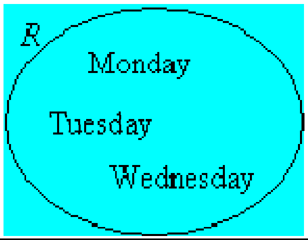
**c)**

Draw and label a Venn diagram to represent the set

*R* = {Monday, Tuesday, Wednesday}.

Solution:

Draw a circle or oval. Label it *R* . Put the elements in *R*.

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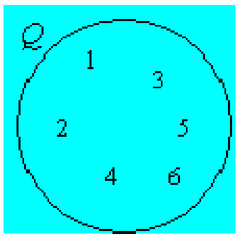
**d)**

Given the set $Q = \{x : 2x - 3 < 11, x \text{ is a positive integer } \}$. Draw and label a Venn diagram to represent the set $Q$.

## Solution:

Since an equation is given, we need to first solve for $x$.

$2x - 3 < 11 \Rightarrow 2x < 14 \Rightarrow x < 7$




So, $Q = \{1, 2, 3, 4, 5, 6\}$

Draw a circle or oval. Label it $Q$ .

Put the elements in $Q$.