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Paper # Applied Calculus

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Q# 1

(a) The function $g(t)$ is defined by

$g(t) = 0 \qquad t < 0$

$t^2 \qquad 0 \leq t \leq 3$

$2t + 3 \qquad 3 < t \leq 4$

$12 \qquad t > 4$

(a) State any point of discontinuity of the function is at $t = 0$ and 4 -

2 1

$\Rightarrow$ First at $t = 0$

$g(t) = t^2$

$g(0) = 0^2 = 0$

From R.H.L :-

$= \lim g(1+h) = \lim_{h\to 0} (1+h)^2$

$= \lim_{h\to 0} 1 + h^2 + 2h$

by Applying limits :-

$= 1 + 0^2 + 2(0)$

$= \underline{1}$

For L.H.L

$\lim_{h\to 0} g(1-h) = 2t+3$

For L.H.L

$= \lim_{h \to 0} 2(1-h) + 3$

$= \lim_{h \to 0} 2 - 2h + 3$

Apply limit

$= 2 - 2(0) + 3$

$= 5$

R.H.L $\neq$ L.H.L $=$ $g(t) = 5$

Now at $t = 4$

$g(4) = 2(4) + 3$

$= 8 + 3$

$= 11$

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(4)

For R·H·L

$$\lim_{h\to 0} g(1+h) = \lim_{h\to 0} 2(1+h)+3$$

$$= \lim_{h\to 0} 2+2h+3$$

by Applying limits

$$= 2+2(0)+3 = 5$$

For L·H·L

$$\lim_{h\to 0} g(1-h) = 12.$$

$g(4) =$ R·H·L $\neq$ L·H·L

point of discontinuity is at $t=4$

Q # 1 (b)

$$\lim_{t \to 3} g(t) = \lim_{t \to 3} (t)^2$$

= By Applying limit

$= (3)^2$

$= 9$

Q#2

$$y(x) = x^2 + \sin x$$

Solution By Maclaurin's series expansion we have

$$f(x) = f(0) + x f'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \cdots\cdots$$

Now

$$f(x) = y(x) = x^2 + \sin x$$

$$f'(x) = 2x + \cos x$$

$$f''(x) = x - \sin x$$

$$f'''(x) = -\cos x$$

Thus

$$f(0) = (0)^2 + \sin(0) = 0$$

$$f'(0) = 2(0) + \cos(0) = 1$$

$$f''(0) = 0 - \sin(0) = 0$$

$$f'''(0) = -\cos(0) = -1$$

Hence by Maclaurin's Expansion

$$f(x) = y(x) = 0 + x(1) + \frac{x^2(0)}{2!} + \frac{x^3(-1)}{3!}$$

$$= 0 + x + 0 - \frac{x^3}{3!} + \cdots\cdots$$

$$= x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$$

is the required Maclaurin's Expansion -

(8)

Q# 3 (i)

(i) Find y'' given

$1 + xy = x^2 + y^2$

Solution :- $1 + xy = x^2 + y^2 \rightarrow (1)$

diff eq (1) b/s w.r.t "x"

$= \frac{d}{dx}(1 + xy) = \frac{d}{dx}(x^2 + y^2)$

$= \frac{d}{dx}(1) + \frac{d}{dx}(xy) = \frac{d}{dx}x^2 + \frac{d}{dx}y^2$

$= 0 + (x \cdot \frac{d}{dx}y + y \cdot \frac{d}{dx}x)$

$= 2x + 2y \cdot \frac{dy}{dx}$

(9)

$= x \cdot \frac{d}{dx} + y(1) = 2 + 2y \cdot \frac{dy}{dx}$

$= x \cdot y' + y = 2x + 2y \cdot y'$

$= x \cdot y' - 2y \cdot y' = 2x - y$

$= y'(x - 2y) = (2x - y)$

$= y' = \frac{2x - y}{x - 2y}$

= Again diff w.r.t "x"

$= y'' = \frac{d}{dx}\left(\frac{2x - y}{x - 2y}\right)$

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$$y'' = \frac{d}{dx}\left(\frac{2x-y}{x-2y}\right)$$

$$= \frac{(x-2y)\frac{d}{dx}(2x-y) - (2x-y)\,{}^{d}/_{dx}(x-2y)}{(x-2y)^2}$$

$$= \frac{(x-2y)(2)\left(-\,{}^{dy}/_{dx}\right) - (2x-y)\left(1-2\,{}^{dy}/_{dx}\right)}{(x-2y)^2}$$

$$= \frac{(2x-4y)(-y') - \left(2x - 4x\,{}^{dy}/_{dx} - 4 + 2y\,y'\right)}{(x-2y)^2}$$

$$= y''(x-2y)^2 = 2xy' + 2yy' - 2x + y$$

$$= y''(x-2y)^2 = \frac{2x-y}{x-2y}(2x-2y) - 2x + y$$

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(11)

$$y'' = \frac{\frac{2x-y}{x-2y}(2x+2y) - 2x + y}{(x-2y)^2}$$ Ans.

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(12)

Q#3 (ii)

$$y = x^3(1+x)^9 e^{6x}$$

Taking ln on both sides

$$\ln y = \ln\left(x^3(1+x)^9 \cdot e^{6x}\right)$$

$$\ln y = \ln x^3 + \ln(1+x)^9 + \ln e^{6x}$$

Now diff w.r.t "x"

$$\frac{1}{y}\frac{dy}{dx} = 3\cdot\frac{1}{x} + 9\cdot\frac{1}{1+x}\frac{d}{dx}(1+x) + \frac{1}{e^{6x}}\frac{d}{dx}e^{6x}$$

$$\frac{dy}{dx} = y\left(\frac{3}{x} + \frac{9}{1+x} + 6(1)\right)$$

$$\frac{dy}{dx} = x^3(1+x)^9 \cdot e^{6x}\left(3/x + \frac{9}{1+x} + 6\right) \quad \text{Ans}$$