Assi Assignment

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Subject Dsp.

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Question no 1 (a)

Detemine the response $y(n)$ $n \geq 0$ ...

$$y(n) - 3y(n) - 4y(y-2) = x(n) + 2x(n-h)$$

to the put $x(n) = 4^n u(n)$.

Solution.

$$y_h(n) = C_1(-1)^n + C_2(4)^n$$

$$y_p(n) = K(4)^n u(n).$$

upon substitution we obtain..

$$Kn(4)^n u(n) - 3K(n-1)(4)^{n-1} u(n-1) - 4K(n-2)(4)^{n-2} u(n-2)$$

$$= (4)^n u(n) + 2(4)^{n-1} u(n-1).$$

To determine K we evaluate this equation for any $n \geq 2$. web select $n = 2$ from we obtain

$K = 6/5$ therefore.

$$y_p(n) = \frac{6}{3} n(4)^n u(n).$$

The total solution to the difference equation.

$$y(n) = C_1(-1)^n + C_2(4)^n + \frac{6}{3} n(4)^n \quad n \geq 0.$$

where the constants C_1 and C_2 are determined such that the intial condition are satisfied.

$$y(0) = 3y(-1) + 4y(-2) + 1$$

$$y(1) = 3y(0) + 4y(-1) + 6.$$

$$= 13y(-1) + 12y(-2) + 9.$$

We evaluate at $n=0$ and $n=1$

$y(0) = c_1 + c_2.$

$y(1) = -c_1 + 4c_2 + 24/5$

we can equate these two sets of relations to obtain c_1 and c_2.

we can simplify the computations above by setting $y(-1) = y(-2) = 0.$

then we have.

$c_1 + c_2 = 1$

$-c_1 + 4c_2 + \frac{24}{5} = 9$

hence $c_1 = -1/25$ and $c_2 = 26/25$

Finally we have there zero none state response to the forcing f/n $x(n) = (4)^n u(n)$

$y_{zs}(n) = -\frac{1}{25}(-1)^n + \frac{26}{25}(4)^n + \frac{6}{5} n(4)^n \geq 0.$

———— x ———— x ————

Q No 1 (B).

Solution:

$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$

$y(n) - 0.6(n-1) + 0.08y(n-2) = x(n)$

To obtain the homogenous equation.
Set input $x(n) = 0$.

$y(n) - 0.6y(n-1) + 0.08y(n-2) = 0$.

$y_h(n) = \lambda^n$

Subtitue the solution to the homogenous equation.

$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$.

$\lambda^{n-2}[\lambda^2 - 0.6\lambda + 0.08] = 0$.

$\lambda^2 - 0.6\lambda + 0.08\lambda = 0$.

$(\lambda - 0.2)(\lambda - 0.4) = 0$

$\lambda_1 = 0.2 \quad \lambda_2 = 0.4$.

$y_h(n) = c_1(\lambda_1)^n + c_2(\lambda_2)^n$

$y(n) = c_1(0.2)^n + c_2(0.4)^n \rightarrow (1)$

The particular Solution is

$y_p(n) = k(-1)^n u(n)$

$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4k(-1)^{n-2} u(n-1) = 0$

$= (-1)^n u(n) - (-1)^{n-1} u(n-1)$

For $n = 2$, $k(1+4+4) = 2$.

$k = 2/9$

The total solution is.

$y(n) = (c_1 \cdot 2^n + c_2 n 2^n + 2/9 (-1)^n] u(n)$

From intial condition we obtained.

$y(-1) = y(-2) = 0$. $\quad c_1 + 2/9 = 0$.

$\boxed{c_1 = -2/9}$

$2c_1 + 2c_2 - 2/9 = 0$.

$\boxed{c_2 = 1/3}$

——x——x——

Question No 2 (a).

Determine the casual signal $x(n)$

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Sol

$$\frac{X(z)}{z} = \frac{z^2}{(2z-1)(z-1)}$$

$$\frac{X(z)}{z} = \frac{A}{2z-1} + \frac{B}{z-1} + \frac{C}{(z-1)^2}$$

Find A, B and C.

$A = 4$

$B = 3$

$C = -1$

Hence $x(n) = [4(2)^n - 3 - n]u(n)$.

—— x —— x ——

Q NO 2 (b).

Determine the Partial fraction expansion of the props function.

$$X(z) = \frac{1}{1-1.5z^{-1}+0.5z^{-2}}.$$

Solution

First we eliminate nagativity powers by multiply both nomsots and denomecte by z^{+2} thus

$$X(z) \cdot \frac{z^2}{z^2-1.5z+0.5}$$

The poles are $P_3=1$ and $P_2 = 0.5$

$$\frac{X(z)}{(z)} = \frac{z}{(z-1)(z-0.5} = \frac{A_1}{z-1} + \frac{A_2}{z-0.5}.$$

A very simple method to determine A_1 and A_2 is multiply the equation by denomenator term $(z-1)(z-0.5)$.

Thus. $z = (z-0.5)A_1 + (z-1)A_2 \rightarrow ①$

Set : $z = P_1 = 1$ in eq ①.

$1 = 1 - 0.5A_1$

$A_1 = 2.$

Set $z = P_2 = 0.5$ in eq ①.

$0.5 = (0.5-1)A_2$

$A_2 = -1.$

We can determine the coefficient $A_1, A_2 --- A_N$ by multiplying both sides ① by each terms. $(z-P_K)$ $K = 1, 2, 3, 4 --- N.$

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Question No 3 (a).

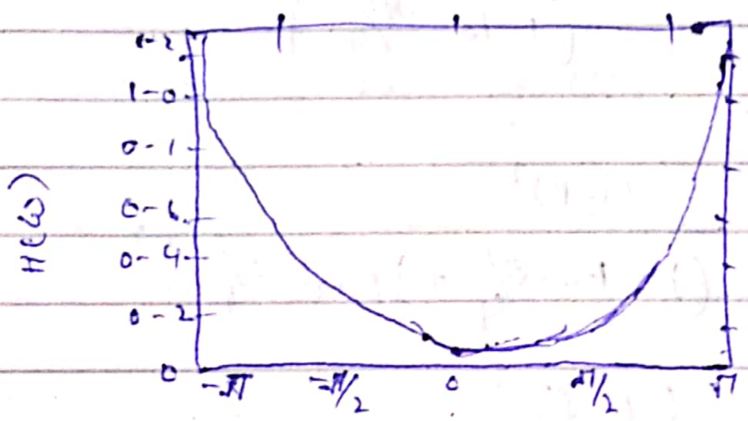
A two-pole low pass has the System response ... $H(z) = \frac{b_0}{(1-pz^{-1})^2}$ ...

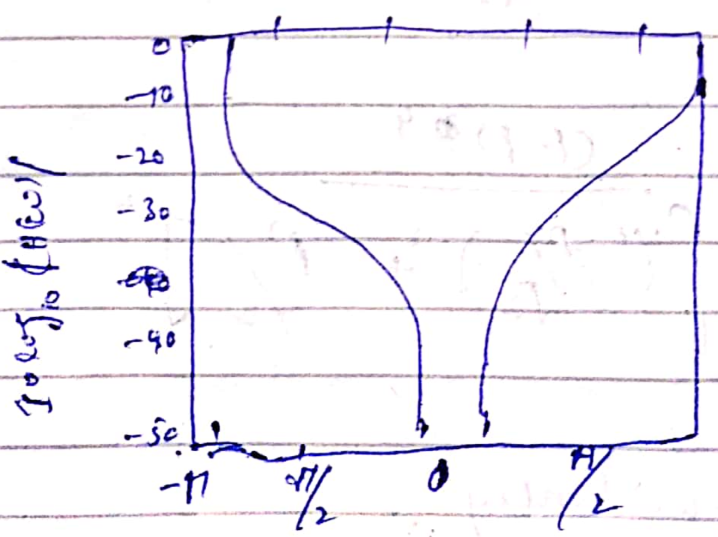
Sol.

At $w = 0$ we have

$$H_2(0) = \frac{b_0}{(1-p)^2} = 1$$

$$b_0 = (1-p)^2$$





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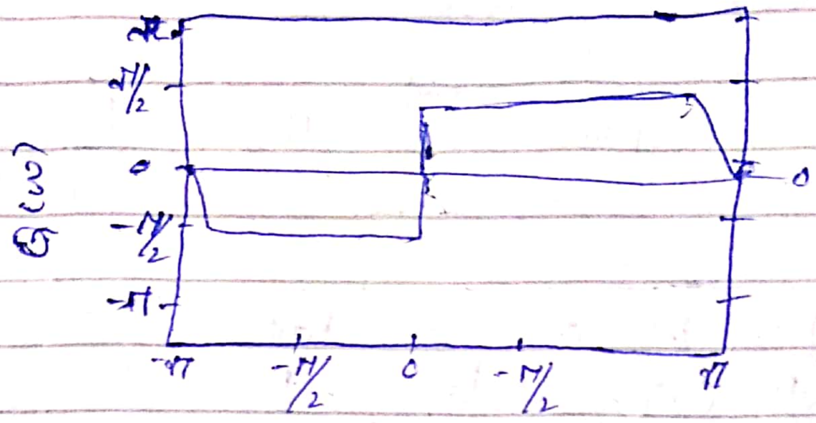


At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-P)^2}{\left(1 - Pe^{-j\pi/4}\right)^2}$$

$$= \frac{(1-P)^2}{\left(1 - P\cos\pi/4 + jP\sin\pi/4\right)^2}$$

$$= \frac{(1-P)^2}{\left(1 - P/\sqrt{2} + jP/\sqrt{2}\right)^2}$$

$$\text{Hence} = \frac{(1-P)^4}{\left[\left(1 - P/\sqrt{2}\right)^2 + P^2/2\right]^2} = 1/2$$

or equilentley

$$\sqrt{2}\,(1-P)^2 = 1 + P^2 - \sqrt{2}P$$

The value of P = 0.32 satisfy this equation. The system f/n for the desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

The same principle can be applied for Design of band pass filters.

x ——————————— x.

Question 3(b):

Design a Two-Pole band pass filter.

Solution:

clearly, The filter Must have poles at $P_{1,2} = r e^{\pm j\pi/2}$ and

zeros at $z=1$ and $z=-1$

consequently the system f/n is:

$$H(z) = \frac{G(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G \frac{(z^2-1)}{(z^2-r^2)}$$

The Gain factor is determined by evaluating the frequency response $H(\omega)$ of the filter at $\omega = \pi/2$. Thus we have,

$$H(\pi/2) = G \frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$. Thus we have.

$$\left[H\left(\frac{4\pi}{9}\right)\right]^2 = \frac{(1-r^2)^2}{4} \cdot \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

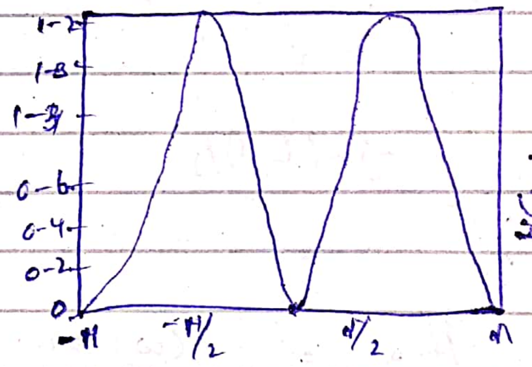
$$= \frac{1}{2}.$$

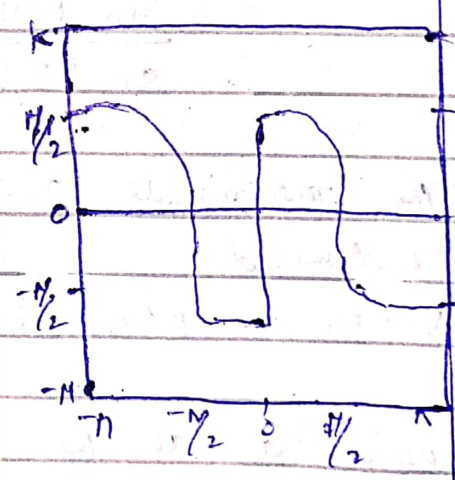
or $1.94(1-r^2)^2 = 1-1.08\, r^2 + r^4$

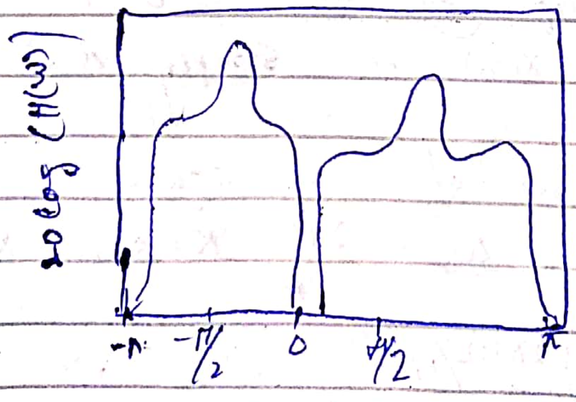
The value of $r^2 = 0.7$ satisfy this equation.

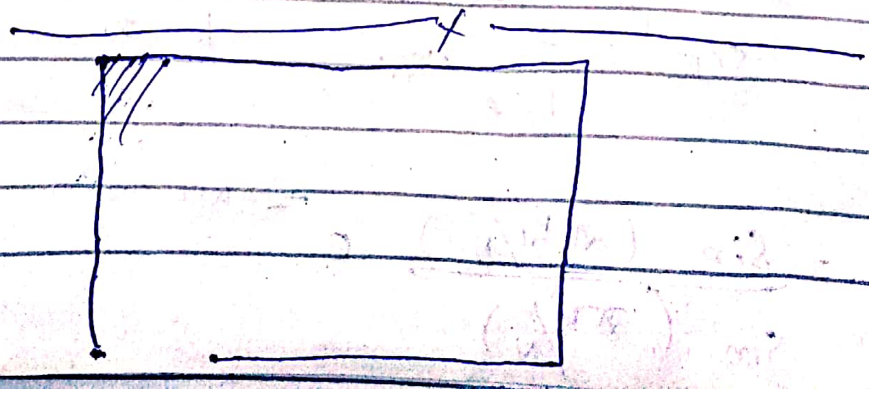
The system f/n for the design filter is.

$$H(z) = \frac{0.15\,(1-z^{-2})}{(1+0.7\,z^{-2})}$$









Question No 4 (a).

A finite duration Sequence of length L is given as.

$$x(n) = \begin{cases} 1 & 0 \le n \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

Determine N point DFT for this Sequence for $N \ge L$.

<u>Sol:-</u>

The fourier Transform of this Sequence is.

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}} \Rightarrow$$

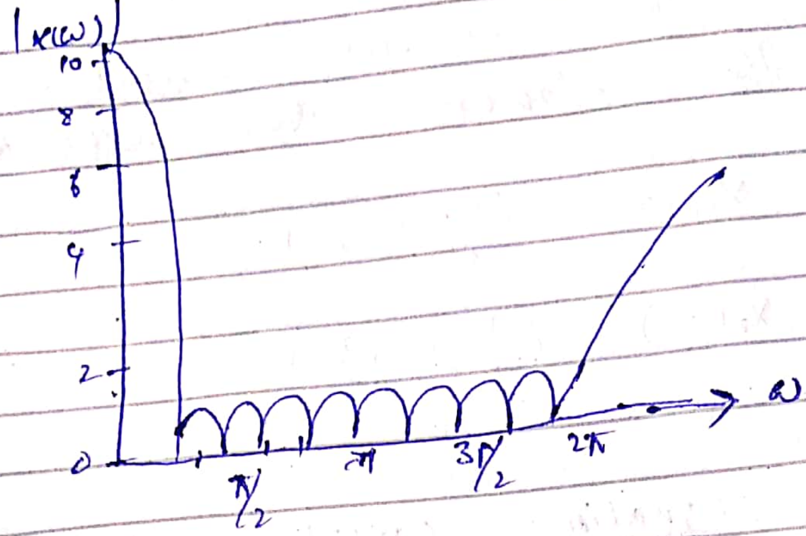
$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}.$$

The magnitude and Phase of $X(\omega)$ are illustrated in figure (1) for $L=0$, The N-point DFT of $x(n)$ is simply $x(\omega)$.

evaluated at the set of N equally spaced frequencies $\omega_k = \frac{2\pi k}{N}$ $K = 0, 1, 2 \dots N-1$

$$X(K) = \frac{1 - e^{-j2\pi KL/N}}{1 - e^{-j2\pi K/N}} \quad K = 0, 1, 2 \dots N-1$$

$$= \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}.$$



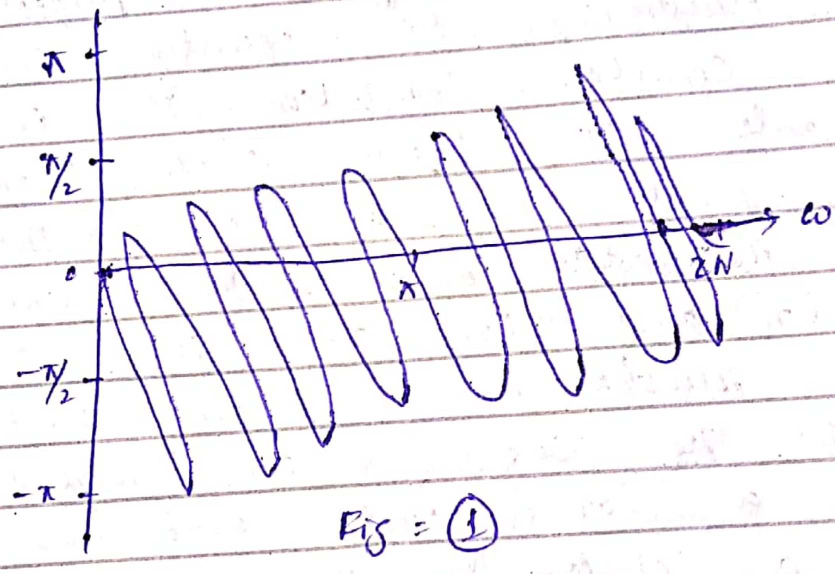


Fig = 1

If N is selected such that N = L then DFT become,

$$X(K) = \begin{cases} L & k = 0 \\ 0 & k = 1, 2, 3 \ldots L-1 \end{cases}$$

Thus there is only one non zero value in the DFT. This is apparent from observation of $X(\omega) = 0$ at frequencies $\omega_k = 2\pi k/L$, $k \neq 0$ then reader should verify that $x(n)$ can be ~~cat~~ recovered from $X(k)$ by performing an L-Point IDFT.

Question No 4. (d).

Compute the DFT of the four point sequence.

$x(n) = (0, 1, 2, 3)$.

Solution.

The first step to determine the matrix W_4. By exploiting the periodicity property of W_4 and the symmetry property.

$$W_N^{K+N/2} = -W_N^{K}$$

the Matrix W_4 may be expressed as.

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}.$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

then

$$X_4 = W_4 X_1 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

the IDFT of X_4 may be determined by conjugation the elements in W_4 to obtain W_4^* and then apply formula.