Introduction To Structural Dynamics And Earthquake Engineering

Submitted by

Kashif Yaqub

Reg.No. 7733

SECTION B

Submitted to

Engr. Yaseen Mahmood

IQRA NATIONAL UNIVERSITY
PESHAWAR

Question # 01

Given Data

- δ_{st} = Deflection due to 7733 lb static load
- Beam is pulled $\frac{1}{2}$" downwords
- $E = 29{,}000$ ksi
- $I = 150\ in^4$

Required Data

- Natural time period of system
- Develop and solve the equation of motion
- Draw graphs to show the variation of displacement with time and the variation of equivalent static force with time

Solution

General EOM for SDOF system is

$$ku + c\dot{u} + m\ddot{u} = p(t)$$

Since system is undamped $(c=0)$ undergoing free vibration $p(t)=0$

Hence general EOM becomes

$$ku + m\ddot{u} = 0 \quad \text{--- (1)}$$

where

$$k = \frac{3EI}{L^3} \Rightarrow \frac{3 \times 29000 \ k/in^2 \times 150 in^4}{(10' \times 12 in)^3}$$

$$k = 7.55208 \ \frac{k}{in}$$

→ In order to eliminate chances of mistake during calculation, it is most appropriate to use fundamental units like lb, ft, sec or kg, m, sec.

So

$$k = 7.55208 \ \frac{k}{in}$$

$$= 7.55208 \times 1000 \times 12$$

$$k = 90625 \ \frac{lb}{ft}$$

Now

$$m = \frac{W}{g}$$

$$m = \frac{7733}{32.2}$$

$$m = 240.15 \text{ slug}$$

→ $\omega_n = \sqrt{\frac{k}{m}} \Rightarrow \sqrt{\frac{90625}{240.15}}$

$$\omega_n = 19.425 \text{ rad/sec}$$

→ $T_n = \frac{2\pi}{\omega_n} \Rightarrow \frac{2\pi}{19.425}$

$$T_n = 0.323 \text{ sec}$$

put in eq ① the values of m and k

$$ku + m\ddot{u} = 0$$

$$90625\, u + 240.15\, \ddot{u} = 0$$

where k is in lb/ft &

m is in lb sec/ft^2

⇒ General solution to EOM for undamped free vibration is

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{v}(0)}{\omega_n} \sin(\omega_n t)$$

$$u(0) = \frac{1}{2''} = \frac{1}{24} \text{ ft} \quad \& \quad \ddot{u}(0) = 0$$

So,

$$u(t) = \left(\frac{1}{24}\right) \times \cos(19.425t) + 0$$

$$= \frac{1}{24} \times \cos(19.425t)$$

Equivalent Static Force at any time "t" is

$$f_s(t) = k \cdot u(t)$$

$$= \frac{90625 \times \cos(19.425t)}{24}$$

$$f_s(t) = 3776 \cos(19.425t)$$

⇒ Amplitude of dynamic displacement, u_0 for undamped free vibration is

$$u_0 = \sqrt{[(u(0))^2 + \left(\frac{\dot{u}(0)}{\omega_n}\right)^2}$$

$$= \sqrt{\left(\frac{1}{24}\right)^2 + 0}$$

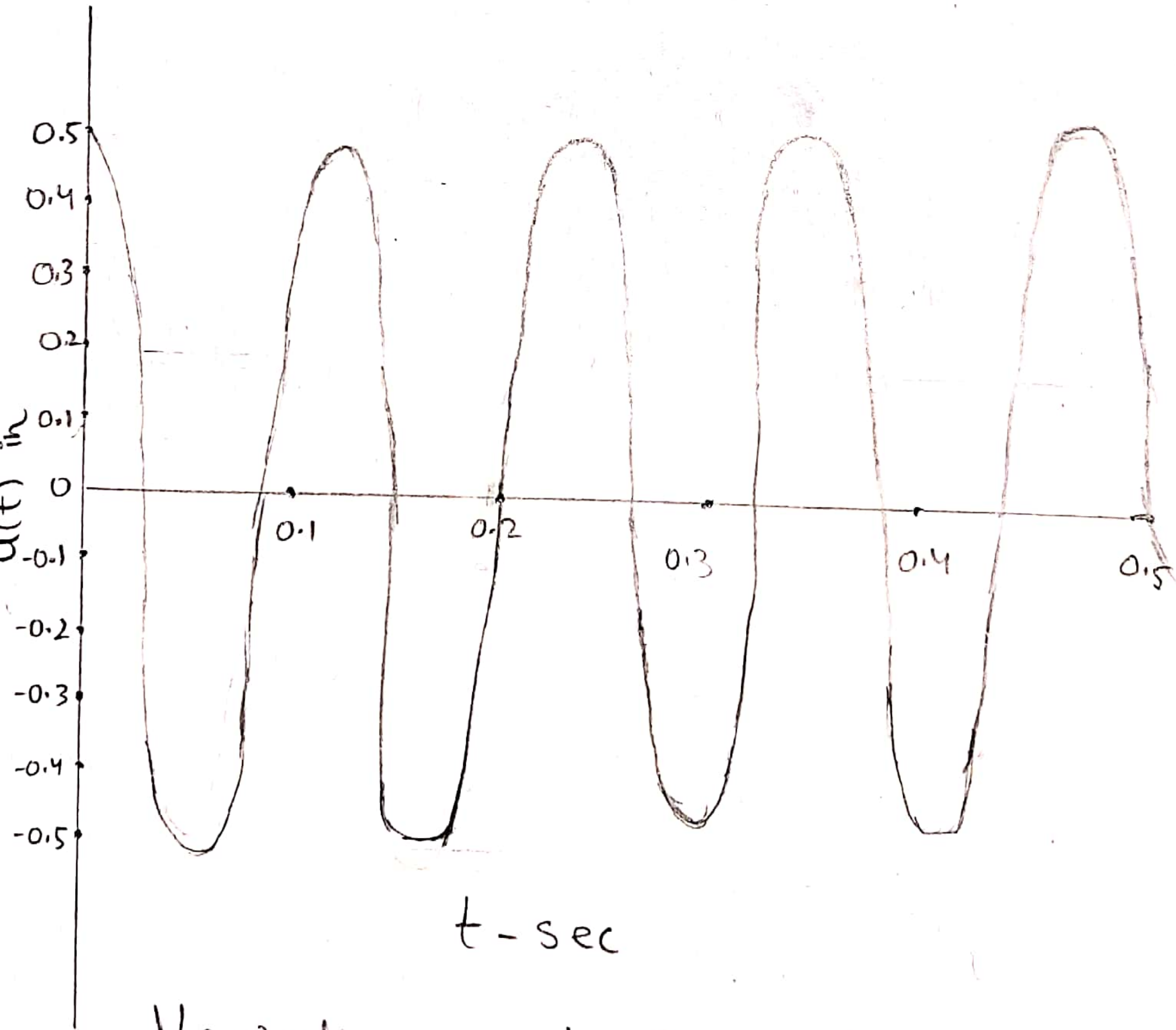
Scanned by CamScanner

$u_o = \frac{1}{24}$ ft

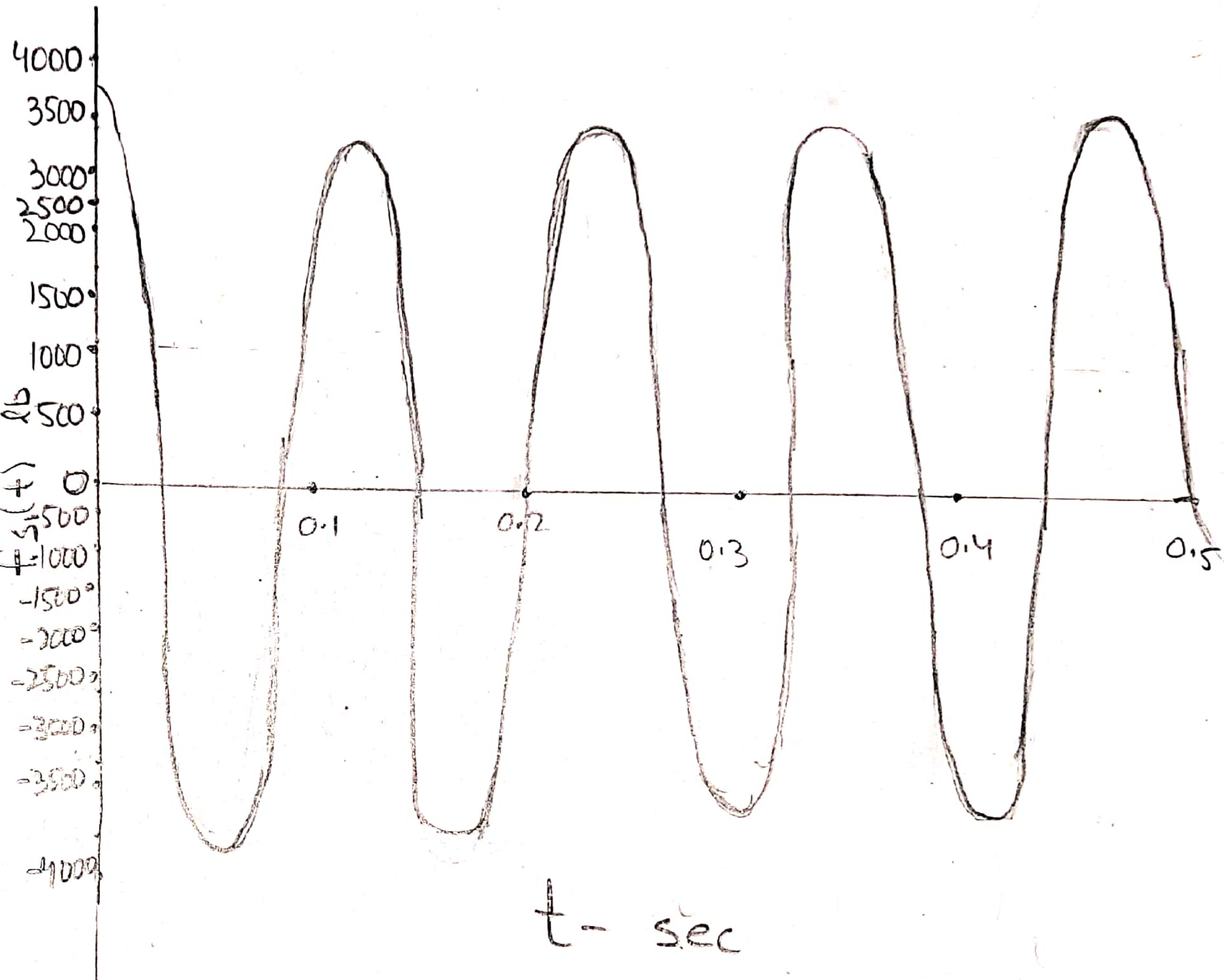
$\Rightarrow$ Amplitude of equivalent static force, f_{so}

$$k u_o = 90625 \times \frac{1}{24}$$

$$k u_o = 3776 \text{ lb}$$



Variation of displacement with time



Variation Of Equivalent Static Forces with time.

Scanned by CamScanner

Question # 02

Given Data

- Using data of beam given in question # 01
- ξ (Damping ratio) of reinforced concrete with considerable cracking = 3-5%
  we consider 3%

Required Data

- Develop and solve the equation showing variation in Equivalent Static Force with time
- Draw graph to show the variation of displacement with time and the variation of equivalent static force with time.

Solution

⇒ EOM for damped free vibration is

$$ku + cu + m\ddot{u} = 0 \quad \text{—} \ (1)$$

From question # 01

$k = 90625 \ \frac{lb}{ft}$ $\quad m = 240.15 \ lb \ \frac{sec^2}{ft}$

and $\omega_n = 19.425 \ rad/sec$

$$C = \zeta \times 2 m \omega_n$$

$$C = (0.03) \times 2(240.15)(19.425)$$

$$C = 279.89 \ lb.sec/ft$$

put the values of c, k & m in eq (1)

$$90625u + 279.89u + 240.15u = 0$$

⇒ Solution to the EOM for damped free vibration is

$$u(t) = e^{-\xi \omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{v}(0) + u(0)\xi\omega_n\right] \times \sin\omega_D t\right]$$

$$\omega_D = 19.425 \ \text{rad/sec}$$

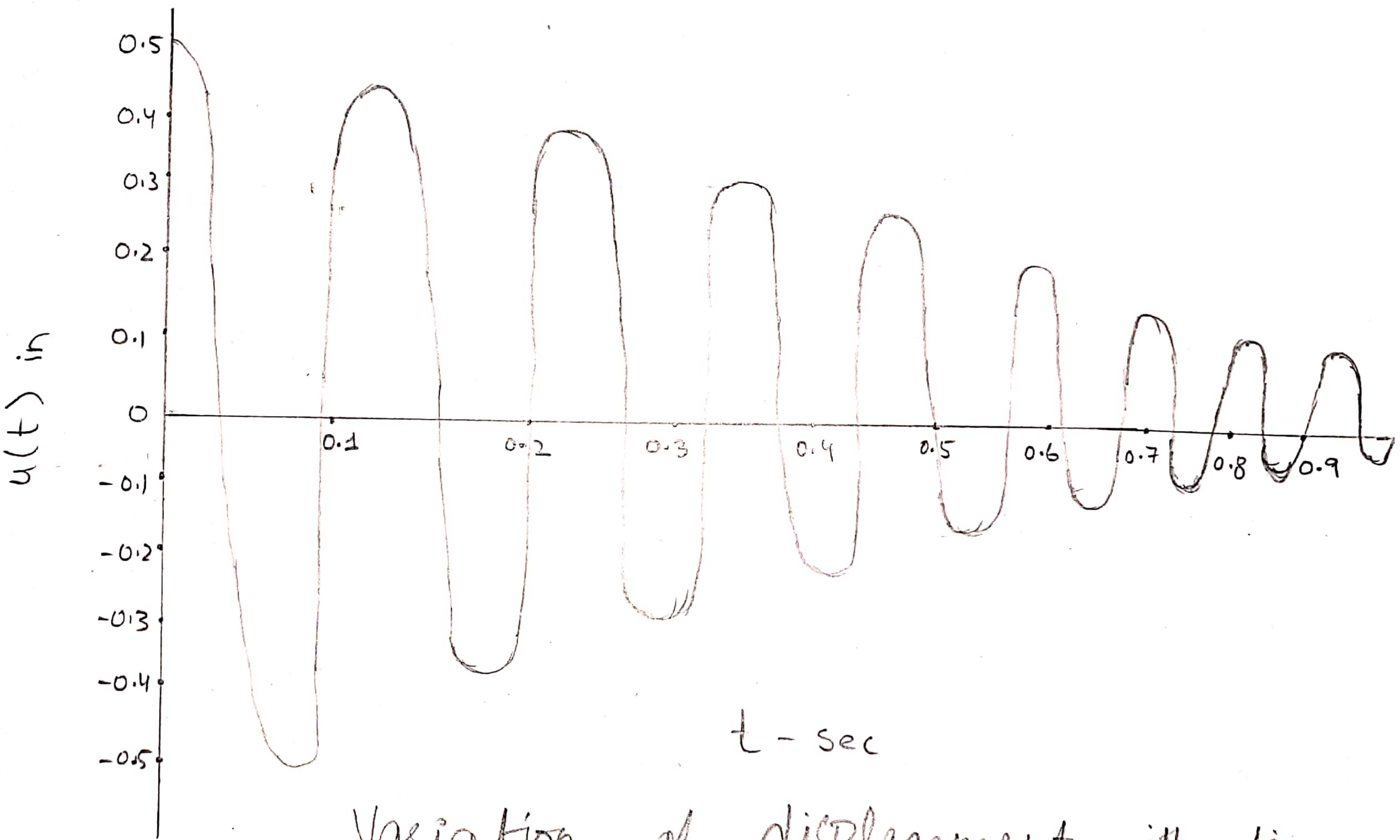
$$u(t) = e^{-0.03 \times 19.425 t}\left[\frac{1}{24} \times \cos(19.425 t + \frac{1}{19.425} \times \left[0 + \frac{1}{24} \times 0.03 \times 19.425\right]\sin(19.425t)\right]$$

$$u(t) = e^{-0.582t}\left[0.041 \times \cos(19.425 t) + 0.00125 \times \sin(19.425 t)\right]$$

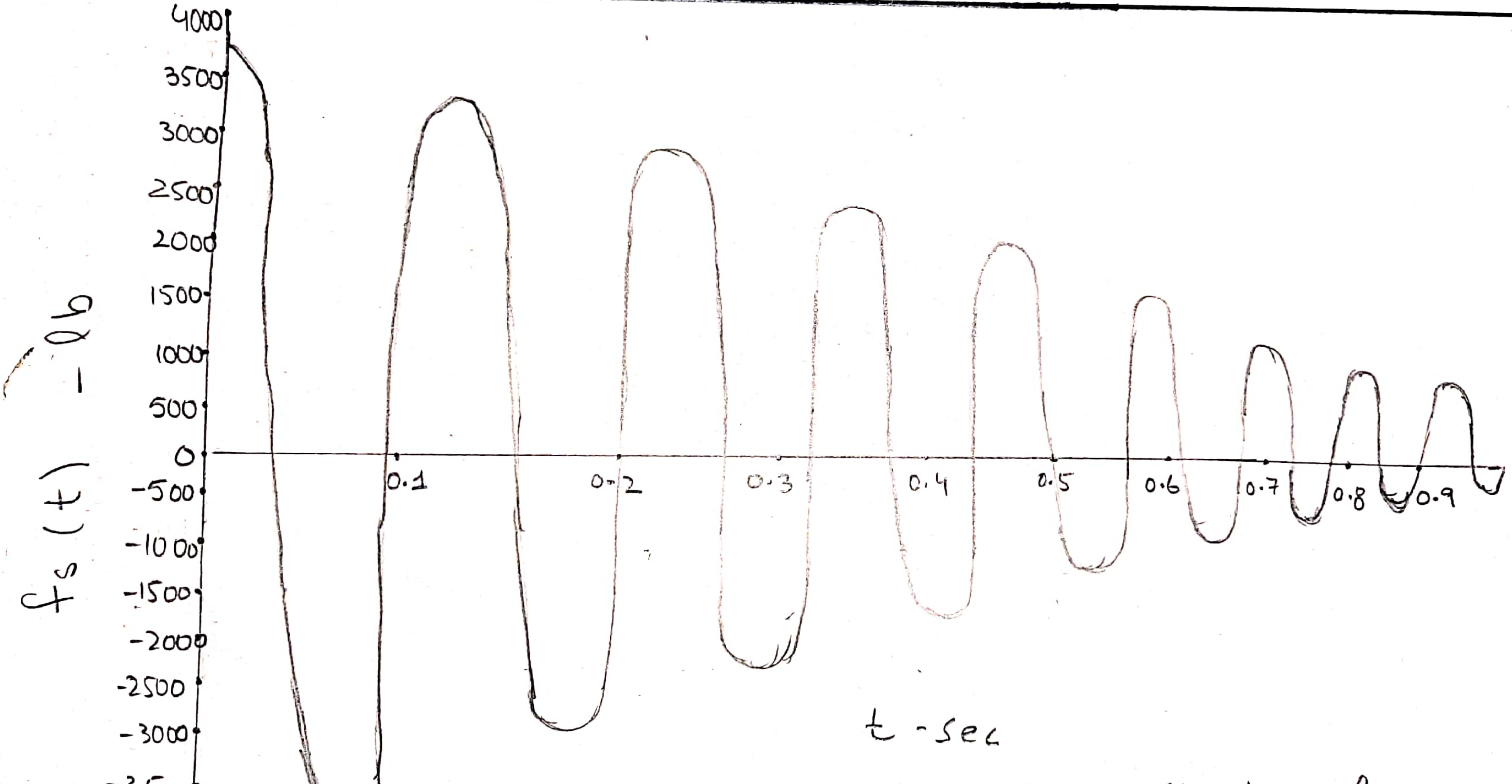
$$f_s(t) = k \cdot u(t) \Rightarrow 90625 \times u(t)$$

$$f_s(t) = e^{-0.582t}\left[(90625 \times 0.041)\cos(19.425 t) + (90625 \times 0.00125)\sin(19.425 t)\right]$$

$$f_s(t) = e^{-0.582t}\left[3715.62\cos(19.425t) + 113.28 \sin(19.425t)\right]$$



Variation of displacement with time



Variation of Equivalent Static Forces with time

Question # 03

Given Data

- Force = 60 kips
- Displacement of tank $= \left(\frac{ID}{1000}\right)''$

$$= \left(\frac{7733}{1000}\right)''$$

$$= 7.733''$$

- Time take to complete 7 cycles = 3.57 sec
- Amplitude of displacement = 2.286 cm

$$= 0.9''$$

Required Data.

a Damping ratio, ξ
b Natural period of undamped vibration
c Stiffness of structures
d Weight of tank
e Damping Co-efficient

b Number of cycles to reduce the displacement amplitude to 0.5"

Solution

Displacement of tank, $u_1 = 7.733"$

After 7 cycles; i.e after $j = 7$,

$$u_{j+1} = u_8 = 0.9"$$

a) Damping Ration, ξ

Damping ratio is given/calculated as;

$$j = \frac{1}{2\pi\xi} \ln\left[\frac{u_1}{u_{1+1}}\right]$$

$$7 = \frac{1}{2\pi\xi} \ln\left[\frac{7.733}{0.9}\right]$$

$$\xi = 0.0488 = 4.88\%$$

b) Natural Period of undamped Vibration, $T_n = ?$

As the 7 cycles of vibrations are completed in 3.57 sec

Time required to complete one cycle, $T_D = \frac{3.57}{7} = 0.51 \text{ sec}$

Now,

$$\omega_D = \omega_n \sqrt{(1-\xi^2)}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{(\omega_n \sqrt{1-\xi^2})}$$

$$\Rightarrow T_D = \frac{T_n}{(1-\xi)^2}$$

$$\Rightarrow T_n = T_D \times \sqrt{1-\xi^2}$$

$$T_n = 0.51 \times \sqrt{1-(0.0488)^2}$$

$$T_n = 0.5094$$

$$T_n = 0.51 \text{ sec}$$

So,
The natural period of undamped vibration, $T_n = 0.51\,sec$

c) Stiffness of Structure, k

$$k = \frac{60 \times \cos 60}{7.687} = 3.91\, k/in$$

$$k = 46920\, lb/ft$$

d) Weight of Tank

Weight of tank, w is calculated as

$$W_n = \sqrt{\frac{k}{m}} \Rightarrow \sqrt{\frac{k}{w/g}} = \sqrt{\frac{k \cdot g}{w}}$$

$$\Rightarrow W_n^2 = \frac{k \times g}{W}$$

$$W = \frac{k \times g}{W_n^2}$$

Also

$$W_n = \frac{2\pi}{T_n}$$

$$W = \frac{k \times g}{\left(4\pi^2 / T_n^2\right)}$$

$$W = k \times g \times \frac{T_n^2}{4\pi^2}$$

$$W = \left[\frac{46920\,lb}{ft} \times \frac{32.2\,ft}{sec^2}\right] \times \frac{(0.51\,sec)^2}{4\pi^2}$$

$$W = 9953.93\,lb$$

or

$$W = 9.95\,k$$

e) Damping Co-efficient, c

It is known that

$$\zeta = \frac{c}{2m\omega_n}$$

$$c = \zeta \times 2m \times \omega_n$$

$$C = \xi \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$C = \frac{(0.0488) \times 4 \times \pi \times \left(\frac{9953.93}{32.2}\right)}{0.51}$$

$$C = 371.71 \text{ lb-sec/ft}$$

b) Number of cycles to reduce the displacement amplitude to 0.5", j

$$j = \frac{1}{2\pi\xi} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$j = \frac{1}{2 \times \pi \times 0.0488} \ln\left[\frac{7.733}{0.5}\right]$$

$$j = 8.93 \text{ or } 9 \text{ cycles.}$$