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Section

B

ID

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Subject

Differential equ.

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Q 1.50

$$x^3 y''' + 2x^2 y' + 2y = 10x + \frac{10}{x}$$

Sol:

There we need some correction because $2x^2 y'$ are not same we should try to solve for $2x^2 y''$

$$x^3 y''' + 2x^2 y'' + 2y = 10x + 10/x$$

$$x^3 \frac{d^3 y}{dx^3} + 2x^2 \frac{d^2 y}{dx^2} + 2y = 10x + 10x^{-1}$$

$$(x^3 D^3 + 2x^2 D + 2)y = 10x + 10x^{-1} \rightarrow \textcircled{1}$$

$$\text{let } x = e^t \Rightarrow t = \ln x$$

$$xD = D$$

$$x^2 D^2 = D(D-1) = D^2 - D$$

$$x^3 D^3 = D(D-1)(D-2) = D^3 - 3D^2 + 2D$$

substituting in eq $\textcircled{1}$

$$(D^3 - 3D^2 + 2D + D^2 - D)y = 10x + 10x^{-1}$$

$$(D^3 - D^2 + 2)y = 10x + 10x^{-1}$$

$$(m^3 - m^2 + 2)y = \frac{10e^t}{e^t} + \frac{10}{e^t}$$

(2)

using synthetic division

$$\begin{array}{r|rrr}
 & 1 & -1 & 02 \\
 -1 & & -1 & 2-2 \\
 \hline
 & 1 & -2 & 2 \underline{0}
 \end{array}$$

now using quadratic equation

$$a = 1, b = -2, c = 2$$

$$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Delta = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$$

$$\Delta = \frac{2 \pm \sqrt{4 - 8}}{2}$$

$$\Delta = \frac{2 \pm \sqrt{-4}}{2} \Rightarrow \frac{2 \pm \sqrt{(-2)^2}}{2}$$

$$\Delta = \frac{2 \pm 2i}{2} = \frac{x(1+i)}{x}$$

$$\frac{x(1-i)}{x}$$

$$y_c = e^{-x} (c_1 \cos x + 2 \sin x)$$

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Now for particular solution

$$y_p = \frac{1}{D^3 - D^2 + 2} \cdot 10e^t + \frac{1}{D^3 - D^2 + 2} \cdot 10e^{-1}$$

$$= \frac{10e^t}{(1)^3 - (1)^2 + 2} + \frac{10e^{-1}}{(1)^3 - (1)^2 + 2}$$

$$= 5e^t + 5e^{-1}$$

General sol:

$$y = y_c + y_p$$

$$y = e^{-x}(c_1 \cos t + c_2 \sin t) + 5e^t + 5e^{-1}$$

Put $e^t = x$ & $t = \ln x$.

$$y = e^{-x}(c_1 \ln x + c_2 \sin x) + 5e^x + 5e^{-x}$$

$\frac{dx}{dt}$

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Q22
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$$x^3 y''' + 4x^2 y'' - 5xy' - 15y = x^4.$$

$$x^3 \frac{d^3 y}{dx^3} + 4x^2 \frac{d^2 y}{dx^2} - 5x \frac{dy}{dx} - 15y = x^4.$$

$$\left(x^3 \frac{d^3}{dx^3} + 4x^2 \frac{d^2}{dx^2} - 5x \frac{d}{dx} - 15 \right) y = x^4.$$

$$(x^3 D^3 + 4x^2 D^2 - 5x D - 15) y = x^4 \rightarrow \textcircled{1}$$

$$\text{let } x = et \Rightarrow t = \ln x.$$

$$xD = \Delta.$$

$$x^2 D^2 = \Delta(\Delta - 1) = \Delta^2 - \Delta.$$

$$x^3 D^3 = \Delta(\Delta - 1)(\Delta - 2) = \Delta^3 - 3\Delta^2 + 2\Delta.$$

Substituting into eq $\textcircled{1}$.

$$\Delta^3 - 3\Delta^2 + 2\Delta + 4\Delta^2 - 4\Delta - 5\Delta - 15 = x^4.$$

$$\Rightarrow \Delta^3 + \Delta^2 - 7\Delta - 15 = e^{4t}.$$

Complementary set: y_c
solve by synthetic eq.

3	1	1	-7	-15
		3	12	15
	1	4	+5	0

(5)

$$\Delta^2 + 4\Delta + 5 = 0$$

$$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2}$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{2} \Rightarrow \frac{-4 \pm 2i}{2}$$

$$\Delta = \frac{-2 \pm i}{1} \Rightarrow$$

$$y_c = e^{4t} (c_1 \cos t + c_2 \sin t)$$

Particular integral is

$$y_p = \frac{1}{F(\Delta)} e^{4t}$$

$$= \frac{1}{\Delta^3 + \Delta^2 - 7\Delta - 15} e^{4t}$$

$$= \frac{1}{4^3 + 4^2 - 7(4) - 15} e^{4t}$$

$$y_p = \frac{1}{37} e^{4t}$$

The general sol. is

$$y = y_c + y_p$$

$$y = e^{4t} (c_1 \cos t + c_2 \sin t) + \frac{1}{37} e^{4t}$$

Again put $t = \ln u$ & $u = \ln u$

$$y = e^{4u} (c_1 \cos \ln u + c_2 \sin \ln u) + \frac{1}{37} e^{4u}$$

Q3:

$$n^2 y'' + 2ny' - 6y = 10n^2$$

$$\text{sol} \rightarrow y(1) = 1 \quad \& \quad y'(1) = -6.$$

$$n^2 \frac{d^2 y}{dn^2} + 2n \frac{dy}{dn} - 6y = 10n^2$$

$$\left(n^2 \frac{d^2}{dn^2} + 2n \frac{d}{dn} - 6 \right) y = 10n^2$$

$$\text{Put } nD \equiv \Delta$$

$$n^2 D^2 = \Delta(\Delta - 1) = \Delta^2 - \Delta.$$

$$n = e^t \quad \& \quad \log n = t.$$

$$(\Delta^2 - \Delta + 2\Delta - 6) y = 10e^{2t}.$$

$$(\Delta^2 + \Delta - 6) y = 10e^{2t}$$

The characteristic equation

$$\Delta^2 + \Delta - 6 = 0$$

$$\Delta^2 + 3\Delta - 2\Delta - 6 = 0$$

$$\Delta(\Delta + 3) - 2(\Delta + 3) = 0$$

$$(\Delta + 3)(\Delta - 2)$$

$$\Delta = -3, \quad \Delta = 2.$$

$$\Delta = 2.$$

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Since Roots are ^{Real.} general & distinct

For $y_c = ?$

$$y_c = c_1 e^{-3t} + c_2 e^{2t}$$

For $y_p = ?$

$$y_p = \frac{I}{\Delta^2 - \Delta - 6} \quad 10e^{2t}$$

$$= \frac{10 e^{2t}}{\Delta^2 - \Delta - 6}$$

$$= \frac{10 \cdot 1}{0} e^{2t} \rightarrow \text{fail.}$$

now

$$10 \frac{1}{\frac{d}{dn} (\Delta^2 + \Delta - 6)} e^{2t}$$

$$y_p = 2te^{2t}$$

General solution.

$$y = y_c + y_p$$

$$= c_1 e^{-3t} + c_2 e^{2t} + 2te^{2t}$$

$$y = c_1 n^{-3} + c_2 n^2 + 2(\log n) n^2 \rightarrow B$$

Put $y(1) = 1$ i.e. $n=1, y=1$ in eq (B)

$$1 = c_1 + c_2 \rightarrow (C)$$

now differentiate eq B wrt x .

$$y' = -3c_1 x^{-4} + 2c_2 x + \frac{2}{x} (x^2 + 4x \log x)$$

$$-6 = -3c_1 + 2c_2 + 2 + 0$$

$$-8 = -3c_1 + 2c_2 \rightarrow \text{---}$$

Multiplying eq (2) with (2) & subtracting from (1).

$$2c_1 + 2c_2 = 2$$

$$-8c_1 + 2c_2 = -8$$

$$\hline$$

$$5c_1 = 10$$

$$\boxed{c_1 = 2}$$

$$-8 = -3(2) + 2c_2$$

$$\boxed{c_2 = -1}$$

now put the value of c_1 & c_2 in eq (B)

$$y = 2x^{-3} - x^2 + 2(\ln x \cdot x(x^2))$$

$$y = \frac{2}{x^3} - x^2 + 2x^2 \log x$$

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Q. 60

$$x^2 y'' + 7xy' + 5y = x^5$$

Sol. 60

$$\rightarrow x^2 \frac{d^2 y}{dx^2} + 7x \frac{dy}{dx} + 5y = x^5$$

$$\left(x^2 \frac{d^2}{dx^2} + 7x \frac{d}{dx} + 5 \right) y = x^5$$

$$\left(x^2 D^2 + 7xD + 5 \right) y = x^5 \rightarrow \textcircled{1}$$

$$\text{let } x = et \Rightarrow t = \ln x$$

$$xD = D$$

$$x^2 D^2 = D(D-1) = D^2 - D$$

Substituting into eq. ①

$$D^2 - D + 7D + 5 = 0$$

$$\Rightarrow (D^2 + 6D + 5) y = e^{5t}$$

Complementary sol: y_c

$$D^2 + D + 5D + 5 = 0$$

$$D(D+1) + 5(D+1)$$

$$(D+1)(D+5)$$

$$D = -1, D = -5$$

$$y_c = C_1 e^{-x} + C_2 e^{-5x}$$

$$y_c = C_1 e^{-x} + C_2 e^{-5x}$$

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Particular integral is

$$y_p = \frac{1}{D^2 + 6D + 5} e^{5t}$$

$$y_p = \frac{1}{25 + 30 + 5} e^{5t}$$

$$= \frac{1}{60} e^{5t}$$

General sol

$$y = c_1 e^t + c_2 e^{-5t} + \frac{1}{60} e^{5t}$$

Replace $x = e^t \Rightarrow t = \ln x$

$$y = c_1 e^{\ln x} + c_2 e^{-5 \ln x} + \frac{1}{60} e^{5 \ln x} \rightarrow \textcircled{B}$$

$x=0$ Put in this eqn

no in eq (B) $e^0 = 1$

Put $y(0) = 2$ i.e. $y = 2$ & $x = 2$

$$2 = c_1 (2)^5 + c_2 (2)^{-5} + \frac{1}{60} (2)$$

$$2 = -32c_1 - 2c_2 + \frac{1}{60} (381)$$

$$2 = -32c_1 - 2c_2 + 8/15$$

$$\frac{22}{15} = -32c_1 - 2c_2 \rightarrow \textcircled{C}$$

now differentiate eq (B) wrt (x)

$$y' = -5c_1 x^{-6} - c_2 x^{-2} + \frac{1}{12} x^4 \rightarrow$$

Put $y'(2) = 2$ i.e. $y' = 2$ at $x = 2$ in above eqn

$$2 = -5c_1 (2)^{-6} - c_2 (2)^{-2} + \frac{1}{12} (2)^4$$

$$2 = -5c_1 (-64) - c_2 (4) + \frac{1}{12} (16)$$

$$2 = 320c_1 + 4c_2 + \frac{4}{3}$$

$$\frac{2}{3} = 320c_1 + 4c_2 \rightarrow \textcircled{D}$$

ming eq (C) with 2 and then in eq (D)
From (D)

$$-\frac{44}{15} = 64c_1 + 4c_2$$

$$-\frac{44}{15} = 64c_1 + 4c_1$$

$$\frac{34}{15} = -256c_1$$

$$c_1 = \frac{34}{15} \times 256$$

$$\boxed{c_1 = 580}$$

Put the value of c_1 in eq (4)

$$\frac{22}{15} = -32(580) - 2c_2$$

$$\frac{22}{15} + 18560 = -2c_2$$

$$c_2 = -9280$$

Now put the value of c_1 & c_2 in eq (B).

$$y = 580x^{-5} - 9280x^{-1} + \frac{1}{60}x^5 \quad \text{Ans}$$

Q. 58 $(x+1)^2 y'' - 3(x+1)y' + 4y = x^2$

let $z = (x+1) \Rightarrow x = z - 1$

$z = (x+1)^2 \Rightarrow z^2 = (x+1)^2$

$x = e^t \Rightarrow \ln x = t$

$\Rightarrow z^2 y'' - 3zy' + 4y = x^2$

$zy' = \Delta$

$zy = \Delta(\Delta-1) = \Delta^2 - \Delta$

~~$\Delta^2 - \Delta + 3\Delta$~~

$(\Delta^2 - \Delta - 3\Delta + 4)y = x^2$

$(\Delta^2 - 4\Delta + 4)y = x^2$

complementary set $y_c =$

$\Delta^2 - 2\Delta - 2\Delta + 4 = 0$

$\Delta(\Delta-2) - 2(\Delta-2)$

$(\Delta-2)(\Delta-2) = 0$

$\Delta = 2, 2$

$$y_c = (C_1 + C_2 n + \dots) e^{2n}$$

$$= (C_1 + C_2 n + \dots) e^{2x}$$

Particular Integral :-

$$y_p = \frac{1}{F(\Delta)} e^{2t}$$

$$= \frac{1}{\Delta^2 - 4\Delta + 4} e^{2t}$$

$$= \frac{1}{(2)^2 - 4(2) + 4} e^{2t}$$

$$\frac{1}{0} e^{2t}$$

$F(\Delta)$ should not equal to zero.

$$\Rightarrow \Delta^2 - 4\Delta + 4$$

$$2\Delta - 4 \rightarrow 0$$

If we put 2 in eq (2) again give zero.

take 2nd time derivatives.

$$= \frac{d}{d\Delta} (2\Delta - 4)$$

$$= 2 \Rightarrow y_p = \frac{1}{2} e^{2t}$$

$$\Rightarrow y = (C_1 + C_2 n) e^{2t} + \frac{1}{2} e^{2t}$$

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$$y_p = \frac{1}{2} e^{2t}$$

The general sol is

$$y = y_c + y_p$$

$$y = (c_1 + c_2 u + \dots) e^{2u} + \frac{1}{2} e^{2t} \overset{\text{Ans}}{\text{Ans}}$$

~~Replace $u = et \Rightarrow t = \ln u$~~

~~$y = (c_1 + c_2 et)$~~