

Name

Shohab Malouk

id

7878

Section

A

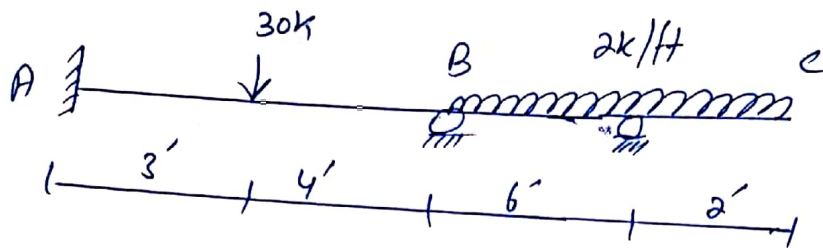
Subject

Structural Analysis II

Q No 1

Q1 ①

Analyze the beam shown in fig-1 by Stiffness method Assume EI is constant.



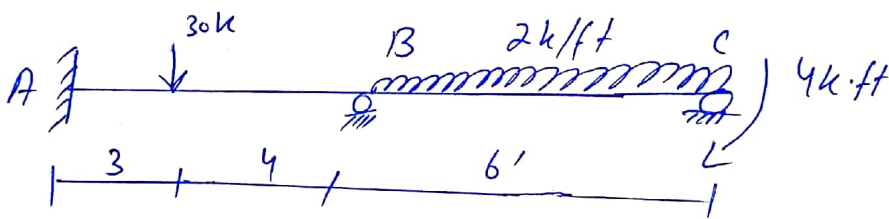
Sol:.

Step #1

Determining kinematic indeterminacy.

$$K \cdot I = 5^{\circ}$$

So we have to reduce the extended portion



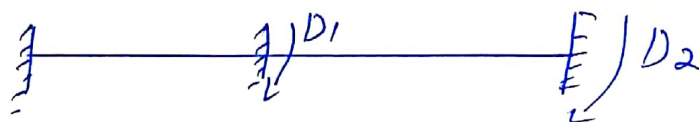
$$\Rightarrow \frac{2(2)}{1} = 4k \cdot ft.$$

Now

$$K \cdot I = -2^{\circ}$$

Step #2

Determine unknown joint displacement.



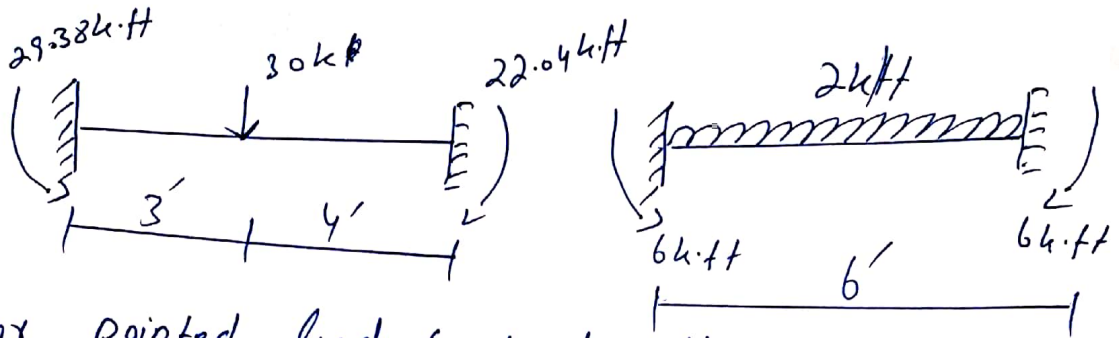
$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}$$

$$\begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$$

Step #3

Q1 (2)

Compute [ADL] matrix



=> for pointed load (not at mid)

=> for Left end:

$$\frac{Pab^2}{L^2} = \frac{(30)(3)^2(4)}{(7)^2} = 22.04 \text{ k.ft}$$

=> for UDL

$$\frac{wL^2}{12} \Rightarrow \frac{(2)(6)^2}{12} = 6 \text{ k.ft}$$

$$ADL_1 = +22.04 - 6 = 16.04 \text{ k.ft}$$

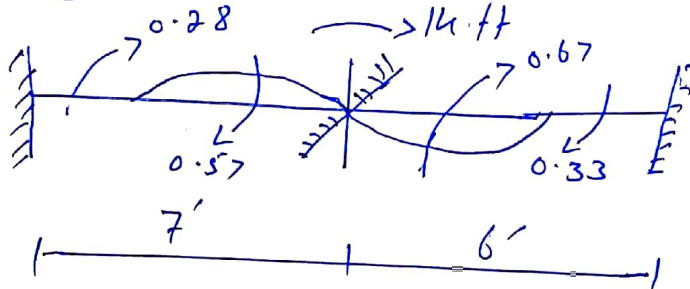
$$ADL_2 = 6 \text{ k.ft}$$

Step #4

compute [S] matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$$

a) $D_1 = 1 \text{ k}$, $D_2 = 0$



$$\begin{array}{c|c} \frac{4EI}{7} = 0.57 & \frac{2EI}{6} = 0.33 \\ \frac{4EI}{6} = 0.67 & \frac{2EI}{7} = 0.28 \end{array}$$

Q1 ③

$$S_{11} = 0.57 + 0.67$$

$$= 1.24EA$$

$$S_{21} = 0.33EA$$

b) $D_1 = 0$ $D_2 = 1k$



$$4EI = 0.67$$

$$2EI = 0.33$$

$$S_{12} = 0.33$$

$$S_{22} = 0.67$$

$$S = \begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix}$$

Step #5

compute $[D]$ matrix

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}^{-1} \times \begin{bmatrix} AD_1 \\ AD_2 \end{bmatrix} = \begin{bmatrix} ADL_1 \\ ADL_2 \end{bmatrix}$$

$$\begin{bmatrix} 1.24 & 0.33 \\ 0.33 & 0.67 \end{bmatrix} \times \text{Adj } A \times \begin{bmatrix} \\ \end{bmatrix}$$

Q1 (4)

$$|S| = (1.24 \times 0.67) - (0.33 \times 0.33) \\ = 0.8308 - 0.1089$$

$$|S| = 0.7219$$

$$\text{Adj } A = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix}$$

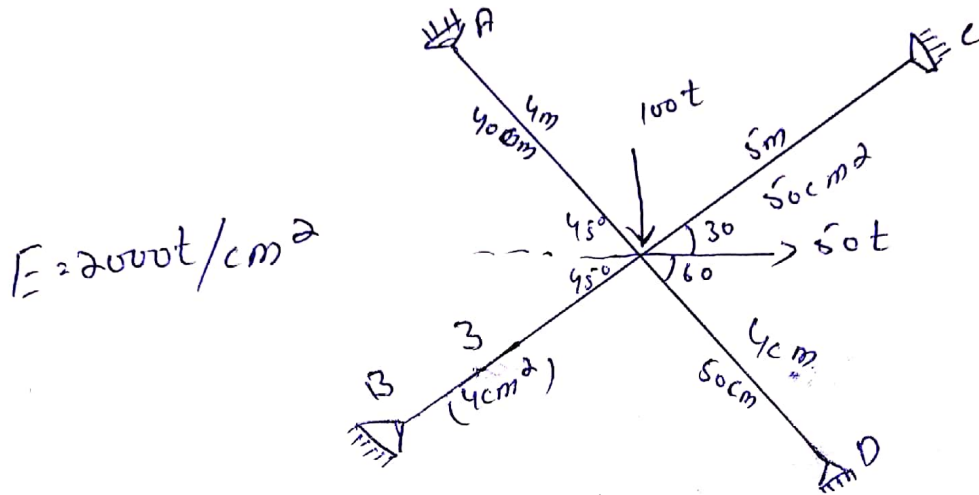
Now

$$\begin{bmatrix} AD_1 - ADL_1 \\ AD_2 - ADL_2 \end{bmatrix} = \begin{bmatrix} 0 - 16.04 \\ 4 - 6 \end{bmatrix} = \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}^R$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.67 & -0.33 \\ -0.33 & 1.24 \end{bmatrix} \times \begin{bmatrix} -16.04 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} -10.08 \\ 2.81 \end{bmatrix}$$

Q2 Analyze the pin-jointed frame shown by Stiffness method Length of the members in m and cross-sectional area of the member in cm^2 are shown in Fig-3 Take $E = 2000 \text{ t/cm}^2$



Sol:- for A:-

$$\sin 45^\circ = \frac{P}{h} = \frac{P}{4}$$

$$\Rightarrow P = 2.828 \text{ m}$$

$$\cos 45^\circ = \frac{b}{4}$$

$$\Rightarrow b = 2.828 \text{ m}$$

for B:-

$$\sin 45^\circ = \frac{P}{3}$$

$$\Rightarrow P = 2.12 \text{ m}$$

$$\cos 45^\circ = \frac{b}{3}$$

$$\Rightarrow b = 2.12 \text{ m}$$

for C :-

$$\sin 30^\circ = \frac{P}{h} = \frac{P}{5}$$

$$\Rightarrow P = 2.5 \text{ m}$$

$$\cos 30^\circ = b/5$$

$$\Rightarrow b = 4.33 \text{ m}$$

$$\text{Now } EA(A) = 2000 \times 40 = 80,000 \text{ t}$$

$$EA(B) = 2000 \times 40 = 80,000 \text{ t}$$

$$EA(C) = 2000 \times 50 = 100,000 \text{ t}$$

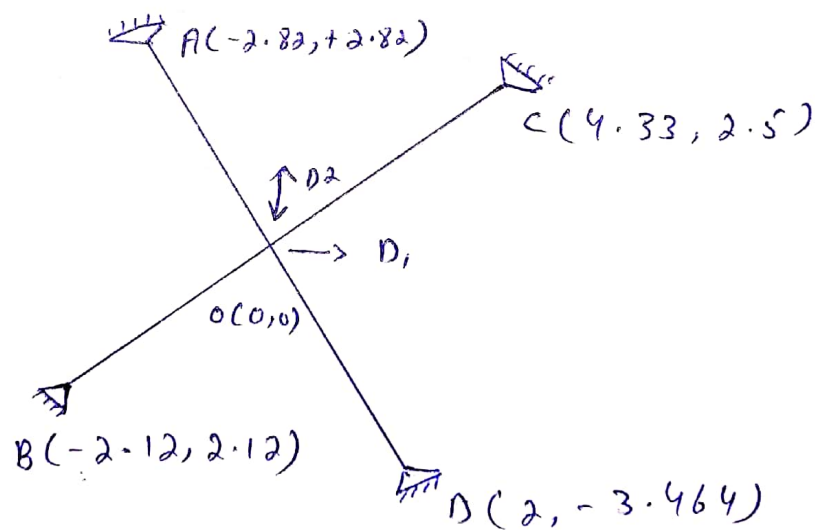
$$EA(D) = 2000 \times 50 = 100,000 \text{ t}$$

Step # 01 $K \cdot I$

$$K \cdot I = 2j - \gamma$$

$$= 2(5) - 8 = 2^\circ$$

Step # 02 Select unknown joint displacement



$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \end{bmatrix}, \begin{bmatrix} AD_1 \\ AD_1 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

Step # 03 ⁽³⁾ $[AMP]_{4 \times 2}$ & $[S]_{2 \times 2}$

i) $D_1 = 1$, $D_2 = 0$

$$AMD = \frac{EA}{L^2} (\kappa_L - \kappa_i)$$

$$AMD_{11} = \frac{80000}{(400)^2} \times (0 + 282) = 141$$

$$AMD_{21} = \frac{80,000}{(300)^2} \times (0 + 212) = 188.44$$

$$AMD_{31} = \frac{100,000}{(500)^2} \times (0 - 433) = 173.2$$

$$AMD_{41} = \frac{100,000}{(400)^2} \times (0 - 200) = -125$$

Now $S_{11} = \sum_{i=1}^m \frac{EA}{L^3} (\kappa_i - \kappa_j)^2$

$$= \frac{80,000}{4} \times (282)^2 + \frac{80,000}{(300)^3} \times (212)^2 + \frac{100,000}{(500)^3} \times (-433)^2 + \frac{100,000}{(400)^3} \times (-200)^2$$

$$S_{11} = 99.405 + 133.167 + 149.991 + 82.5$$

$$S_{11} = 445.063$$

$$S_{12} = S_{21} = \sum_{i=1}^m \frac{EA}{L^3} \times (\kappa_i - \kappa_j) (\gamma_i - \gamma_j)$$

$$= \frac{80,000}{(400)^3} \times (282)(-282) + \frac{80,000}{(300)^3} \times (212)(212) + \frac{100,000}{(500)^3} \times (-433)(0 - 250) + \frac{100,000}{(400)^3} \times (-200)(0 + 346)$$

Q2 (4)

$$S_{12} = S_{21} = 12.237$$

ii) $D_1 = 0$, $D_1 = 1k'$

$$AMD = \frac{EA}{L^2} (y_k - y_j)$$

$$AMD_{12} = \frac{80,000}{400^2} (-282) = -141$$

$$AMD_{22} = \frac{80,000}{300^2} (212) = 188.44$$

$$AMD_{32} = \frac{100,000}{500^2} (-250) = -100$$

$$AMD_{42} = \frac{100,000}{400^2} (346) = 216.25$$

$$\text{Now } S_{22} = \sum_{2 \times 1}^m \frac{EA}{L^3} (y_k - y_j)^2$$

$$= \frac{80,000}{400^3} (-282)^2 + \frac{80,000}{300^3} (212)^2 + \frac{100,000}{500^3} (-250)^2 + \frac{100,000}{400^3} (346)^2$$

$$S_{22} = 469.628$$

Step #04

Q25

$$[D] = [S]^{-2} \times [AD]$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 445.003 & 12.237 \\ 12.237 & 469.628 \end{bmatrix}^{-1} \times \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \end{bmatrix} = \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

Step #05 [AM]

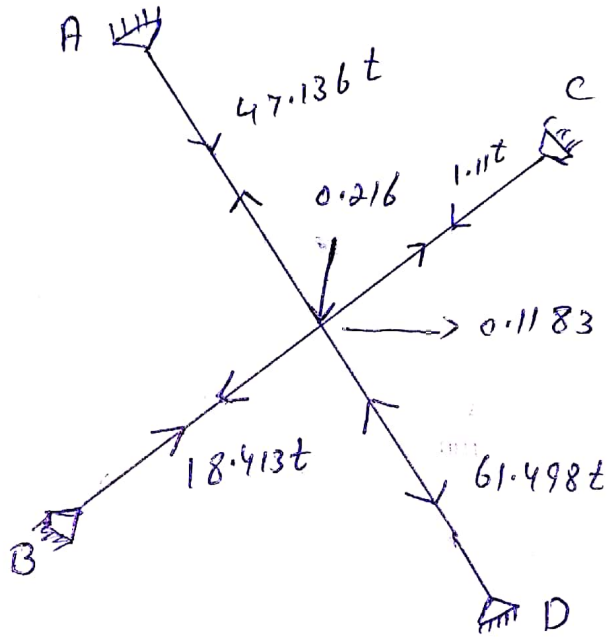
$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 141 & -141 \\ 188.44 & 188.44 \\ -173.2 & -100 \\ -125 & 216.25 \end{bmatrix} \times \begin{bmatrix} 0.1183 \\ -0.216 \end{bmatrix}$$

$$\begin{bmatrix} 141 \times 0.1183 + (-141) \times (-0.216) \\ 188.44 \times 0.1183 + 188.44 \times (-0.216) \\ -173.2 \times 0.1183 + (-100) \times (-0.216) \\ -125 \times 0.1183 + 216.25 \times (0.216) \end{bmatrix}$$

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 16.68 + 30.46 \\ 22.29 - 40.70 \\ -20.49 + 21.6 \\ -14.79 - 46.71 \end{bmatrix}$$

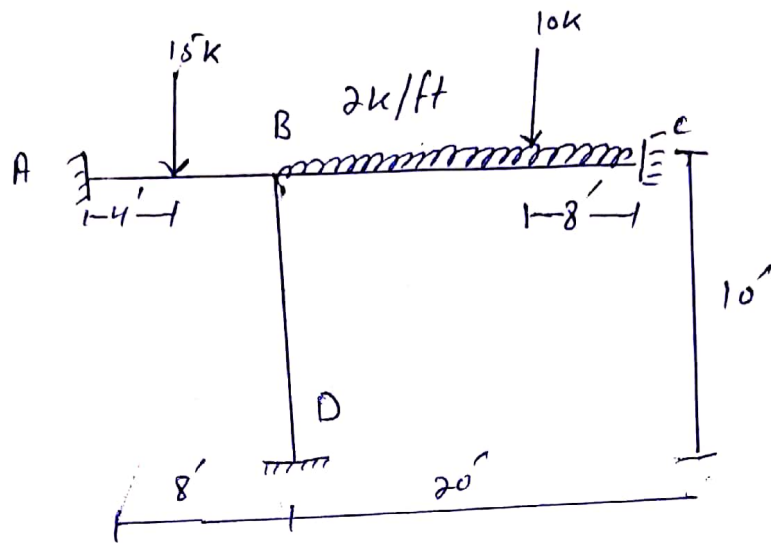
02 (6)

$$\begin{bmatrix} AM_1 \\ AM_2 \\ AM_3 \\ AM_4 \end{bmatrix} = \begin{bmatrix} 47.136t \\ -18.413t \\ 1.11t \\ -61.498t \end{bmatrix}$$



Q No 3:-

Analyze the rigid-joint frame shown in Fig 2 by stiffness method. Assume EI is constant.



Sol:-

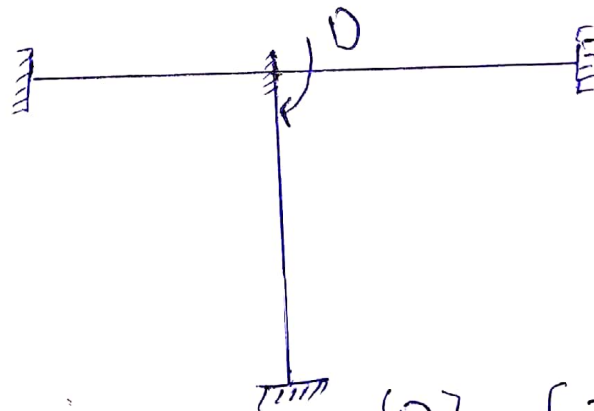
Step #1

Determine kinematic indeterminacy.

$$k \cdot I = 1^{\circ}$$

Step #2

Determine unknown joint displacement.

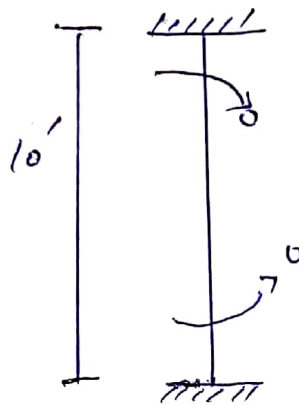
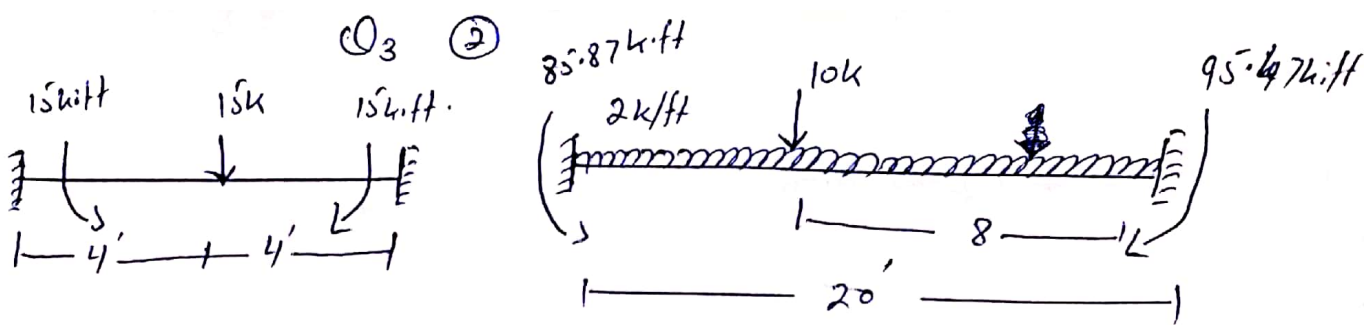


$$[D] = [?]$$

$$[AD] = [0]$$

Step #3 :-

Compute $[ADL]$ matrix.



=> point load at center :-

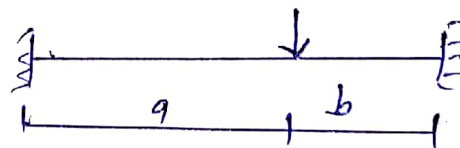
$$\frac{PL}{8} = \frac{(15)(8)}{8} = 15 \text{ kip}\cdot\text{ft}$$

=> uniformly distributed load

$$\frac{WL^2}{12} \Rightarrow \frac{(2)(20)^2}{12} = 66.67 \text{ k}\cdot\text{ft}$$

=> point load (not at mid)

Suppose.



for left end

$$\frac{Pab^2}{L^2} \Rightarrow \frac{(10)(12)(8)^2}{(20)^2} = 19.2 \text{ k}\cdot\text{ft}$$

for Right End

$$\frac{Pa^2b}{L^2} = \frac{(10)(12)^2(8)}{(20)^2} = 28.8 \text{ k}\cdot\text{ft}$$

Q3 ②

So Total moment at left end

$$19.2 + 66.67 = 85.87 \text{ k}\cdot\text{ft}$$

Similarly at right end

$$28.8 + 66.67 = 95.47 \text{ k}\cdot\text{ft}$$

$$\text{So } [AD] = -85.87 + 15 = -70.87 \text{ k}\cdot\text{ft}$$

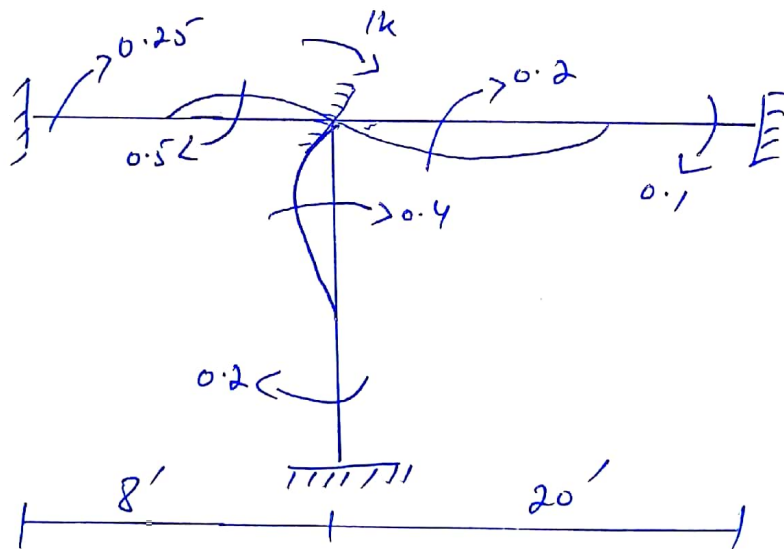
Step # 4

Determine $[S]$ matrix

$$[S] = [S_{ij}]$$

Now,

$$D = 1 \text{ k}$$



$$\Rightarrow \frac{4EI}{8} = 0.5 \qquad \frac{2EI}{8} = 0.25$$

$$\Rightarrow \frac{4EI}{20} = 0.2 \qquad \frac{2EI}{20} = 0.1$$

$$\Rightarrow \frac{4EI}{10} = 0.4 \qquad \frac{2EI}{10} = 0.2$$

Q3 (4)

$$[S] = (0.5 + 0.4 + 0.2) EI \\ = 1.1 EI$$

$$[S] = 1.1 EI$$

Step #5

compute (D) matrix

$$[D] = [S]^{-1} \times [AD] - [ADL]$$

$$[D] = \frac{1}{1.1} \times [0] - [-70.87]$$

$$= \frac{70.87}{1.1}$$

$$[D] = [64.42] \frac{1}{EI}$$