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ID 7962
Section B
Dep Be (civil)
Paper Differential equation

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Question No 1

Part (A)

Sol

$w = \sin(x+ct) + \cos(2x+2ct)$

$$\boxed{\frac{\partial w}{\partial t^2} = \frac{c^2 \partial w}{\partial x^2}}$$

$$\frac{\partial w}{\partial t} = \cos(x+ct) + c - \sin(2x+2ct) + 2c$$

$$\frac{\partial^2 w}{\partial t^2} = -\sin(x+ct) + c^2 - \cos(2x+2ct) + 4c^2 \quad (1)$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct) + 2$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(\cancel{2x}\,2x+2ct)$$

$$= \left[-\sin(x+ct) - 4\cos(x+2ct)\right]$$

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$$\frac{\partial w^2}{\partial t^2} = +c^2\left[-\sin(x+ct) - 4(\cos(2x+2ct))\right]$$

$c^2 \cdot \partial^2 w / \partial x^2$

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Q No 1

Part B

$w = \tan(2x+ct)$

Sol

$$\frac{\partial w^2}{\partial t^2} = \cancel{c}\, 2\sec(2x+ct)\, 2\,\frac{\sec(2x+t)}{\partial t}$$

$$= 2c^2 \sec(2x+ct)\sec(2x+\cancel{ct})\tan(2x+ct)$$

$$\frac{\partial^2 w}{\partial t^2} = 2c^2\sec^2(2x+ct)\tan(2x+ct)$$

$$\frac{\partial w}{\partial x} = 2\sec^2(\cancel{2x}\,2x+ct)$$

$$\frac{\partial^2 w}{\partial x^2} = 2\cdot 2\sec(2x+ct)\cdot\sec(2x+ct)\tan(2x+ct)\cdot 2$$

$$= 8\sec^2(2x+ct)\tan(2x+ct)$$

$$2c^2\sec^2(2x+ct)\tan(2x+ct) \neq c^2 8\sec^2(2x+ct)\tan(2x+c)$$

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So it is not solved more

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Q No 2

Given function is

~~F(x)~~ {

$F(x) = x, \quad -\pi < x < 0$
$\quad\;\; = 2x, \quad 0 \le x \le \pi$

Sol

we have to find the fourier co-efficients, a_0, a_n and b_n

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + \frac{1}{\pi}\int_{0}^{\pi} 2x\,dx$$

$$= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_{0}^{\pi}$$

$$= \frac{1}{\pi}\left[0 - \frac{\pi^2}{2}\right] + \frac{2}{\pi}\left[\frac{\pi^2}{2} - 0\right]$$

$$\boxed{a_0 = \frac{-\pi}{2} + \pi = \pi/2 \quad \rightarrow ①}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\int_{-\pi}^{0}(x\cos nx\,dx) + \frac{1}{\pi}\int_{0}^{\pi}(2x\cos nx)\,dx$$

$$= \frac{1}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(\frac{-\cos nx}{n^2}\right)\right]_{-\pi}^{0}$$

$$+ \frac{2}{\pi}\left[x\left(\frac{\sin nx}{n}\right) - \left(-\frac{\cos nx}{n^2}\right)\right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi}\left\{\frac{\cos(0)}{n^2} - \frac{\cos n\pi}{n^2}\right] + \frac{2}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos(0)}{n^2}\right]$$

$$= \frac{1}{\pi}\cdot\left[\frac{1-(-1)^n + 2(-1)^n - 2}{n^2}\right] = \frac{(-1)^n - 1}{\pi n^2}$$

So

$$\boxed{a_n = \begin{cases} \dfrac{-2}{\pi n^2} & \text{if } n \text{ is add} \\ 0 & \text{if } n \text{ is even} \end{cases}}$$

Q NO 3

Solve the initial value problem

$y'' - 4y' + 13y = 8\sin 3x$

$y(0) = 1$ and $y'(0) = 2$

Sol

Associated Homogenous equation.

Put $y = m$ in (2)

$m^2 - 4m + 13 = 0$

Use Quadratic formula

$a = 1, \quad b = -4, \quad c = 13$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$m = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(13)}}{2(1)}$$

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$$= \frac{4 \pm \sqrt{16 - 52}}{2}$$

$$= \frac{4 \pm \sqrt{-36}}{2}$$

$$= \frac{4 \pm \sqrt{36i}}{2}$$

$$= \frac{4 \pm 6i}{2}$$

$$2 \pm 3i$$

$$m_1 = 2 + 3i$$
$$m_2 = 2 - 3i$$

$$y_c = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) \rightarrow (A)$$

Let

$$y_p = A\cos 3x + B\sin 3x \rightarrow (x)$$

Diff = w.r.t to x

$$y'_p = -3A\sin 3x + 3B\cos 3x$$

Again Diff w.r.t to x

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$y''_p = 9A\cos3x - 9B\sin3x$

Put ①

$\Rightarrow (-9A\cos3x - 9B\sin3x) - 4(-3A\sin3x + 3B\cos3x)$
$+ 13(A\cos3x + B\sin3x) = 8\sin3x$

$\Rightarrow -9A\cos3x - 12B\cos3x + 13A\cos3x - 9B\sin3x$
$+ 12A\sin3x + 13B\sin3x = 8\sin3x$

$\Rightarrow (-9A - 12B + 13A)\cos3x + (-9B + 12A + 13B) \Rightarrow$
$\sin3x = 8\sin3x$

$\Rightarrow (4A - 12B)\cos3x + (4B + 12A)\sin3x = 8\sin3x$

comparing co-efficient.

$\sin3x \Rightarrow \quad 4B + 12A = 8 \rightarrow (a)$
$\cos3x \Rightarrow 4A - 12B = 0 \Rightarrow 4A = 12B$

$$\boxed{\Rightarrow A = 3B \rightarrow (b)}$$

Put (b) in (a)

$4B + 12(3B) = 8$
$4B + 36B = 8$
$40B = 8$

$$\boxed{B = 1/5 \Rightarrow (i)}$$

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Put (a) in (b)

$\Rightarrow \boxed{A = 3/5} \Rightarrow$ (a) (b) (d)

$y_p = \frac{3}{5}\cos 3x + \frac{1}{5}\sin 3x \rightarrow$ (B)

The Gr. sol is

$y = y_c + y_p$

$y = e^{2x}(c_1\cos 3x + c_2\sin 3x) + \frac{3}{5}\cos 3x$
$+ \frac{1}{5}\sin 3x \rightarrow$ (C)

Now we need to find the values of c_1 and c_2 for this

Put $x = 0$ and $y = 1$ in (C)

$1 = (c_1(1) + c_2(0)) + \frac{3}{5}(1) + \frac{1}{5}(0)$

$1 = c_1 + 3/5$

$c_1 = 1 - 3/5$

$\boxed{c_1 = 2/5} \rightarrow **$

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Diff (C) w.r.to "x^2"

$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x) + c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x)$

$-\frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x \rightarrow$ (D)

Put $y' = 2,\ x = 0$ in (D)

$y' = c_1(2e^{2x}\cos 3x - 3e^{2x}\sin 3x)$
$+ c_2(2e^{2x}\sin 3x + 3e^{2x}\cos 3x)$
$-\frac{6}{5}\sin 3x + \frac{3}{5}\cos 3x$

Put $y' = 2,\ x = 0$

$2 = c_1(2e^{2(0)}\cos 3(0) - 3e^{2(0)}\sin 3(0))$
$+ c_2(2e^{2(0)}\sin 3(0)\, 3e^{2(0)}\cos 3(0))$
$-\frac{6}{5}\sin 3(0) + \frac{3}{5}\cos 3(0)$

$2 = c_1(2) + c_2(3) - 0 + \frac{3}{5}$

$2 = 2c_1 + 3c_2 + 3/5$

Put $c_1 \quad 2/5$

$2 = 4/5 + 3c_2 + 3/5$

$2 = 7/5 + 3c_2$

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$3c_1 = 2 - \frac{7}{5}$

$3c_2 = 3/5$

$\boxed{c_2 = 3/15 \rightarrow \cancel{***}}$

put (**) and (***) in (C)

$y = e^{2x}(2/5 \cos 3x + 3/15 \sin 3x) + 3/5 \cos 3x + 1/5 \sin 3x$

$\boxed{y = 2/5\, e^{2x} \cos 3x + 3/15\, e^{2x} \sin x + 3/5 \cos 3x + 1/5 \sin 3x}$

Required general solution.

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Q No 4

$(D^2 - DD')z = \cos x \cos 2y$

Sol

It is already in symobolic form.

$(D^2 - DD')z \cos x \cos 2y \rightarrow (a)$

put A.E $D^2 - DD' = 0$

As we know

$\frac{D}{D'} = m$ i.e $D = m, D' = 1$

$\Rightarrow m^2 - m = 0$

$m = 0, 1$

There for $C.F = f_1(y) + f_2(y+x)$

From eq (a)

$$P.I = \frac{1}{D^2 - DD'} \cos x \cos 2y$$

$$= \frac{1}{2} \cdot \frac{1}{D^2 - DD'} \cos x \cos 2y$$

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

$$C.F = \phi_1(y-x) + x\, \phi_2(y-x)$$

$$P.I = \frac{1}{D^2 + 2DD' + D'^2} \left[2(y-x) + \sin(x-y)\right]$$

$$= \frac{1}{(D+D')^2} \left[2(y-x) + \sin(x-y)\right]$$

By General Method.

$m = -1 \; ; \; y - x = c$

$$= \frac{1}{D+D'} \int \left[2c + \sin(-c)\right] dx$$

$$= \frac{1}{D+D'} \left[\cancel{2x}\; 2cx - (\sin c)x\right]$$

Replacing c by $y-x$

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$$\int (2xc - x\sin c)\,dx = cx^2 - \frac{x^2}{2}\sin c$$

Replacing c by y-x

$$x^2(y-x) - \frac{x^2}{2}\sin(y-x) = x^2y - x^3 + \frac{x^2}{2}\sin(x-y)$$

Hence the required solution is.

$$z = C.F + P.I = F_1(y-x) + xF_2(y-x) + x^2y - x^3 + \tfrac{1}{2}x^2\sin(x-y)$$