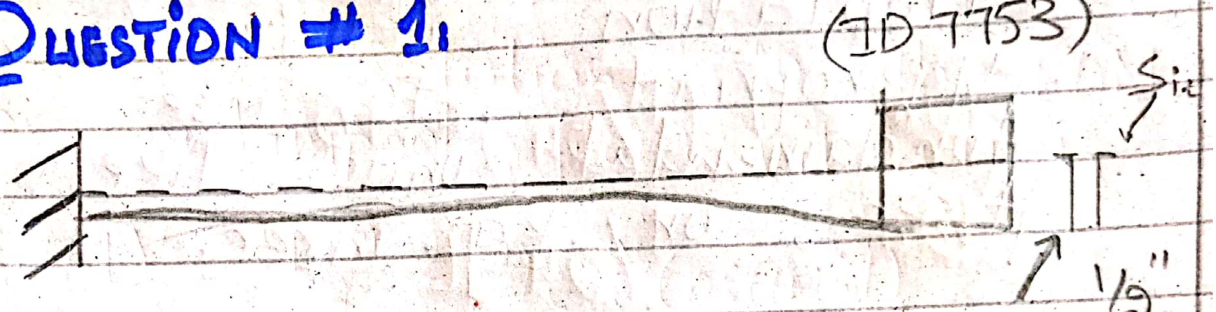
ID : 7753
SEC: "A"

QUESTION # 1.



SOLUTION:

The general E.O.M for CDOF system is

$$ku + Cu + mu = P(t)$$

In our case system is undamped (C=0) undergoing free vibration.

Hence general EOM becomes

$$ku + mu = 0$$

$$k = 3EI/L^3$$

$$k = 3 \times 29000 \; k/in^2 \times 150 \; in^4$$

$K = 7.55$ k/in

inorder to eliminate the chances of mistakes during calculation it is more appropriate to use fundamental units like lb, ft, sec, or kg, m, sec.

7.55 k/in $= 90625$ lb/ft

$m = 7753$ lb $/ 32.2$ ft.sec

$m = 240.8$

$W_n = \sqrt{K/m}$

$W_n = \sqrt{\frac{90625}{240.8}}$

$W_n = 19.39$ rad/sec

$T_n = 2\pi / W_n = 2\pi / 19.39$

$T_n = 0.325$ sec.

subsituting the corresponding value in eq (1).

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$90625 + 240.8 = 0$

where K is in lb/ft
and m is lb/sec/ft^2

General Solution to the EOM for undamped free vibration.

[redacted] = ~~1/2"~~ [redacted] ~~Cos(wnt)~~

$$u(t) = U(0)\cos(\omega_n t) + \frac{\ddot{U}(0)}{2U_n}\sin(\omega_n t)$$

$$U(0) = 1/2'' = 1/24 \text{ ft} \;\&\; \ddot{U}(0) = 0$$

$$U(t) = \left(\frac{1}{24} \times \cos(19.39t) + 0\right)$$

$$U(t) = 1/24 \times \cos(19.39t)$$

Equivalent static force at any time t is

$$F_s(t) = K \cdot U(t)$$

$$F_s(t) = \frac{90625 \times \cos(19.39t)}{24}$$

$= $ [redacted] $= 3776\cos(19.39t)$

Amplitude of dynamic displacement is for undamped free vibration

$$u_0 = \sqrt{(u(0))^2 + (u(0)\,\omega_n)^2}$$

$$u_0 = \sqrt{(1/24)^2 + 0}$$

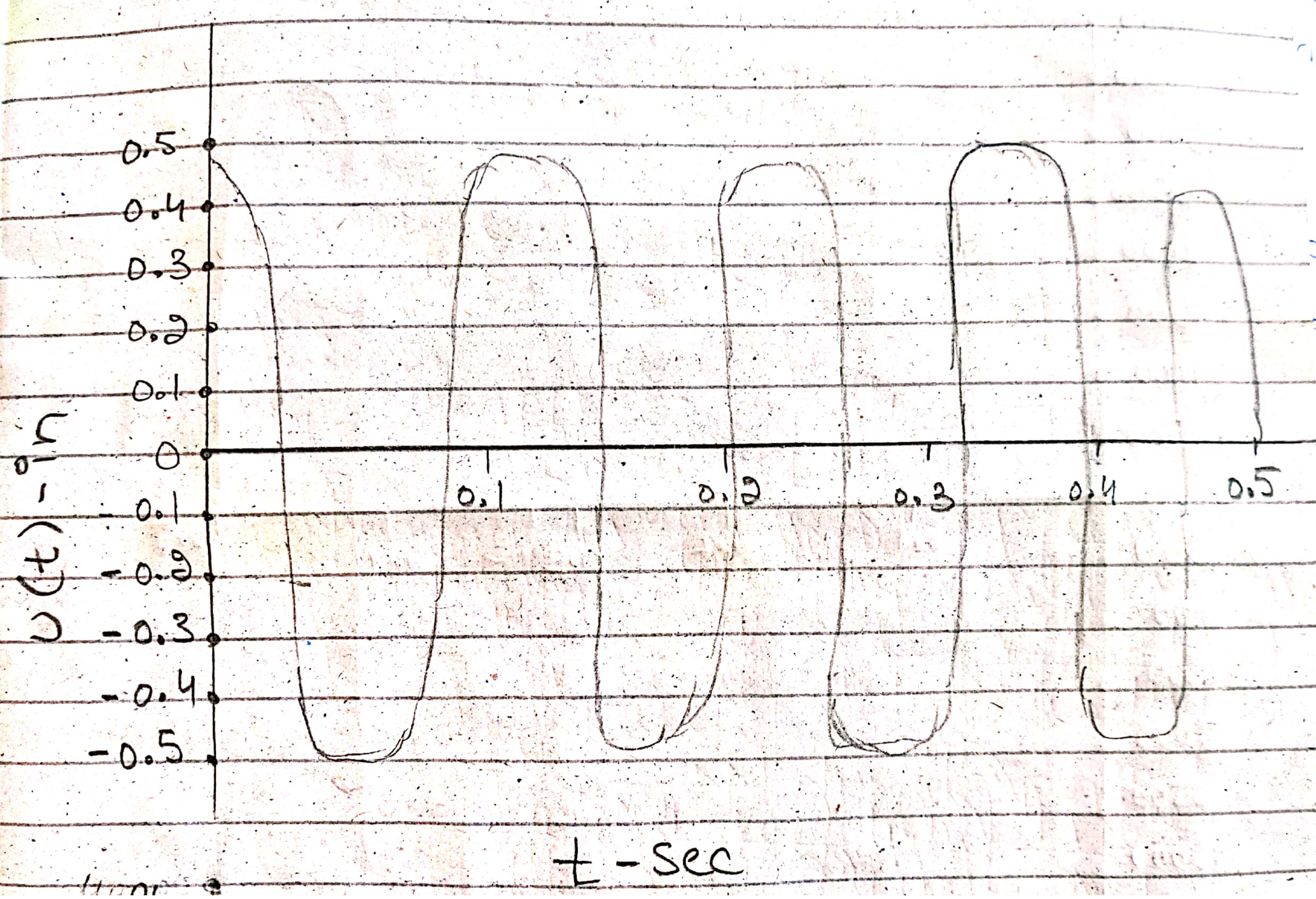
$$u_0 = 1/24 \text{ ft}$$

Aplitude of equivalent static force :

$$Ku_0 = 90625 \times 1/24$$

~~$Ku_0 = 377.64$ lb.~~

$$Ku_0 = 3776.4 \text{ lb.}$$



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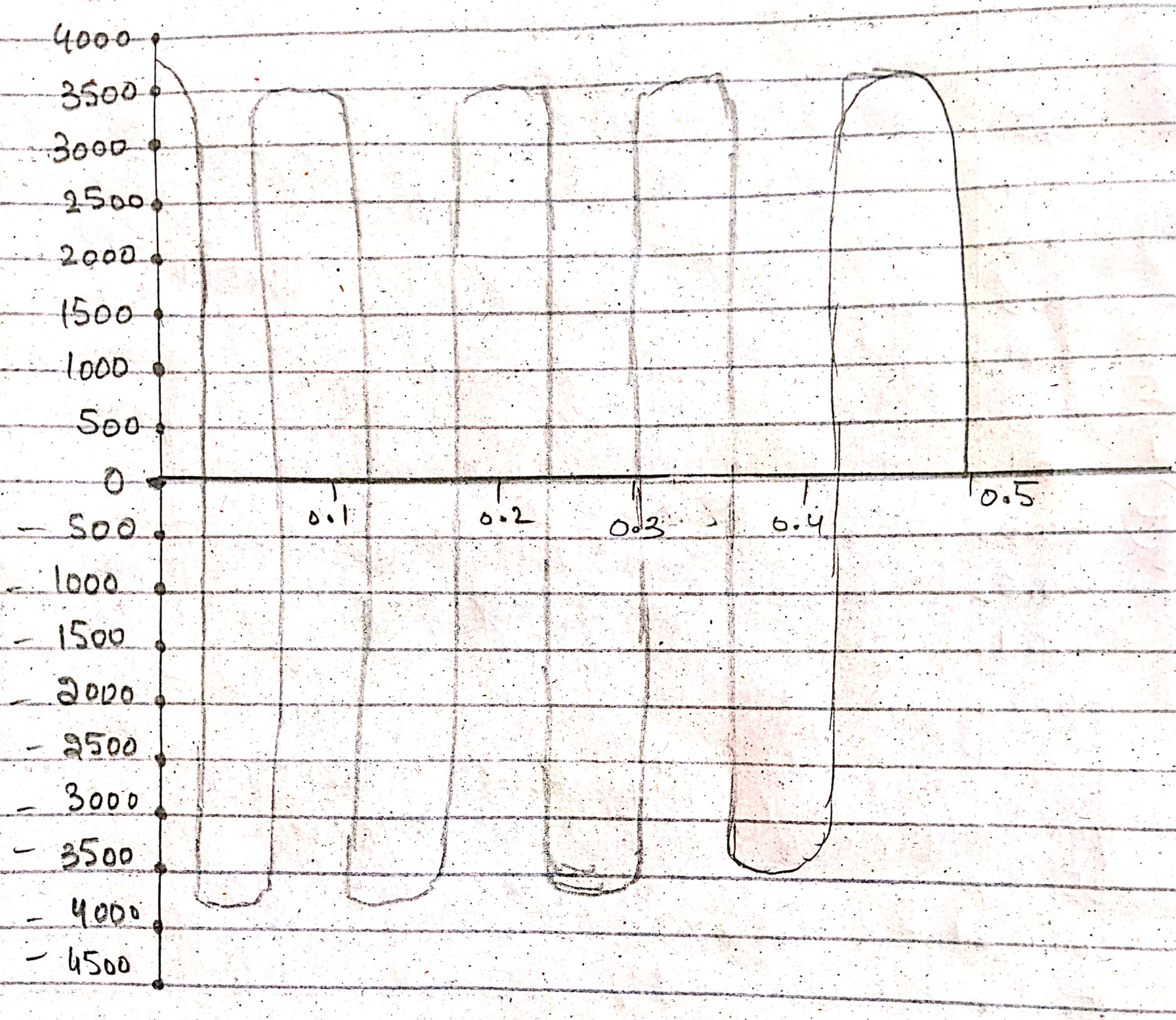

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QUESTION # 2:

ζ (Damping Ratio) of reinforcement concrete with considerable cracking = 3-5% = 3%

Using Data of beam given in Question # 1.

REQUIRED DATA:

Develop & solve equation showing variation in equivalent static force with time.

Draw graph to show the variation of displacement with time & variation of equivalent static force with time.

SOLUTION:

EOM for damped free vibration is

$$ku + c\dot{u} + m\ddot{u} = 0 \quad \text{---} (1)$$

from eq (1)

$k = 90625 \ lb/ft$ &

$m = 240.8 \ lb\,sec^2/ft$

$\omega_n = 19.39 \ rad/sec.$

$C = \zeta \times 2m\omega_n$

$C = (0.03) \times 2(240.8 \times 19.39)$

$C = 280.2 \ lb.sec/ft$

Putting values in eq ①.

$90625u + 280.2 + 240.8 = 0$

solution to the EOM for damped forces vibration is.

$$u(t) = e^{-\zeta\omega_n t}\left[u(0)\cos(\omega_n t) + \frac{1}{\omega_D}\left(\dot{u}(0) + u(0)\zeta\omega_n\right)\sin(\omega_n t)\right]$$

$\omega_D = 19.39 \ rad/sec.$

$$u(t) = e^{-0.03 \times 19.39t}\left[\tfrac{1}{24} \times \cos(19.39t) + \frac{1}{19.39} \times \left[0 + \tfrac{1}{24} \times 0.03 \times 19.39 \times \sin(19.39t)\right]\right]$$

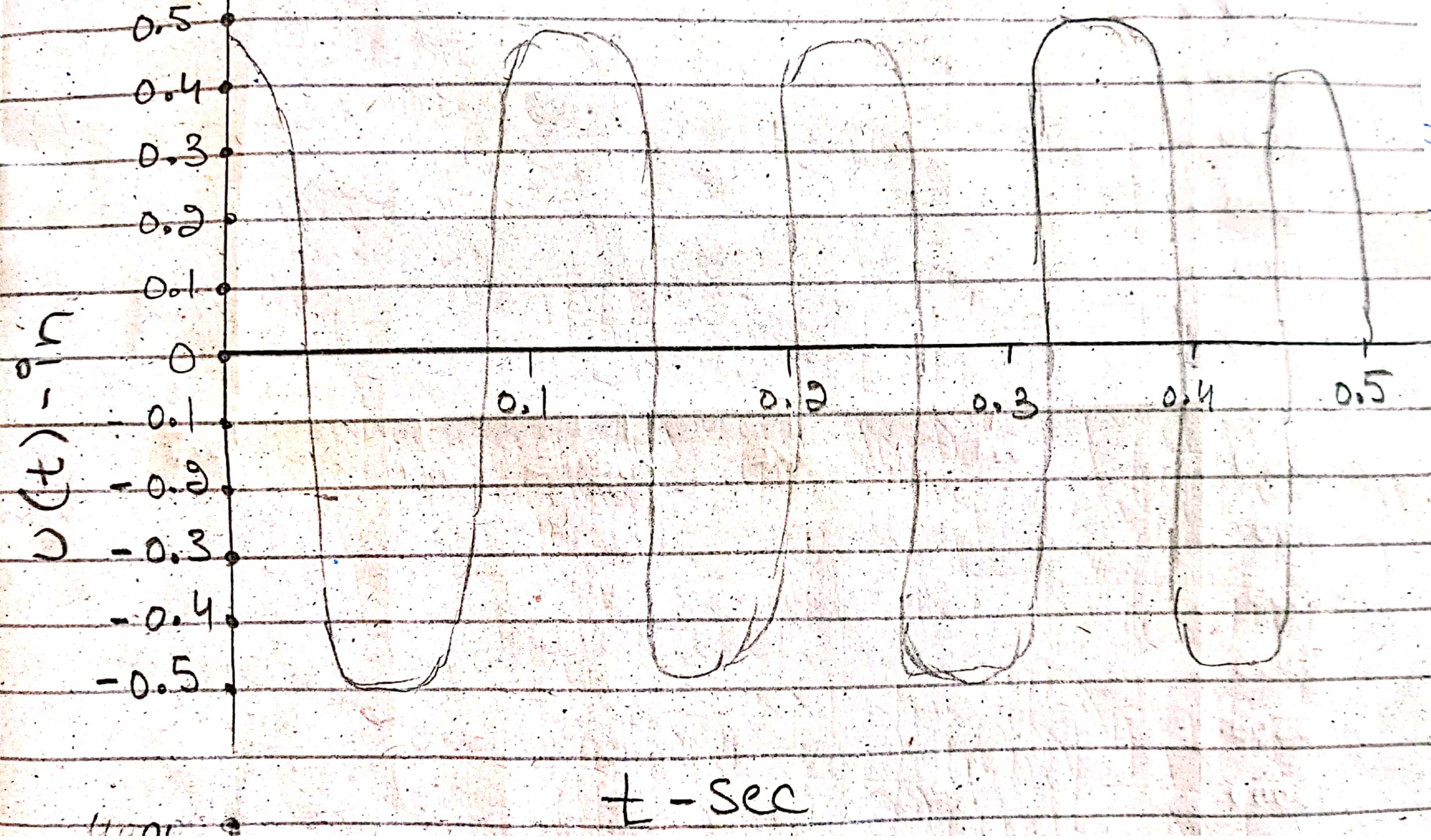
$$U(t) = e^{-0.582t}\left[0.041 \times \cos(19.39t) + 0.00125 \times \sin(19.39t)\right]$$

~~$U(t) = e^{-0.582t}[$~~

$$F_s(t) = K \cdot U(t) \Rightarrow 90625 \times U(t)$$

$$F_s(t) = e^{-0.582t}\left[(90625 \times 0.041)\cos(19.39t) + (90625 \times 0.000125) + \sin(19.39t)\right]$$

$$F_s(t) = e^{-0.582t}\left(3715.62\cos(19.42t) + 113.28\sin(19.39)t\right.$$



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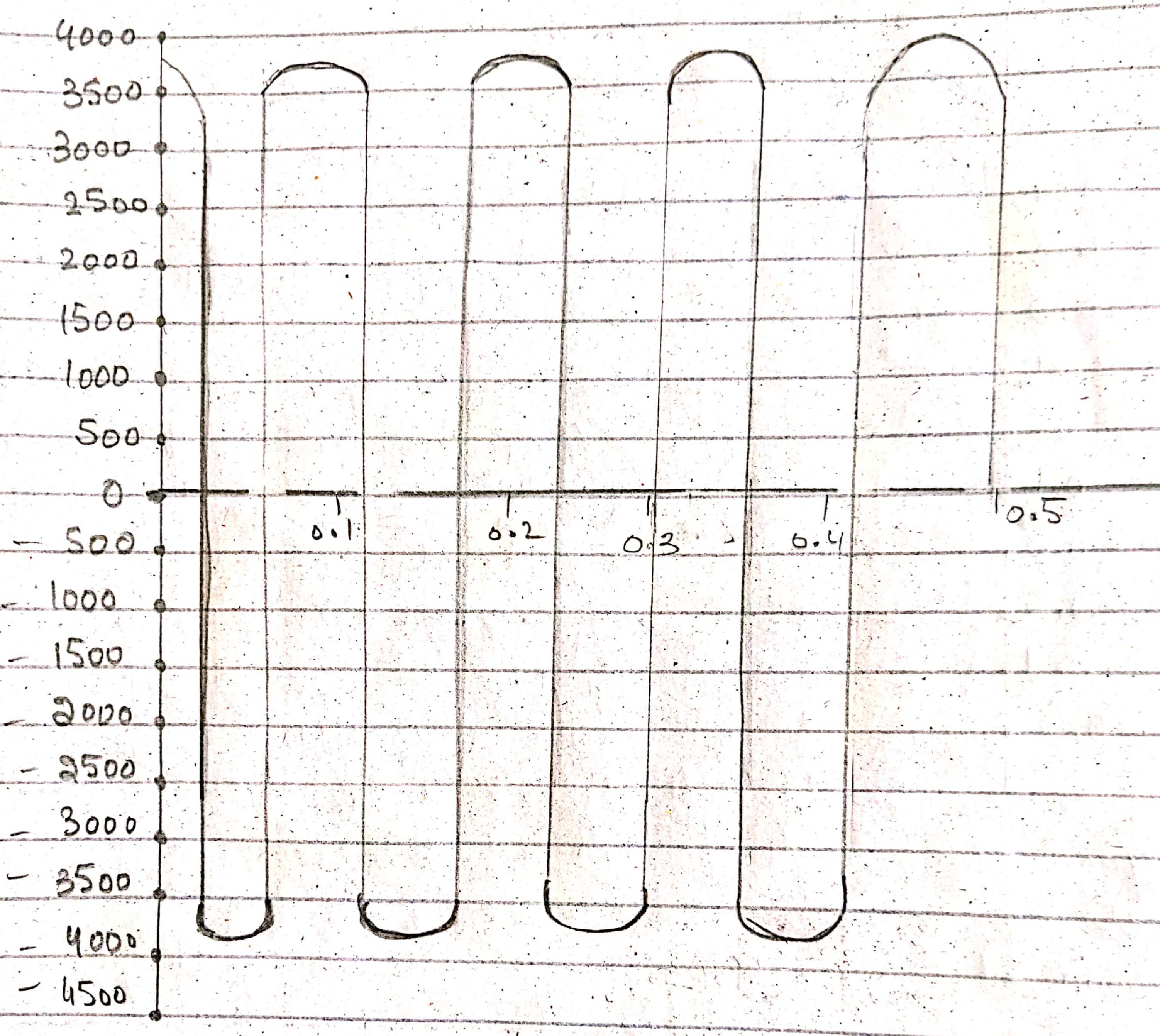

4000
3500
3000
2500
2000
1500
1000
500
0
- 500
- 1000
- 1500
- 2000
- 2500
- 3000
- 3500
- 4000
- 4500
0.1
0.2
0.3
0.4
0.5

QUESTION #3:

GIVEN DATA:

Force = 60 kips

$U_1 = 7753/1000 = 7.753$ in

After: $j = 7$ (cycles)

completed = 3.57 sec

$U_{J+1} = 2.286$ cm = 0.9 in.

ignore the verticle vibration.

REQUIRED DATA:

- Damping ratios.
- Natural period of undamped vibration
- Stiffness of Structures.
- Weight of tank
- Damping Co-efficient.
- number of cycle to reduce the displacement to 0.5"

SOLUTION:

★ **DAMPING RATIO = ζ = ?**

As;

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{U_1}{U_{j+1}}\right]$$

By putting value

$$7 = \frac{1}{2(3.14)\zeta} \ln\left[\frac{7.753}{0.9}\right]$$

$$\zeta * (7 \times 2 \times 3.14) = 0.93$$

$$\zeta * (43.96) = 0.93$$

$$\zeta = 0.93 / 43.96$$

$$\zeta = 0.0211$$

$$\zeta = 2.11\ \%$$

* $T_n = ?$

As "Seven" cycles are completed in 3.57 sec, Thus time required to complete one cycle.

$= 7/3.57 = 1.96$ sec.

$T_D = 1.96$ sec.

Now:

$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$

$\Rightarrow T_D = T_n / \sqrt{1-\zeta^2}$

$\Rightarrow T_n = T_D (\sqrt{1-\zeta^2})$

$= 1.96 (\sqrt{1-(0.0462)^2})$

$T_n = 1.957$ sec "Natural period of undamped vibration"

* **STIFFNESS OF STRUCTURE $K = ?$**

As $K = \frac{F \cdot \cos\theta}{2}$ $\left(\begin{array}{l} F = 60 \text{kips} \\ \theta = 60° \end{array} \right)$

$K = 60 \cdot \cos(60°)/2$

$K = 15$ k/in

$k = 18000 \; lb/ft$

* Weight of Tank W=?

As;

$$\omega_n = \sqrt{k/m} = \sqrt{k/(w/g)} = \sqrt{k \cdot g/w}$$

$$\Rightarrow \omega_n^2 = k \cdot g/w \Rightarrow \left(W = k \cdot g/\omega_n^2\right)$$

By putting values of $\omega_n = 2\pi/T_n$

$$W = k \cdot g/(4\pi^2/T_n^2) = k \cdot g \left(\frac{T_n^2}{4\pi^2}\right)$$

$$W = \frac{18000 \; lb}{ft} \cdot \frac{32.2 \; ft}{sec} \left(\frac{(1.957)^2}{4(3.14)^2}\right)$$

$$W = 56284.75 \; lb$$

$$W = 56.284 \; Klb$$

* DAMPING CO-EFFICIENT; C=?

$$\zeta = \frac{c}{2m\omega_n}$$

$$\Rightarrow c = \zeta(2m\omega_n) = \zeta(2m(2\pi/T_n))$$

$$c = \frac{0.0462\left(2\left(\frac{56284}{32.2}\right)\right)(2(3.14))}{1.957}$$

$C = 518.886 \ lb.sec/ft$

* Number of Cycles to reduce displacement altitude from 6.872 in to 0.5 in"

$j = ?$

$$j = \frac{1}{2\pi\zeta} \ln\left(\frac{U_1}{U_{j+1}}\right)$$

$$= \frac{1}{2(3.14)(0.0462)} \ln\left[\frac{7.753}{0.9}\right]$$

$j = 7$ cycles