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Subject: Intro to structural dynamics and earthquake engg.

Submitted To: Sir. Engr, Yaseen Mahmood.

Q# 1

Sol:- ~~The g~~

Beam is pulled 0.5 inch downward

$E = 29,000$ ksi

$I = 150 \text{ in}^4$. δ

st = Deflection due to static load.

Ib = 7168

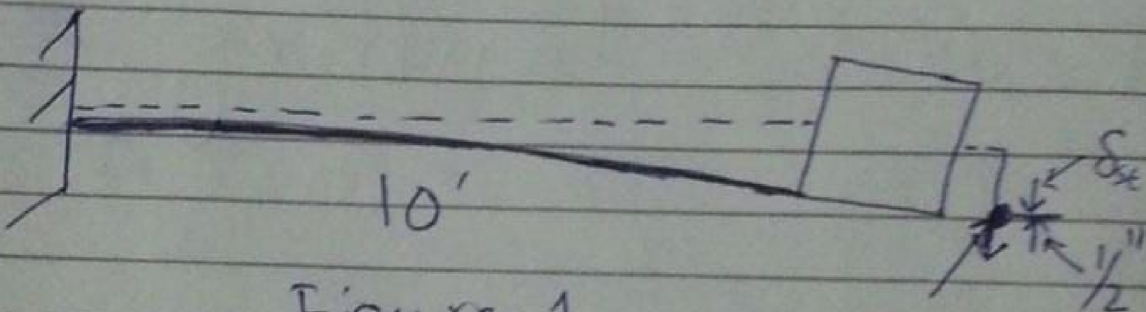


Figure-1

Sol: The general E.O.M for SDOF system is

$$ku + c\dot{u} + m\ddot{u} = p(t)$$

In our case system is undamped ($c=0$) undergoing free vibration ($P(t)=0$)

Hence general EOM becomes $ku + m\ddot{u} = 0$ .... (1)

$$K = 3EI/L^3$$

$$= \frac{3 \times 29000 \ K/in^2 \times 150 \text{ in}^4}{(10 \times 12 \text{ in})^2}$$

$$\boxed{k = 7.55\ k/in}$$

In order to eliminate the chances of mistake during calculation, it is more appropriate to use fundamental units like lb, ft sec or kg, m, sec

$$\boxed{k = 7.55\ k/in = 90625\ lb/ft}$$

$$m = \frac{7168\ lb\,sec^2}{32.2\,ft}$$

$$\boxed{m = 222.60\ slug}$$

$$\omega_n = \sqrt{k/m} = \sqrt{\frac{90625}{222.60}}$$

$$\boxed{\omega_n = 20.177\ rad/sec}$$

$$T_n = 2\pi/\omega_n = \frac{2\pi}{20.177}$$

$$\boxed{T_n = 0.311\ sec}$$

Substituting the corresponding values in eq(1)

$$90625u + 222.60\ddot{u} = 0$$

Where "k" is in lb/ft and 'm' is in lb sec/ft^2

General solution to the EOM for undamped free vibration is,

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n}\sin(\omega_n t)$$

$$u(0) = 1/4'' = 1/48 \text{ ft and } \dot{u}(0) = 0$$

$$u(t) = (1/48) \times \cos(20.177t) + 0 =$$

$$(1/48) \times \cos(20.177t)$$

Equivalent static force at any time 't' is

$$f_s(t) = k.u(t) = \frac{90625 \times \cos(22.177t)}{48}$$

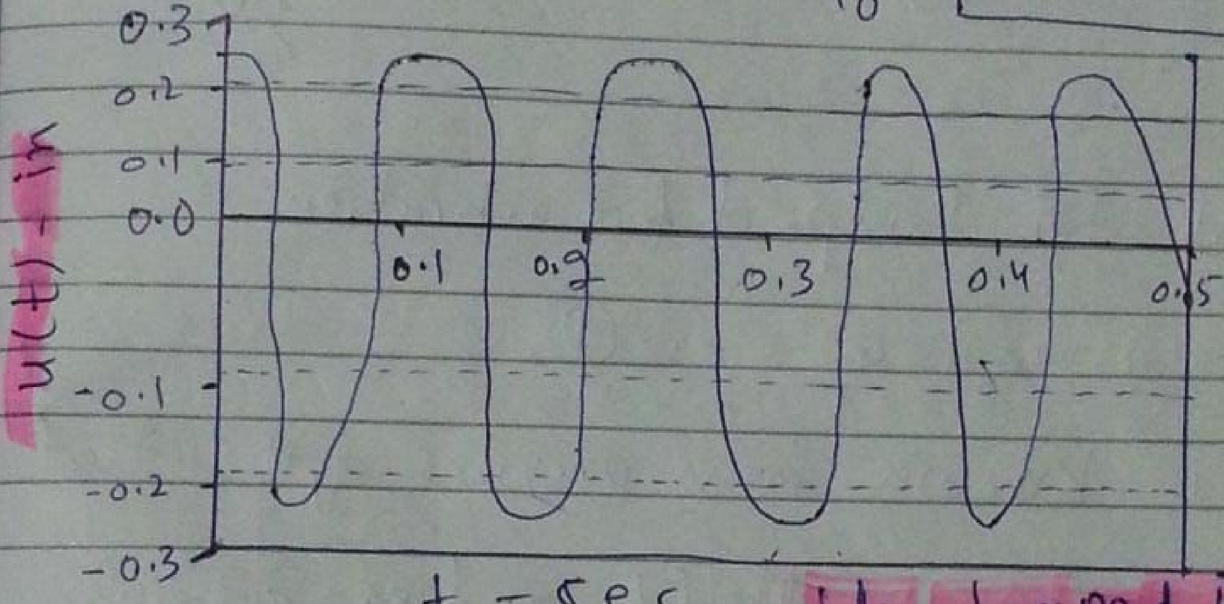
$$f_s(t) = 1888\cos(22.177t)$$

Amplitude of dynamic displacement u_0 for undamped free vibration is

$$u_0=\sqrt{\left[(u(0))^2+\left(\dot{u}(0)/\omega_n\right)^2\right]}=\sqrt{\left((1/48)^2+0\right)}=1/48\ \text{ft}$$

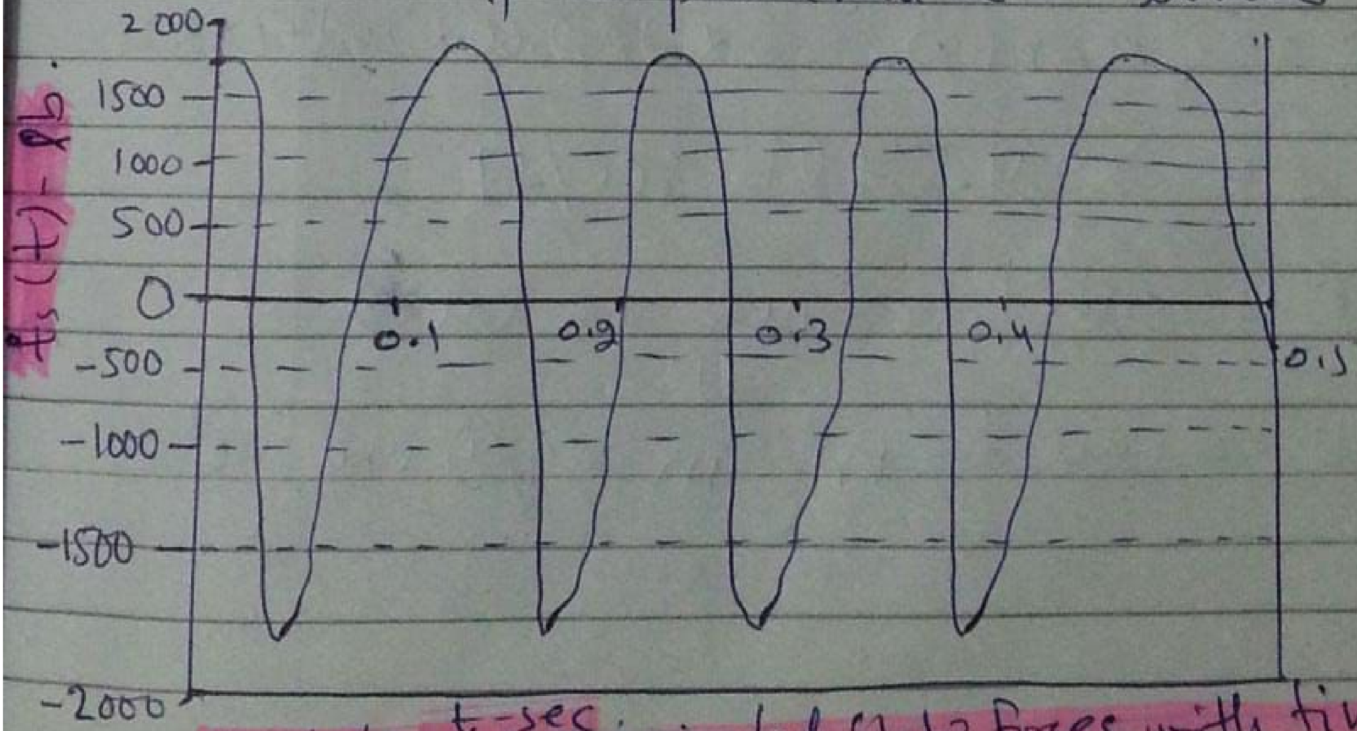
Amplitude of equivalent static force,

$$K u_0 = 90625 \times 1/48 = \boxed{1888\ \text{lb}}$$



Un-damped Free Vibration

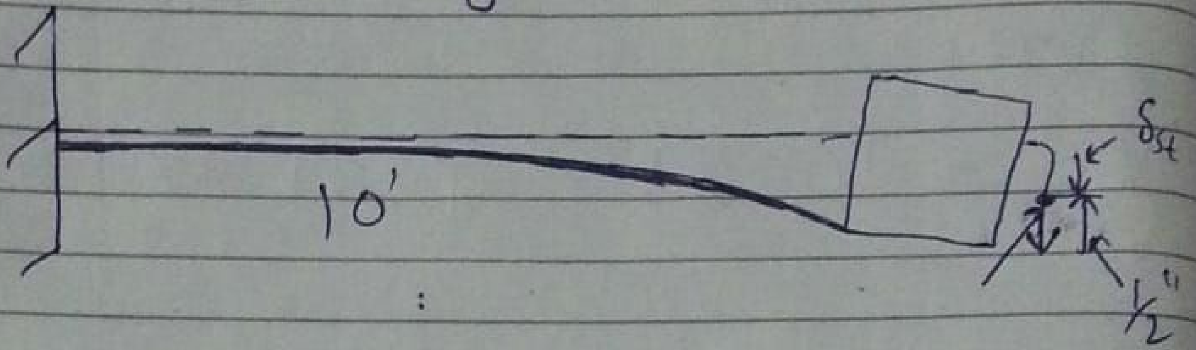
Variation of displacement with time



Variation of equivalent Static forces with time

Q# 2. For the beam's data given in question # 1 develop and solve the equation of motion for vibration resulting at free end.......

Sol:- Beam given in q# ①



E.O.M for damped free vibration is

$ku + c\dot{u} + m\ddot{u} = 0$ .... ①

It is known from previous q# 1 that

$K = 90625 \text{ lb/ft}$ and $m = 222.60 \text{ lb.sec}^2/\text{ft}$

$c = \zeta \times 2m\omega_n = 2 \times 222.60 \times 20.177 \times 0.025$

$c = 224.57 \text{ lb. sec/ft}$

By substituting values of

K, c and m in eq ① we get

$$90625u + 224.57\dot{u} + 222.60\ddot{u} = 0$$

solution to the E.O.M for damped free vibration is:

$$u(t) = e^{-\xi \omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\dot{u}(0)\xi\omega_n\right]\cdot \sin(\omega_D t)\right)$$

$$\omega_D = 20.177 \text{ rad/sec}$$

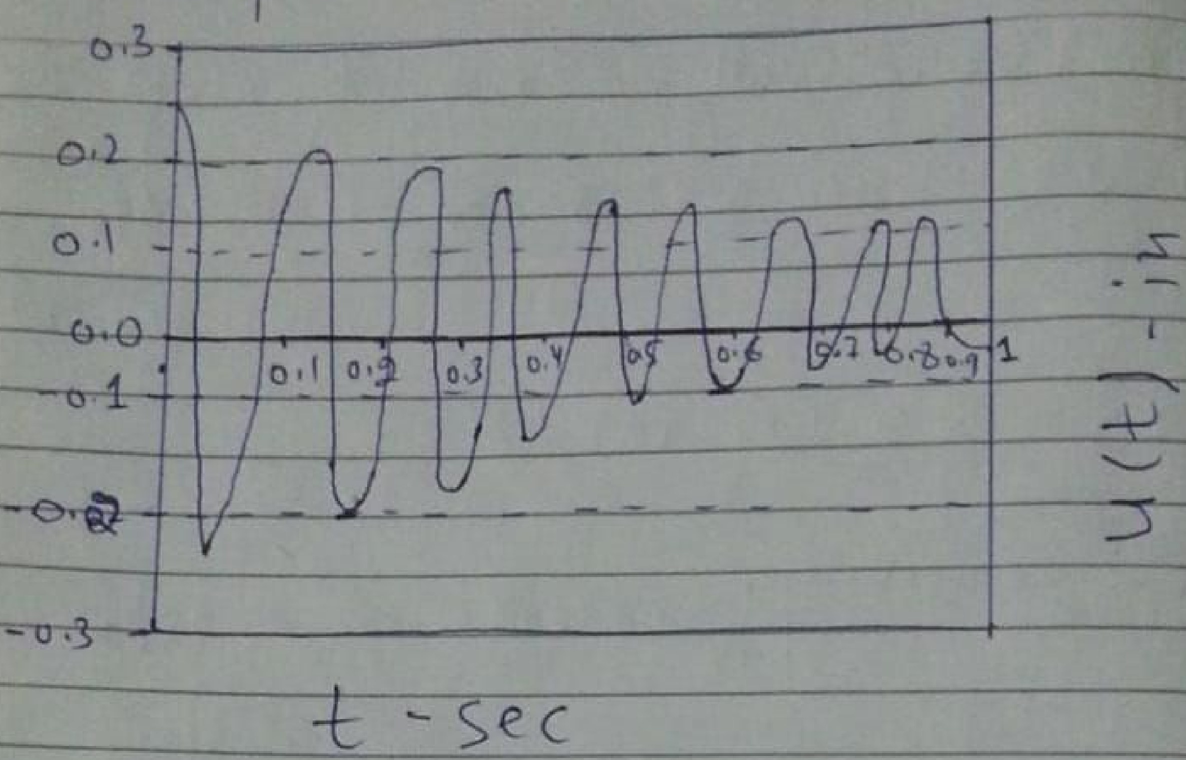
$$u(t) = e^{-0.025\times 20.177t}\left[\frac{1}{48}\times\cos(20.177) + \frac{1}{20.177}\times\left[0 + \frac{1}{48}\times 0.025\times 20.177\times\sin(20.177t)\right]\right]$$

$$u(t) = e^{-0.504t}\left[0.0208\times\cos(20.177) + 5.2083\times 10^{-4}\times \sin(20.177t)\right]$$

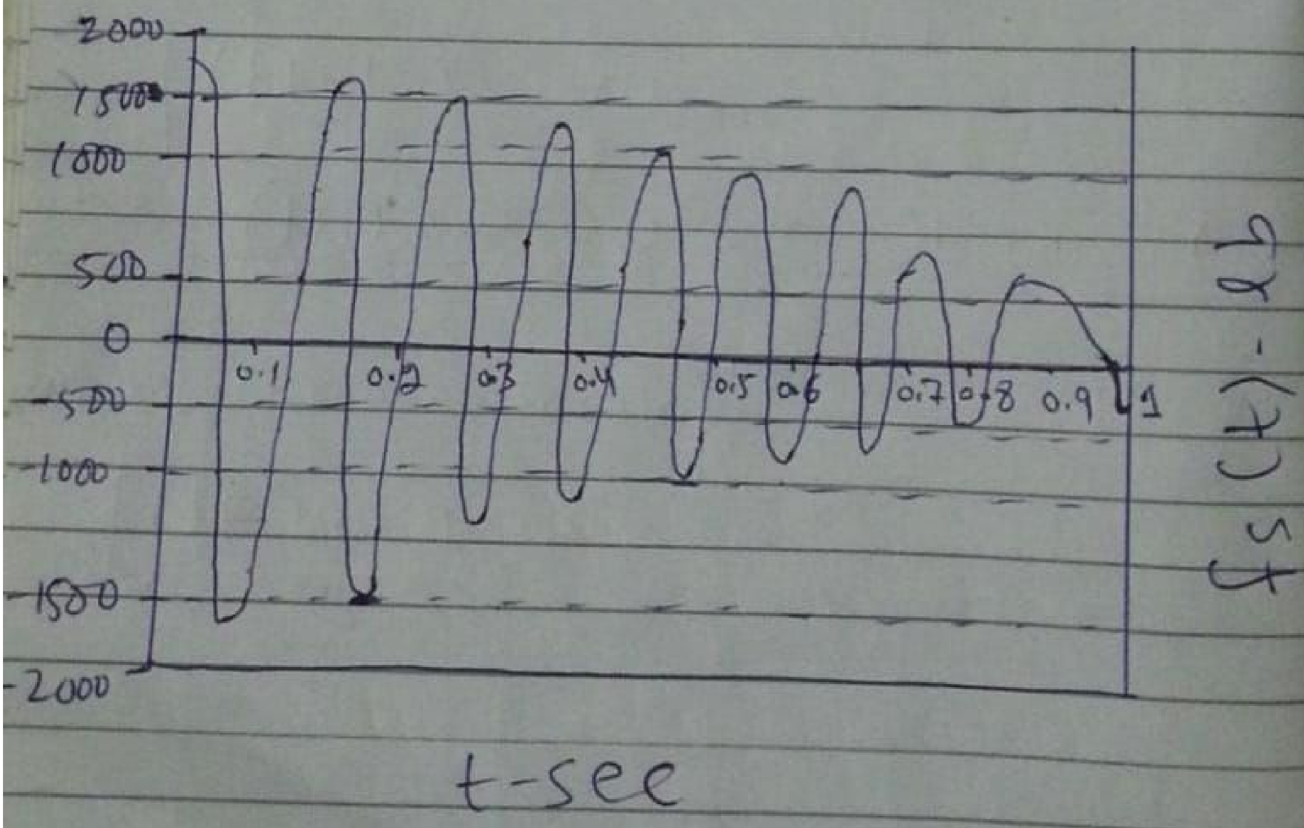
$$f_s(t) = K.u(t) = 90625\times u(t)$$

$$f_s(t) = e^{-0.0504t}\left[0.0208\cos(20.177t) + 0.0005208 \times\sin(20.177t)\right]$$

Damped Free Vibration.

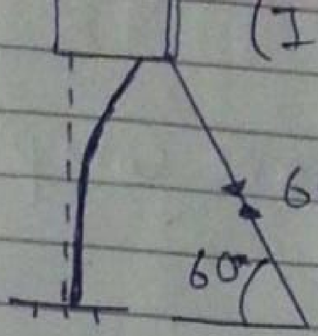


Variation of displacement with time



Vartion of Equivalent Static Forces with time

Q#3.



$(ID/1000) = \frac{7168}{1000} = 7.168$ in.

Required Data:-

a) Damping Ratios
b) Natural Period of un-damped vibration
c) stiffness of structures
d) Weight of tank
e) Damping Co-efficient
f) Number of cycles to reduce the displacement amplitude to 0.5"

Sol:-

$u_1 = 7.168$ in

After $j = 7$, $u_{j+1} = u_6 = 0.9$

a) ζ = Damping Ratio = ?

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$\Rightarrow 7 = \frac{1}{2\pi\zeta} \ln\left[\frac{7.168}{0.9}\right]$$

$$\boxed{\Rightarrow \zeta = 0.181 = 1.81\%}$$

b) $T_n = ?$

∵ 7 cycles of vibrations are completed in 3.57 seconds

⇒ Time required to complete one cycle

$$= \frac{3.57}{7}$$

$$\boxed{T_D = 0.51 \text{ sec}}$$

Now

$$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$$

$$2\pi / \omega_D = \frac{2\pi}{(\omega_n \sqrt{1-\zeta^2})}$$

$$= T_D = \frac{T_n}{\sqrt{1-\zeta^2}}$$

$$\Rightarrow T_n = T_D \times \sqrt{1-\zeta^2}$$

$$\Rightarrow T_n = 0.51 \times \sqrt{1-(0.181)^2}$$

$$\boxed{\Rightarrow T_n = 0.5015 = 0.51 \text{ sec}}$$

c) $k = ?$

$$k = \frac{60 \times \cos 60^\circ}{2} = 15\ k/in$$

$$\boxed{K = 18000\ lb/ft}$$

d) Weight of tank, $W = ?$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{\left(\frac{W}{g}\right)}} = \sqrt{\frac{K \times g}{W}}$$

$$\Rightarrow \omega_n^2 = K \times g / W$$

$$W = K \times \frac{g}{\omega_n^2}$$

Also $\omega_n = \frac{2\pi}{T_n}$

$$\Rightarrow W = \frac{kg}{\left(\frac{4\pi^2}{T_n^2}\right)} = Kg \times \frac{T_n^2}{4\pi^2}$$

$$W = \frac{\frac{18000\,lb}{ft} \times \frac{32.2\,ft}{sec^2} (0.51\,sec)^2}{4\pi^2}$$

$$\boxed{W = 74875 \text{ lb} = 74.9 \text{ k}}$$

e) Dumping Co-efficient, c = ?

It is known that $\zeta = \frac{c}{2m\omega_n}$

$$\Rightarrow c = \zeta \times 2m\omega_n = \zeta \times 2m \times (2\pi / T_n)$$

$$c = \frac{0.181 \times 4 \times \pi \times \left(\frac{74875}{32.2}\right)}{0.51}$$

$$\boxed{c = 10370.4 \text{ lb.sec/ft}}$$

f) No. of cycles to reduce the displacement amplitude to 0.5"in

$J = ?$

$$J = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$\Rightarrow J = \frac{1}{2 \times \pi \times 0.181} \ln\left[\frac{7.168}{0.5}\right]$$

$$\boxed{j = 12.605 \text{ or } 13 \text{ cycles}}$$