# Department of Electrical Engineering
# Final – Term Assignment Spring 2020
# Date: 24/06/2020

## Course Details

**Course Title:** Numerical Analysis **Module:** ______

**Instructor:** ______ **Total Marks:** 50

## Student Details

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|---|---|---|---|
| Q1. | (a) | Find the root of the equation given below by Bisection method, accuracy must be up to three decimal places $$x^3 - x^2 + x - 7 = 0$$ | Marks 10<br>CLO 1 |
| Q2. | (a) | Use Regula-Falsi method to compute the root of the following equation in the interval [0, 1] after third iteration. $$f(x) = \cos x - xe^x$$ | Marks 07<br>CLO 1 |
| | (b) | Use Regula-Falsi (method of false position) to solve the following equation, accuracy must be up to four decimal places. $$x^3 - 4x - 9 = 0$$ | Marks 07<br>CLO 2 |
| Q3. | (a) | Find the real root of the following equation using Newton-Raphson method in the interval [2,3] after third iteration. $$x^3 - 3x - 5 = 0$$ | Marks 08<br>CLO 2 |
| | (b) | Solve the following equation by using Muller's method, only perform three iterations. ($x_0 = 0.5$, $x_1 = 1$, $x_2 = 0$) $$x^3 - 7x^2 + 14x - 6$$ | Marks 08<br>CLO 2 |
| Q4. | (a) | Using Gaussian Elimination method, solve the following system of equations $$2x - y + 2z = 2$$ $$x + 10y - 3z = 5$$ $$x - y - z = 3$$ | Marks 10<br>CLO 1 |

1

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Q1 Ans #
SOL #

$$F(x) = x^3 - x^2 + x - 7 = 0$$

STEP 1 * Assume limit

$f(1) = (1)^3 - (1)^2 + (1) - 7 = 0$

$= 1 - \cancel{1} + \cancel{1} - 7$

$= \boxed{-6}$

$F(2) = (2)^3 - (2)^2 + (2) - 7$

$= 8 - 4 + 2 - 7$

$= \boxed{-1}$

$F(3) = (3)^3 - (3)^2 + 3 - 7$

$= 27 - 9 + 3 - 7$

$F(3) = \boxed{14}$

$[2,3] = F(2) \times F(3) = (-1)(14) = -14 < 0$

$C = \frac{2+3}{2} = \frac{5}{2} = \boxed{2.5}$

STEP NO2

(2)

Mid point #

$$C = \frac{2+3}{2} = \frac{5}{2} = \boxed{2.5}$$

$$f(2.5) = (2.5)^3 - (2.5)^2 + 2.5 - 7$$
$$= \boxed{4.875}$$

$$F(2) \times f(2.5) = (-1) \times (4.875) = -4.875 < 0$$

STEP NO3

Mid point

$$C = \frac{2+2.5}{2} = \frac{4.5}{2} = \boxed{2.25}$$

$$F(2.25) = (2.25)^3 - (2.25)^2 + (2.25) - 7$$
$$= \boxed{1.5181}$$

$$F(2) \times F(2.25) = (-1 \times 1.5781) = (-1.5781) < 0$$

STEP NO4

Mid point

$$C = \frac{2+2.25}{2} = 2.125$$

$$f(2.125) = (2.125)^3 - (2.125)^2 + (2.0625) - 7$$
$$= \boxed{-0.4177}$$

$$F(2) \times f(2.0625) = (-1) \times (-0.4177)$$
$$= \boxed{+0.4177 > 0}$$

Root of the equation lies in limit (2, 2.125), i.e

$$\boxed{0.20507}$$

Q2(a) Ans#    3

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$F(x) = \cos x - xe^x$

$[0,1] =$

$f(0) = \cos(0) - (0)e^0$

$= 1 - 0 = 1$

$F(1) = \cos(1) - (1)e^1$

$= 0.999 - 2.7182$

$= -1.7192$

Here $a = 0$ $\quad b = 1$

$F(a) = 1 \quad F(b) = -1.7192$

Formula #

$\boxed{1st}$ $\quad \dfrac{a\,f(a) - b\,f(b)}{F(b) - f(a)} = \dfrac{0(1) - 1(-1.7192)}{-1.7192 - 1}$

$= \dfrac{+1.7192}{-2.7192} = \boxed{-0.6322}$

$f(-0.6322) = \cos(-0.6322) - (-0.6322)e^{-(0.6322)}$

$= 0.9999 - (-0.6322)(0.5314)$

$= 0.9999 + 0.3359$ (4)

$= 1.3358$

again step2 #

$a = -0.6322 \qquad b = 1$

$f(a) = 1.3358 \qquad F(b) = -1.7192$

famula

$$= \frac{af(a) - bf(b)}{f(b) - f(a)}$$

$$= \frac{-0.6322(1.3358) - (1)(-1.7192)}{-1.7192 - 1.3358}$$

$$= \frac{-0.8444 + 1.7192}{-3.055} = \frac{2.5636}{-3.055} = -0.839$$

$$f(-0.8391) = \cos(-0.8391) - (-0.8391)e^{-0.8391}$$

$$= 0.9998 - (-0.8391)(0.4320)$$

$$= 0.9998 + 0.3625$$

$$= 1.3625$$

step3 # $\quad a = -0.8391 \qquad b = 1$

$f(a) = 1.3623 \qquad F(b) = -1.7192$

$$-\frac{0.8391(1.3623) - (1)(-1.7192)}{-1.7192 - 1.3623}$$

$$= \frac{1.1431 + 1.7192}{3.0815} = \frac{2.862}{3.0815} = \boxed{0.9287}$$

Ans

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5

Q2:(b)

$f(x) = x^3 - 4x - 9$

$f(0) = (0)^3 - 4(0) - 9$

$= -9$

$f(1) = (1)^3 - 4(1) - 9$

$= 1 - 4 - 9$

$= -12$

$f(2) = (2)^3 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$ (negative)

$f(3) = (3)^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$ (Positive)

Root lies b/w $[2, 3]$

First approx: $a = 2 \quad b = 3$

using formula:

$$x_1 = \frac{a f(a) - b f(b)}{f(b) - f(a)}$$

$$= \frac{2 f(2) - 3 f(3)}{f(3) - f(2)} = \frac{2(-9) - 3(6)}{6 - (-9)} = \frac{-36}{15}$$

$\boxed{= -2.4}$

$\cancel{f(12) = (12)^3 - 4(12) - 9}$

$\cancel{= 1671}$

Junaid ur Rehman 11484 6

$f(-2.4) = (-2.4)^3 - 4(-2.4) - 9.$

$= -5.76 - (-9.6) - 9$

$= \boxed{-5.16.}$

Root lies b/w $f(-2.4)$ & $f(3)$ =

$a = (-2.4)$ $b = 3$

$f(-2.4) = -5.16$ $f(3) = 6.$

$$= \frac{a f(a) - b f(b)}{f(b) - f(a)}$$

$$= \frac{(-2.4)(-5.16) - (3)(6)}{6 - (-2.4)} = \frac{12.384 - 18}{6 + 2.4} = \frac{-5.016}{8.4}$$

$= -0.5971$

$f(-0.5971) = (-0.5971)^3 - 4(-0.5971) - 9$

$= -0.2128 - (-2.284) - 9.$

$= -6.9288$

Root lies b/w $(-0.5971, 3)$

$a = -0.5971$ $b = 3$

$f(a) = -6.9288$ $f(b) = 6.$

$$= \frac{-0.5971(-6.9288) - 3(6)}{6 - (-6.9288)} = \frac{4.1371 - 18}{12.9288}$$

$= \boxed{-2.7448}$

7

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11484

Q3 (a) Ans #

$$f(x) = x^3 - 3x - 5$$

$$f'(x) = 3x^2 - 3$$

Since root lies b/w $[2, 3]$

initial point $x_0 = \frac{2+3}{2} = \frac{5}{2} = 2.5$.

NRM Formula =

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{(x_n^3 - 3x_n - 5)}{3x_n^2 - 3}$$

$$x_{n+1} = \frac{x_n(3x_n^2 - 3) - (x_n^3 - 3x_n - 5)}{3x_n^2 - 3}$$

$$x_{n+1} = \frac{3x_n^3 - 3x_n - x_n^3 - 3x_n - 5}{3x_n^2 - 3}$$

$$x_{n+1} = \frac{2x_n^3 - 6x_n - 5}{3x_n^2 - 3}$$

Iteration 1 $x_0 = 2.5$.

$$x_{0+1} = \frac{2(2.5)^3 - 6(2.5) - 5}{3(2.5)^2 - 3} = \frac{31.25 - 15 - 5}{18.75 - 3}$$

$$= \frac{11.25}{15.75} = 0.7142$$

8

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Iteration 2

$$x_{1+1} = \frac{2(x_1)^3 - 6x_1 - 5}{3x_1^2 - 3}$$

$$x_2 = \frac{2(0.7142)^3 - 6(0.7142) - 5}{3(0.7142)^2 - 3}$$

$$x_2 = \frac{0.7286 - 4.2852 - 5}{1.5302 - 3}$$

$$x_2 = \frac{-8.5566}{-1.4697} = \cancel{6.\ldots} \; 5.8220$$

Iteration 3

$$x_{2+1} = \frac{2(x_2)^3 - 6(x_2) - 5}{3x_2^2 - 3}$$

$$= \frac{2(5.8220)^3 - 6(5.8220) - 5}{3(5.8220)^2 - 3}$$

$$= \frac{394.68 - 34.93 - 5}{101.68 - 3}$$

$$= \frac{354.67}{98.687} = 3.594$$

9

Q3(b)

$f(x) = x^3 - 7x^2 + 14x - 6$

$x_0 = 0.5 \quad x_1 = 1 \quad x_2 = 0.$

$f(x_0) = (0.5)^3 - 7(0.5)^2 + 14(0.5) - 6.$

$f(0.5) = 0.125 - 1.75 + 7 - 6.$

$f(0.5) = -0.625$

$f(x_1) = x_1^3 - 7x_1^2 + 14x_1 - 6.$

$= (1)^3 - 7(1)^2 + 14(1) - 6.$

$= 1 - 7 + 14 - 6.$

$f(1) = 2.$

$f(x_2) = x_2^3 - 7x_2^2 + 14x_2 - 6.$

$= (0)^3 - 7(0)^2 + 14(0) - 6.$

$f(0) = -6.$

$h_1 = x_1 - x_0 = 1 - 0.5 = 0.5$

$h_2 = x_2 - x_1 = 0 - 1 = -1$

$\delta_1 = \dfrac{f(x_1) - f(x_0)}{h_1} = \dfrac{2 - (-0.625)}{0.5} = \dfrac{2.625}{0.5} = 5.25$

$\delta_2 = \dfrac{f(x_2) - f(x_1)}{h_2} = \dfrac{-6 - (-0.625)}{-1} = 5.375$

$a = \dfrac{\delta_2 - \delta_1}{h_2 + h_1} = \dfrac{5.375 - 5.25}{-1 + 0.5} = \dfrac{0.125}{-1.5} = -0.0833$

(10)

$b = a \times h_2 + \delta_2 = -0.0833(-1) + 5.375$

$= 5.4583$

$c = f(x_2) = -6$

$x_3 = x_2 + \dfrac{-2c}{b \pm \sqrt{b^2 - 4ac}}$

$= \dfrac{0}{1} + \dfrac{-2(-6)}{5.4583 \pm \sqrt{(5.4583)^2 - 4(-0.0833)(-6)}}$

$= \dfrac{12}{5.4583 \pm \sqrt{29.79 \cancel{0.3333} (1.999)}}$

$= \dfrac{12}{5.4583 \pm 5.2717}$

$= \dfrac{12}{0.1866} = 64.30$

or

$= \dfrac{12}{10.73} = \boxed{1.11}$

Relative error $\varepsilon_a = \left| \dfrac{x_3 - x_2}{x_3} \right| \times 100\% = \left| \dfrac{1.11 - 0}{1.11} \right| \times 100\%$

$= 100\%$ error

Now $x_0 = x_1 = 1$

$x_1 = x_2 = 0$

$x_2 = x_3 = 1.11$

(11)

2nd iteration

$f(x_0) = (1)^3 - 7(1)^2 + 14(1) - 6$

$= 1 - 7 + 14 - 6 = 2$

$f(x_1) = (0)^3 - 7(0)^2 + 14(0) - 6 = -6$

$f(x_2) = (1.11)^3 - 7(1.11)^2 + 14(1.11) - 6 =$

$= 1.367 - 8.624 + 15.54 - 6$

$= 2.283$

$h_1 = x_1 - x_0 = -6 - 2 = -8$

$h_2 = x_2 - x_1 = 2.283 - 0 = 2.283$

$\delta_1 = \dfrac{f(x_1) - f(x_0)}{h_1} = \dfrac{-6-2}{-8} = 1$

$\delta_2 = \dfrac{f(x_2) - f(x_1)}{h_2} = \dfrac{2.283 - (-6)}{2.283} = \dfrac{8.283}{2.283} = 3.628$

$a = \dfrac{\delta_2 - \delta_1}{h_2 + h_1} = \dfrac{3.628 - 1}{2.283 + (-8)} = \dfrac{2.628}{-5.716} = -0.459$

$b = a \times h_2 + \delta_2 = (0.459)(2.283) + 3.628 = 2.580$

$c = f(x_2) = 2.283$

$$x_3 = x_2 + \frac{-2c}{b \pm \sqrt{b^2 - 4ac}}$$

(12)

$$= \cancel{2.283}\ 0 + \frac{-2(2.283)}{2.580 \pm \sqrt{(2.580)^2 - 4(\cancel{2}0.459)(2.283)}}$$

$$= \frac{4.566}{2.580 \pm \sqrt{6.656 + 4.1915}}$$

$$= \frac{4.566}{2.580 \pm \sqrt{10.84}}$$

$$= \frac{4.566}{2.580 \pm 3.290}$$

$$= \frac{4.566}{5.873} = 0.7744.$$

$$\varepsilon_a = \left| \frac{x_3 - x_2}{x_3} \right| \times 100\%.$$

$$= \left| \frac{0.7744 - 0}{0.7744} \right| \times 100\%. = 100\%.$$

Now,

$x_0 = x_1 = 0$

$x_1 = x_2 = 1.11$

$x_2 = x_3 = 0.7744$

3rd iteration

$f(x_0) = (0)^3 - 7(0)^2 + 14(0) - 6 = -6$

C3

$f(x_1) = (1.11)^3 - 7(1.11)^2 + 14(1.11) - 6 = 2.283$

$f(x_2) = (0.7744)^3 - 7(0.7744)^2 + 14.(0.7744) - 6$

$= 0.4618 - 3.874 + 10.416 - 6$

$= \boxed{0.953}$

$h_1 = x_1 - x_0 = 1.11 - 0 = \boxed{1.11}$

$h_2 = x_2 - x_1 = 0.7744 - 1.11 = \boxed{-0.335}$

$\delta_1 = \dfrac{f(x_1) - f(x_0)}{h_1} = \dfrac{2.283 - (-6)}{1.11} = \dfrac{8.28}{1.11} = \boxed{7.462}$

$\delta_2 = \dfrac{f(x_2) - f(x_1)}{h_2} = \dfrac{0.953 - 2.283}{-0.335} = \cancel{3.970}$ $3.970$

$a = \dfrac{\delta_2 - \delta_1}{h_2 + h_1} = \dfrac{3.970 - 7.462}{-0.335 + 1.11} = -4.505$

$b = a \times h_2 + \delta_2 = (-4.505) \times (-0.335) + 3.970 = 5.479$

$c = f(x_2) = 0.953.$

$x_3 = x_2 + \dfrac{-2c}{b \pm \sqrt{b^2 - 4ac}}$

$= 1.11 + \dfrac{-2(0.953)}{5.479 \pm \sqrt{(5.479)^2 - 4(-4.505)(0.953)}}$

$$= +1.11 + \frac{(-1.906)}{5.479 \pm 3.584}$$

$$= 1.11 + \frac{(-1.906)}{9.333}$$

$$= \frac{9.333(1.11) - (1.906)}{9.333}$$

$$= \frac{10.3596 - 1.906}{9.333}$$

$$x_3 = \boxed{8.45363}$$

$$\varepsilon_a = \left| \frac{x_3 - x_2}{x_3} \right| \times 100\%$$

$$= \left| \frac{8.4536 - 1.11}{8.4536} \right| \times 100\%$$

$$= 0.868 \times 100\% = 86.8\%$$

Qn 4 (a) In matrix form

(15)

$$\begin{bmatrix} 2 & -1 & 2 \\ 1 & 10 & -3 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 3 \end{bmatrix}$$

In Augmented form

$$\begin{bmatrix} 2 & -1 & 2 & : & 2 \\ 1 & 10 & -3 & : & 5 \\ 1 & -1 & -1 & : & 3 \end{bmatrix}$$

Applying operations

$R_2 \to 2R_2 - R_1$

$R_3 \to 2R_3 - R_1$

$$\begin{bmatrix} 2 & -1 & 2 & : & 2 \\ 0 & 19 & 4 & : & 8 \\ 0 & -1 & -4 & : & 4 \end{bmatrix}$$

Applying operations

$\cancel{R_3 \to R_3 - R_1}$

$R_3 \to 19R_3 + R_2$

$$\begin{bmatrix} 2 & -1 & 2 & : & 2 \\ 0 & 19 & 4 & : & 8 \\ 0 & 0 & -70 & : & 84 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 2 \\ 0 & 19 & 4 \\ 0 & 0 & -70 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 2 \\ 8 \\ 84 \end{bmatrix}$$

Writing in equation form.

$2x - y + 2z = 2$ — (i)

$19y + 4z = 8$ — (ii)

$-70z = 84$

$$z = \frac{.84}{-70} = -1.2$$

Putting in ~~eq (2)~~ eq (ii)

$$19y + 4(-1.2) = 8.$$

$$19y + (-4.8) = 8.$$

$$19y = 8 + 4.8 = 12.8.$$

$$y = \frac{12.8}{19} = 0.673$$

Putting values of y, & z in eq (i).

$$2x - (0.673) + 2(-1.2) = 2.$$

$$2x - 0.673 + (-2.4) = 2.$$

$$2x - 3.073 = 2$$

$$2x = 5.073$$

$$x = \frac{5.073}{2} = 2.5365$$

$$\boxed{x = 2.536 \quad y = 0.673 \quad z = -1.2}$$