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Sec B

Subject # Intro to structural Dynamics & Earthquake Engg

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QNO # 01

Given Data:

$E = 29{,}000\,ksi$

$I = 150\,in^4$

δ_{st} = Deflect due to 7719 lb static load

Beam is pulled ½" downwords

Required Data:

Natural time period of System

Develop & solve the equation of motion

Draw graphs to show the variation of displacement with time, & the variation of equivelent static forces with time.

Sol:

General EOM for SDOF system

is $ku + c\dot{u} + m\ddot{u} = p(t)$

Since system is undamped $(C=0)$

undergoing free vibration $p(t)=0$

Hence general EOM becomes

$$ku + m\ddot{v} = 0 \quad \text{—— ①}$$

$$k = \frac{3EI}{L^3} \Rightarrow \frac{3 \times 29000\ k/in^2 \times 150\ in^4}{(10 \times 12\ in)^3}$$

$$k = 7.55\ k/in \Rightarrow k = 7.55208$$

→ In order to eliminate chances of mistake during calculation, it is more appropriate to use fundamental units like lb, ft sec or kg, m, sec

$$k = 7.55208\ k/in \Rightarrow 90625\ lb/ft$$

$$m = \frac{W}{g} \Rightarrow \frac{7719}{32.2} = 239.72\ slug$$

$$w_n = \sqrt{\frac{k}{m}} \Rightarrow \sqrt{\frac{90625}{239.72}}$$

$$w_n = 19.443\ rad/sec$$

$$T_n = \frac{2\pi}{w_n} \Rightarrow \frac{2\pi}{19.443} = 0.323\ sec$$

Put m & k in eq ①

$$90625 u + 239.72 \ddot{u} = 0$$

where k is in lb/ft & m is in lb sec/ft²

⇒ General solution to EOM for undamped free vibration is

$$u(t) = u(0) \cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n} \sin(\omega_n t)$$

$$u(0) = \frac{1}{2}'' = \frac{1}{24} \text{ ft} \; \& \; \dot{u}(0) = 0$$

$$u(t) = \left(\frac{1}{24}\right) \times \cos(19.443t) + 0 = \left(\frac{1}{24}\right) \times \cos(19.443t)$$

Equivalent Static Force at any time "t" is

$$f_s(t) = k \cdot u(t) = \frac{90625 \times \cos(19.443t)}{24}$$

$$= 3776 \cos(19.443t)$$

Amplitude of dynamic displacement, u_0 for undamped free vibration is

$$u_0 = \sqrt{(u(0))^2 + \left(\frac{v(0)}{\omega_n}\right)^2}$$

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$$= \sqrt{\left(\frac{1}{24}\right)^2 + 0}$$

$$= \frac{1}{24} \text{ ft}$$

Amplitude of equivalent static force, f_{so}

$$ku_o = 90625 \times \frac{1}{24}$$

$$\boxed{ku_o = 3776}$$

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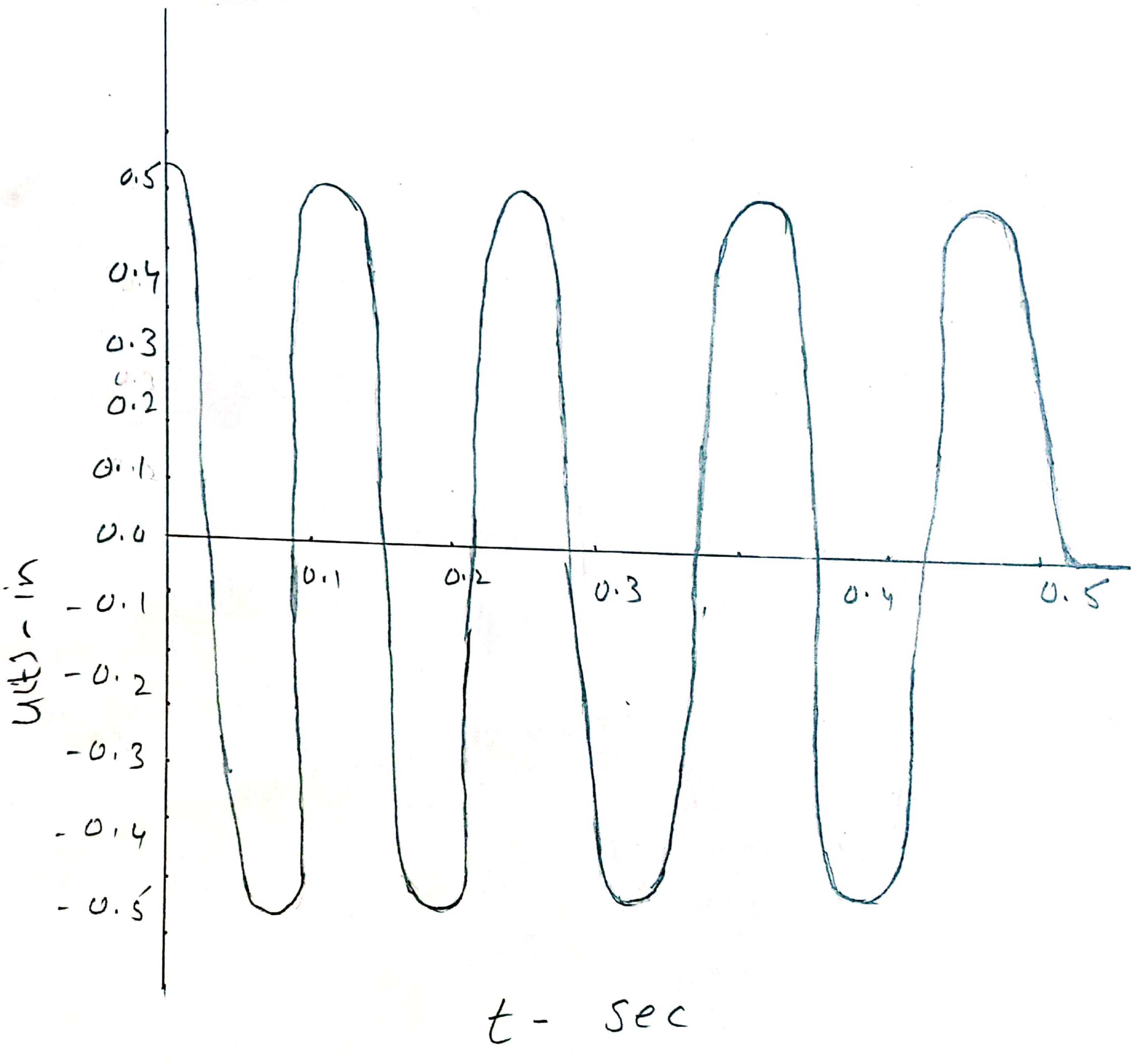
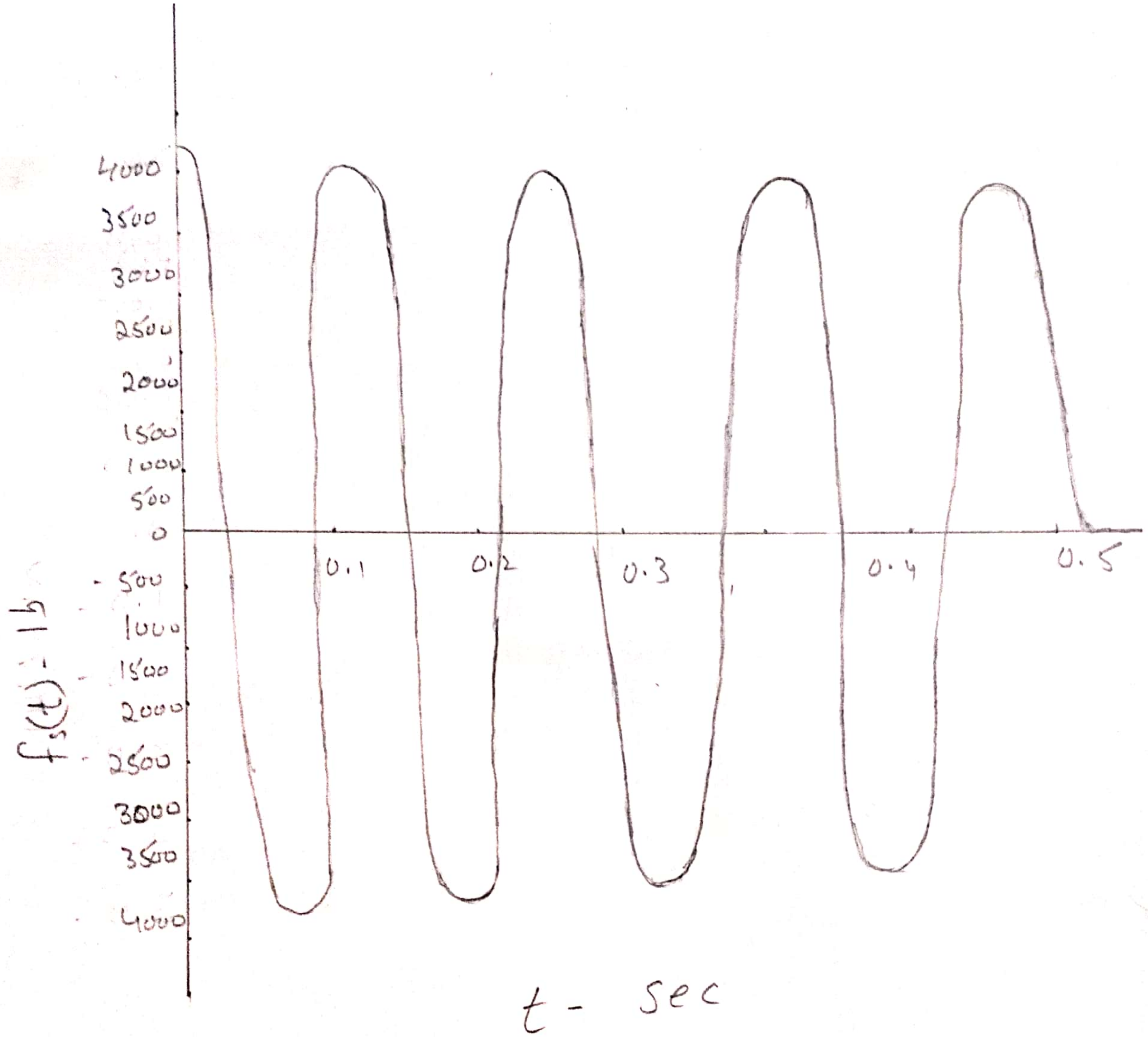


Vibration of displacement with time.



Vibration of Equivalent static force with time

QNO # 02

Given Data:

ζ (Damping ratio) of Reinforced Concrete with Considerable cracking

$= 3-5\%$

$= 3\%$

Using Data of beam given in Question # 1

Required

-> Develope & solve the equation Showing variation in Equivalent Static Force with time

-> Draw graph to show the variation of displacement with time & the variation of equivilent static force with time

Sol:.

EOM for demped free vibrations is $ku + cu + mu = 0$ — (i)

from Question 1

$K = 90625$ lb/ft & $m = 239.72$ lb sec²/ft

$\omega_n = 19.443$ rad/sec

$C = \xi \times 2m\omega_n$

$C = (0.03) \times 2(239.72)(19.443)$

$C = 279.65$ lb-sec/ft

Put values in eq (1)

$90625 + 279.65u + 239.72u = 0$

Solution to the EOM force damped free vibration is

$u(t) = e - \xi\omega_n t\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}[\dot{u}(0) + u(0)\,\xi\omega_n\right].\sin\omega_D t$

$$\omega_D = 19.443 \text{ rad/sec}$$

$$u(t) = e^{-0.03 \times 19.443 t}\left[\frac{1}{24} \times \cos(19.443) + \frac{1}{19.443} \times \right.$$

$$\left[0 + \frac{1}{24} \times 0.03 \times 19.443 \times \sin(19.443t)\right)$$

$$u(t) = e^{-0.583t}\left[0.041 \times \cos(19.443t) + 0.00125\right.$$

$$\left. \times \sin(19.443t)\right]$$

$$f_s(t) = K \cdot u(t) \Rightarrow 90625 \times u(t)$$

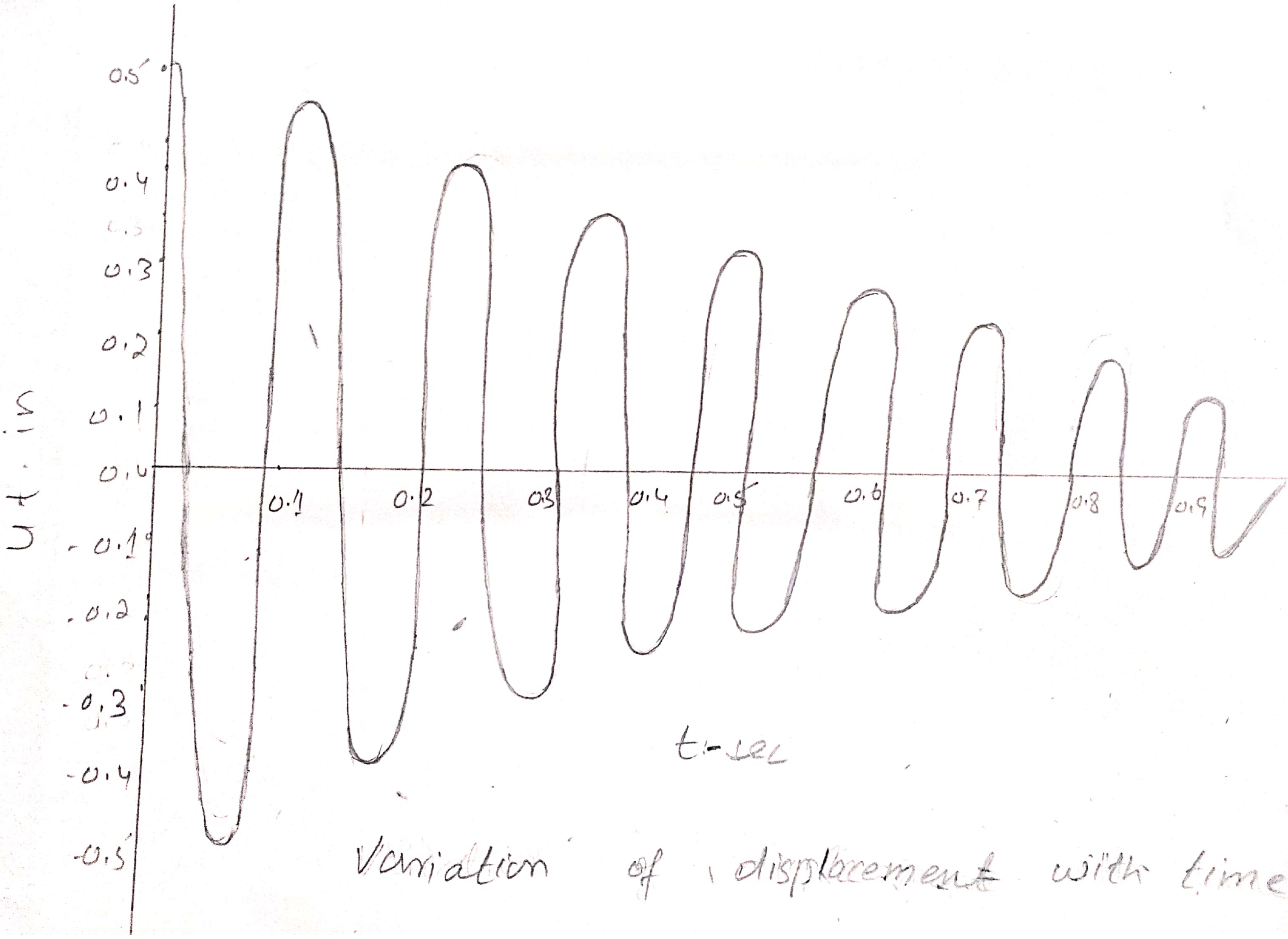
$$f_s(t) = e^{-0.583t}\left[(90625 \times 0.041) \cos(19.443t)\right)$$

$$+ (90625 \times 0.00125) \sin(19.443t))$$

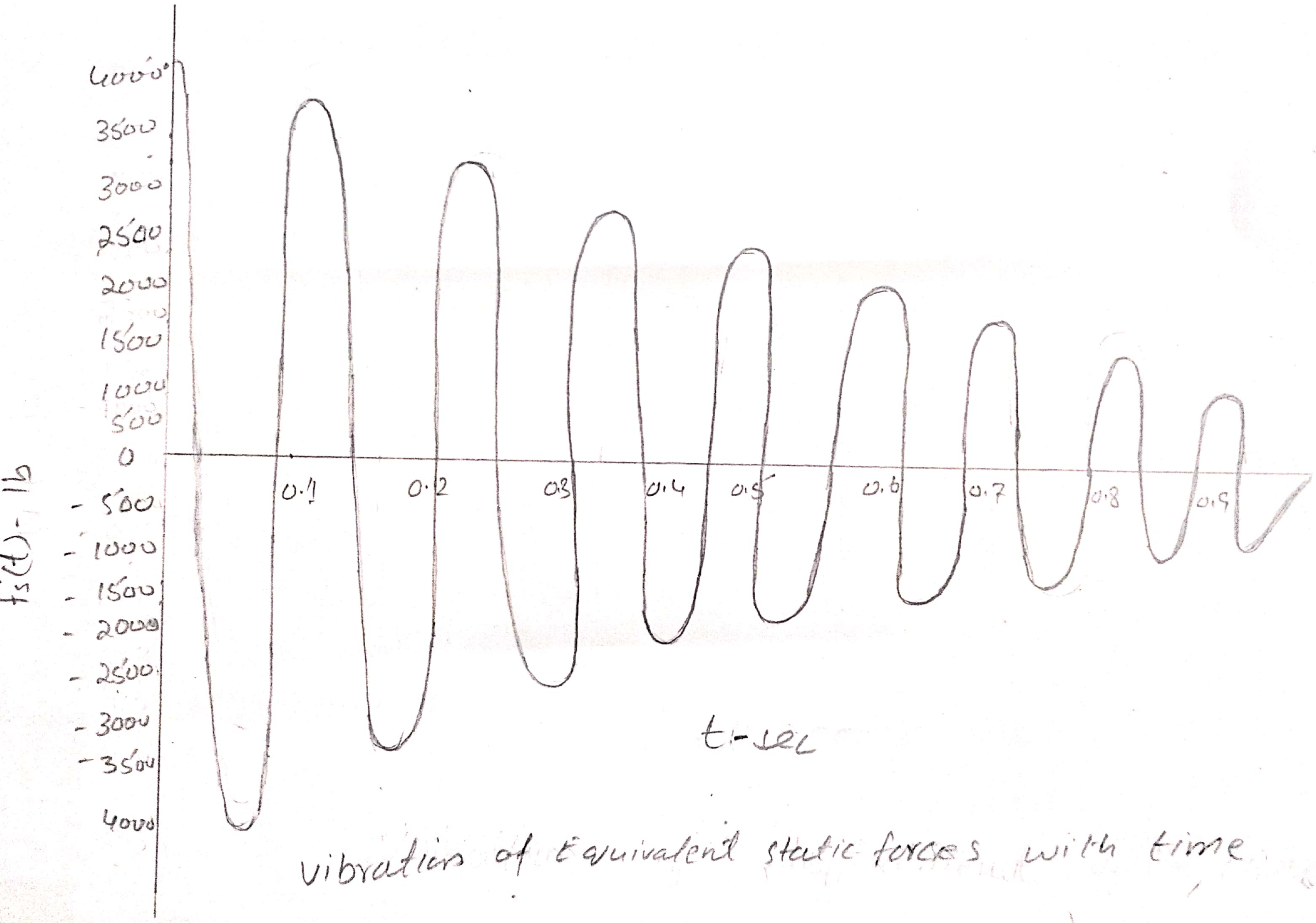
$$f_s(t) = e^{-0.583t}$$

$$\left[3715.62 \cos(19.443t) + \right.$$

$$113.28 \sin(19.443t))$$



Variation of displacement with time



vibration of Equivalent static forces with time

Q No # 03

Given Data:

Force = 60 kips

Displacement of tank $= \left(\frac{ID}{1000}\right)''$

$= \left[\frac{7719}{1000}\right]'' = 7.719$

Cycles = 7

Time taken to complet 7 cycles

= 3.57 sec

amplitude of displacement = 2.286 cm

= 0.9''

Required Data:.

(a) Damping ratios
(b) Natural period of undamped vibration
(c) Stiffiness of structure
(d) weight of tank.

(e) Damping coefficient

(f) Number of cycles to reduce the displacement amplitude to 0.5"

Solution:

→ Displecement of tank, $u_1 = 7.719''$

→ After 7 cycles.

i.e, After $j = 7$, $u_{j+1} = u_8$

$= 0.9''$

(a) Damping ratio = ?

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_1 + 1}\right]$$

$$7 = \frac{1}{2\pi\zeta} \ln\left[\frac{7.719}{0.9}\right] = 0.0488$$

$\Rightarrow$ 4.88%

(b) Natural period of undamped vibration = T_n = ?

As the 7 cycles of vibration are Completed in 3.57 sec

$\Rightarrow$ Time required to complet one cycle, $T_D = \frac{3.57}{7} = 0.51$ sec

$T_D = 0.51$ sec

Now

$$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{(\omega_n\sqrt{1-\zeta^2})}$$

$$\Rightarrow T_D = \frac{T_n}{(1-\zeta)^2}$$

$$\Rightarrow T_n = T_D \times \sqrt{1-\zeta^2}$$

$$\Rightarrow T_n = 0.51 \times \sqrt{1-(0.0488)^2}$$

$T_n = 0.51$ sec

(C) Stiffness of structure, $k = ?$

$$k = \frac{60 \times \cos 60^\circ}{7.719} \Rightarrow 3.89\ k/in$$

$k = 3.89\ k/in \Rightarrow 46680\ lb/ft.$

(d) weight of tank $W = ?$

$$\omega_n = \sqrt{k/m} = \sqrt{\frac{k}{W/g}} = \sqrt{\frac{k \cdot g}{W}}$$

$$\Rightarrow \omega_n^2 = \frac{k \times g}{W}$$

$$W = \frac{k \times g}{\omega_n^2}$$

Also $\omega_n = \dfrac{2\pi}{T_n}$

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$$W = \frac{kg}{\left(\frac{4\pi^2}{T_n^2}\right)}$$

$$W = kg \times \frac{T_n^2}{4\pi^2}$$

$$W = \left[46680\ lb/ft \times 32.2\ ft/sec^2\right] \times \frac{0.51\ sec^2}{4\pi^2}$$

$$W = 9903.013\ lb$$

$$W = \boxed{9.90\ K}$$

(e) Damping co-efficient, $c = ?$

It is known that

$$\zeta = c/2m\omega_n$$

$$c = \zeta \times 2m\omega_n$$

$$= \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$\Rightarrow C = \frac{(0.0488) \times 4 \times \pi \times \left(\frac{9903.013}{32.2}\right)}{0.51}$$

$$\Rightarrow C = 369.80 \ lb.sec/ft$$

(F) Number of cycles to reduce the displacement amplitude to 0.5''. J = ?

$$J = \frac{1}{2\pi\zeta} \ln\left(\frac{u_i}{u_{i+1}}\right)$$

$$\Rightarrow J = \frac{1}{2\pi \times 0.0488} \ln\left(\frac{7.719}{0.5}\right)$$

$$= j = 8.93$$ OR

9 cycles.

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