DAY: DATE:

NAME : M. ASIM KHAN

CLASS : BE(C)

SECTION : A

ID # 7708

SUBJECT : INTRO. TO STRUCTURE DYNAMIC & EARTH QUAKE ENGINEERING

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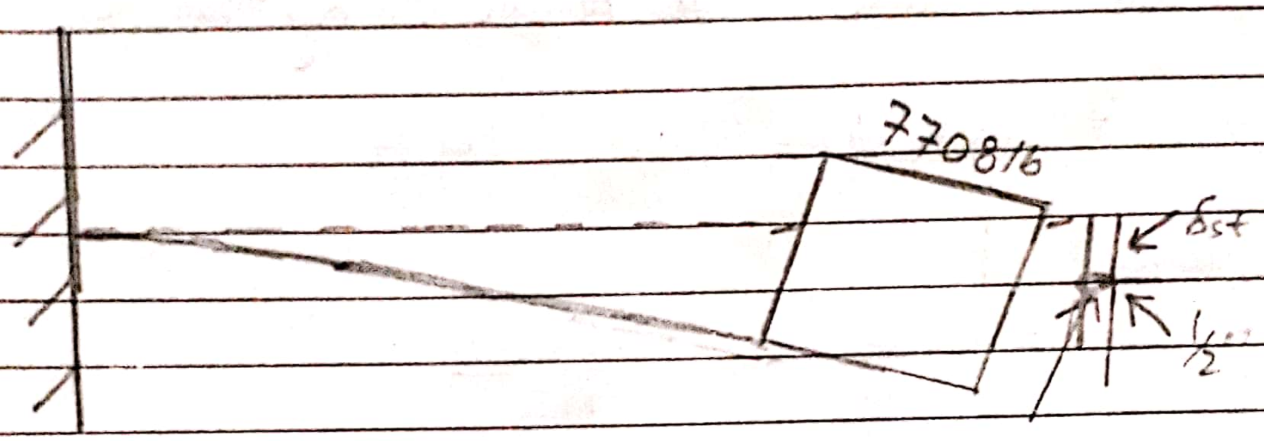
# Question : No.1




## SOLUTION

The general E.O.M for SDOF system is

$$ku + c\dot{u} + m\ddot{u} = p(t)$$

In our case undamped (c=0) undergoing free vibration (P(t)=0)

Hence general EOM becomes $ku + m\ddot{u} = 0$ ...(1)

$$k = 3EI/L^3$$

$$= \frac{3 \times 29000 \frac{k}{in^2} \times 150 in^4}{(10 \times 12 in)^3}$$

$$k = 7.55 k/in$$

In order to eliminate the chance of mistake during calculation, it is

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more appropriate to use fundamental units like lb, ft sec or kg, m, sec.

$k = 7.55\, k/in = 90625\, lb/ft$

Now

$$\omega_n = \sqrt{\frac{k}{m}} \Rightarrow \sqrt{\frac{90625}{m}}$$

$$m = \frac{7708\, lb\,sec^2}{32.2\, ft} = 239.37\, slug$$

Now put value of m in $\omega_n$ formula

$$\omega_n = \sqrt{\frac{90625}{239.37}} = 19.45\ rad/sec$$

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{19.45} = 0.323\ sec$$

Substituting the corresponding values in eq-1

$$90625u + 239.37\ddot{u} = 0$$

Where k is in lb/ft and 'm' is in lb sec/ft

General Solution to the EOM for undamped

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free vibration is.

$$u(t) = u(0)\cos(\omega_n t) + \dot{u}(0)/\omega_n \sin(\omega_n t)$$

$$u(0) = 1/2'' = \frac{1}{24} ft \text{ and } \dot{u}(0) = 0$$

$$u(t) = \left(\frac{1}{24}\right)\cos(19.45t) + 0 = \left(\frac{1}{24}\right)\cos(19.45t)$$

Equivalent static Force at any time "t" is

$$f_s(t) = k.u(t) = \frac{90625\cos(19.45t)}{24}$$

$$f_s(t) = 3776\cos(19.45t)$$

Amplitude of dynamic displacement, $u_o$ for undamped free vibration is

$$u_o = \sqrt{\left[(u(0))^2 + \left[\frac{\dot{u}(0)}{\omega_n}\right]^2\right]}$$

$$= \sqrt{\left(\left(\frac{1}{24}\right)^2 + 0\right)} = \frac{1}{24} ft$$

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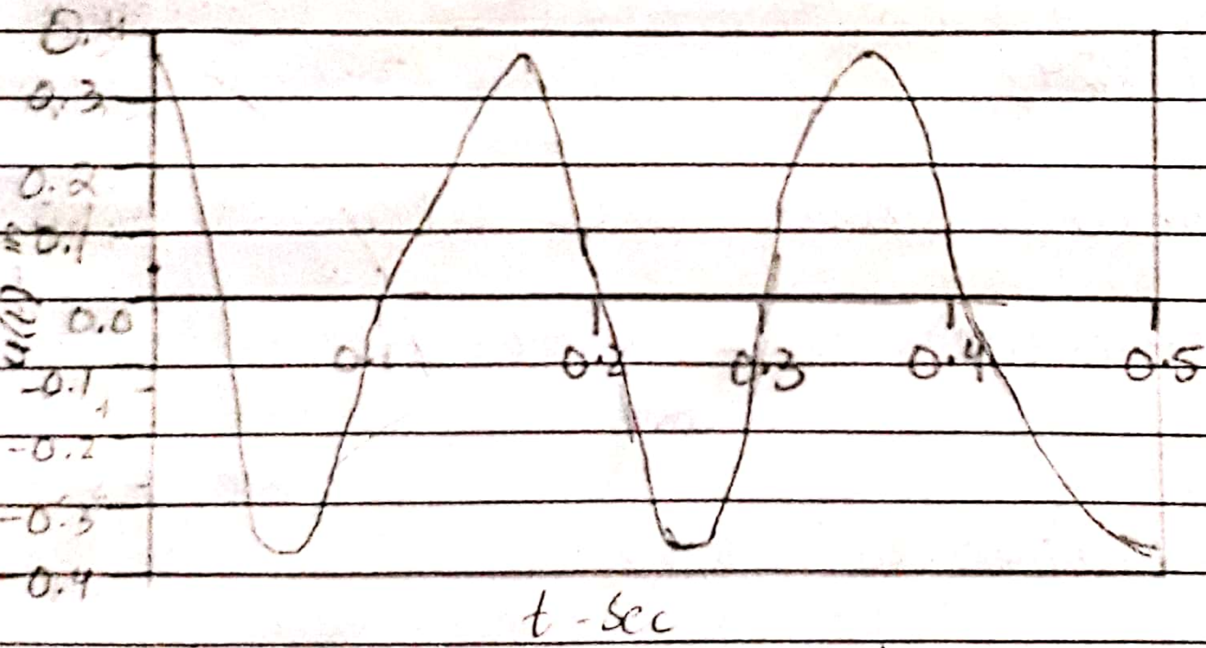
Amplitude for equivalent Static force, $f_{so}$

$$Ku_0 = 90625 \times \frac{1}{24} = 377615$$

# GRAPH

## Variation of Displacement with time




$$u(t) = \left(\frac{1}{24}\right) \cos(19.45(t))$$

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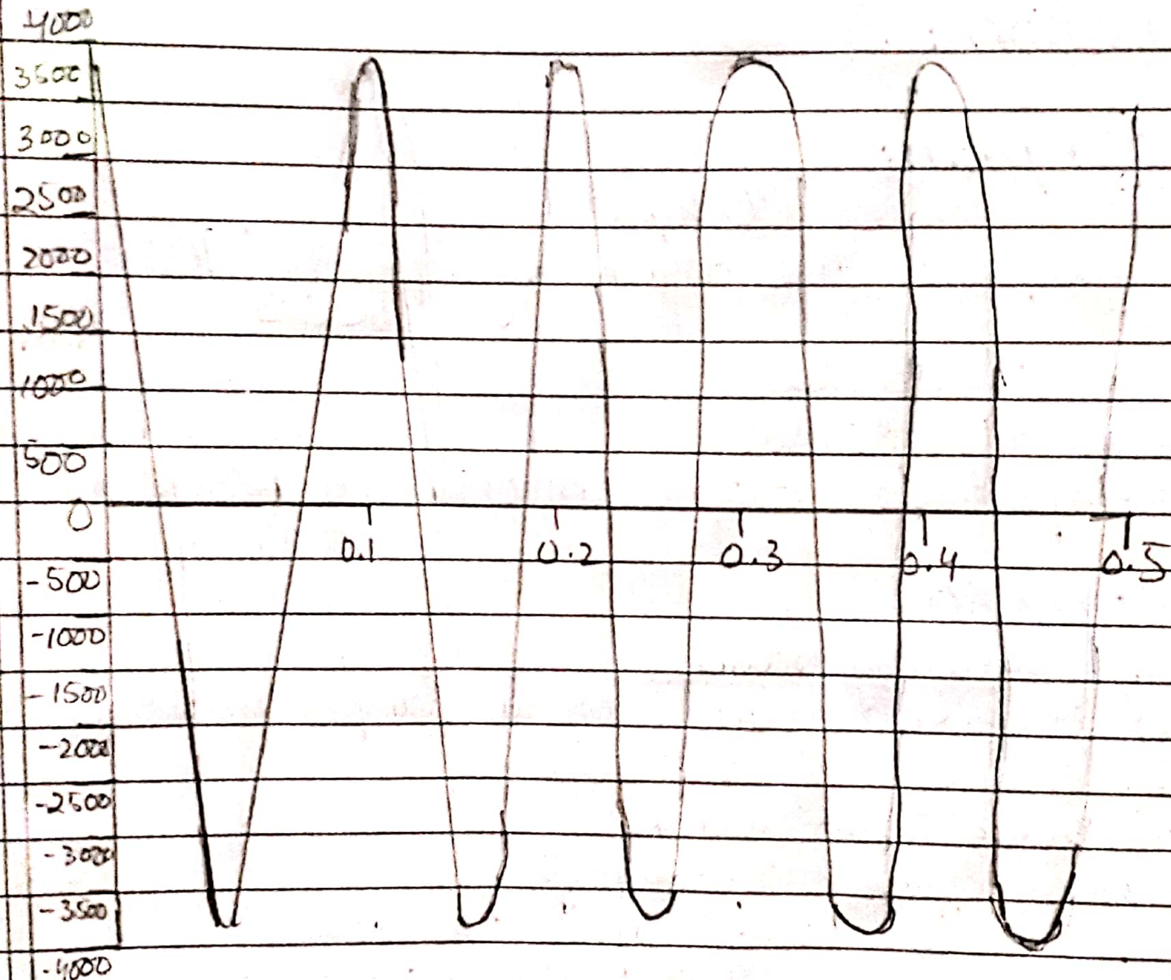


Variation of Equivalent static Forces with time

$f(t) = 3776 \cos(19.45t)$

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# Question = 2

## GIVEN DATA

Damping ratio of reinforced Concrete with considerable cracking = 3-5%

→ So we take $\zeta_0 = 3\%$.

→ Other data are taken from Question = 1

## REQUIRED DATA

⇒ Develop and solve the equation of motion for vibration at free end?

⇒ Develop an equation showing variation in Equivalent static forces with time = ?

## SOLUTION

As we know that EOM (equation of motion) for damped free vibration is;

$$ku + c\dot{u} + m\ddot{u} = 0 \rightarrow (1)$$

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As we know that from question no 1 i.e

$K = 90625\ lb/ft$

$m = 239.37\ lb.se^2/ft$

$w_n = 19.45\ rad/sec$

As we know that

$C = \zeta \times 2 m w_n$

$C = (0.03) \times 2 \times 239.37 \times 19.45$

$C = 279.34\ lb.sec/ft$

By putting values in eq (1) we get

$90625 u + 279.34 \dot{u} + 239.37 \ddot{u} = 0$

Solution to EOM for damped free vibration is;

$$u(t) = e^{-\zeta w_n t}\left(u(0)\cos(w_D t) + \frac{1}{w_D}\left[\dot{u}(0) + u(0)\zeta w_n\right] \cdot \sin(w_D t)\right)$$

$w_D = 19.46\ rad/sec$

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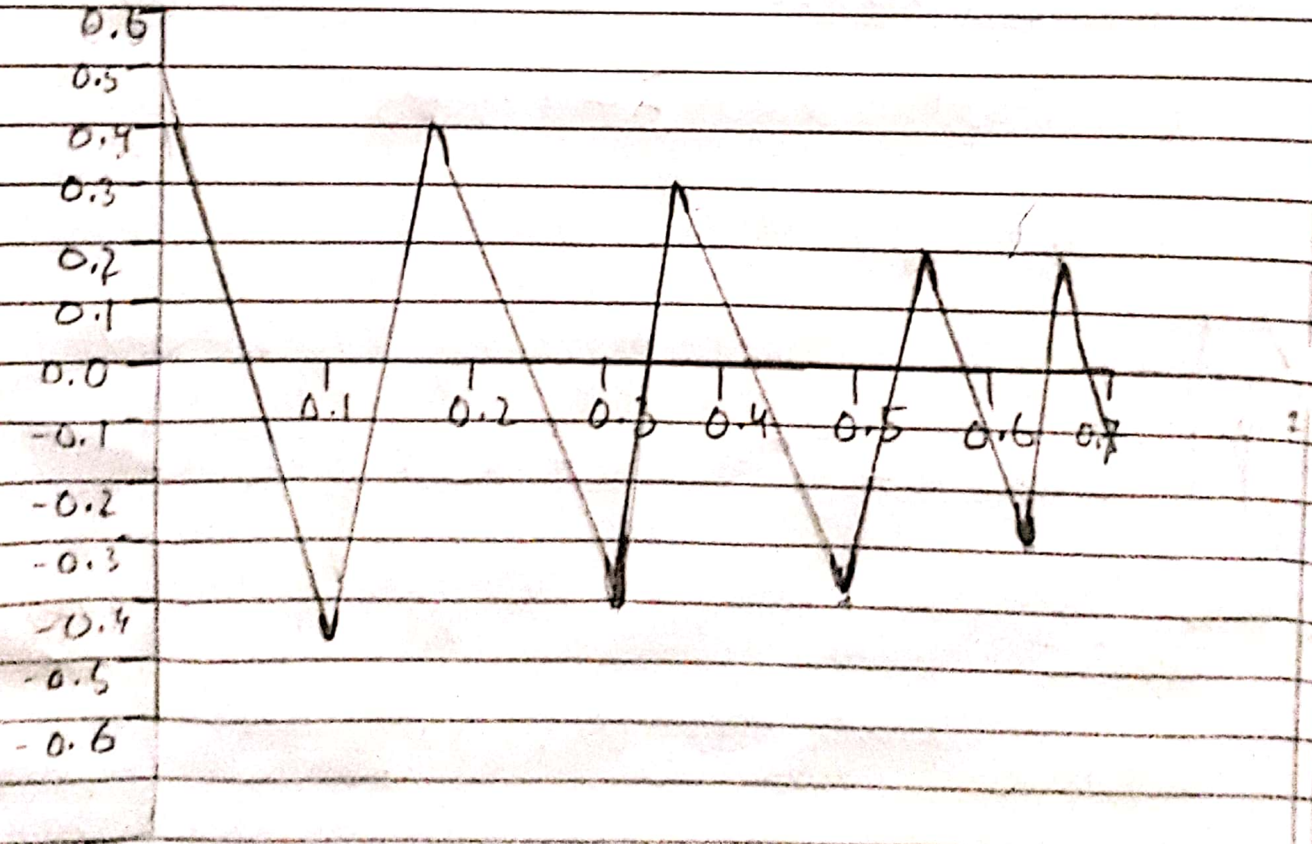
$$u(t) = e^{-(0.03\times 19.45\times t)}\left[\frac{1}{24}\times \cos(19.45t) + \frac{1}{19.45}\left(0 + \frac{1}{24}\times 0.03\times 19.45 \times \sin(19.45t)\right)\right]$$

$$u(t) = e^{-0.584t}\left[0.042\cos(19.45t) + 0.024\sin(19.45t)\right]$$

$$f_s(t) = Ku(t) = 90625\times u(t)$$

$$f_s(t) = e^{-0.584t}\left[3806.25\cos(19.45t) + 2175\sin(19.45t)\right]$$




Variation of Displacement Vs time

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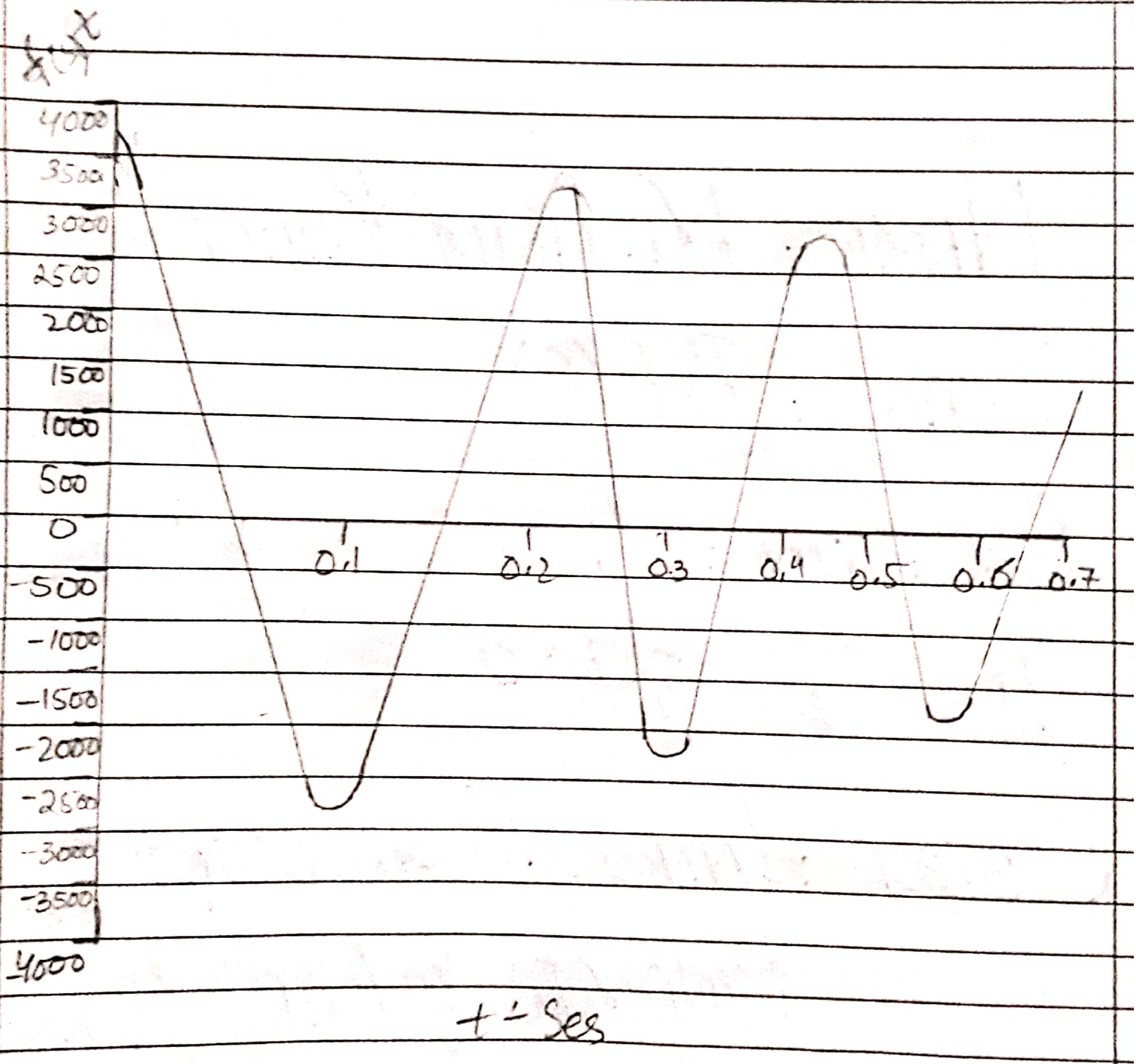



Variation of Equivalent Static Forces with Time

# Question : 3

## GIVEN DATA:

Force = 60 kips

displace the tank by = $\left(\frac{7708}{1000}\right)$ inch

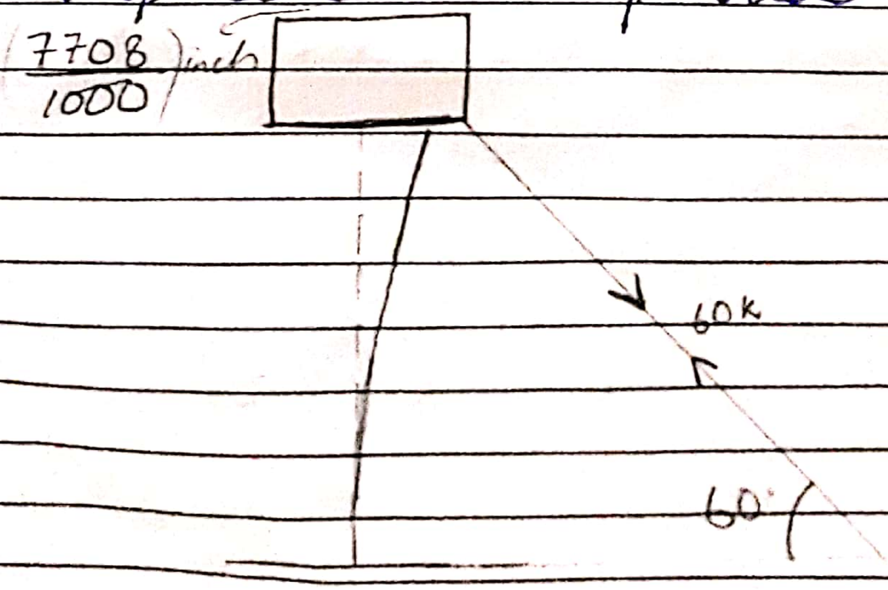
T = 7 cycles

Completion t = 3.57 sec

amplitude displacement = 2.286 cm
= 0.9 inch

## REQUIRED

a) Damping ratios
b) Natural period of un damped vibration
c) Stiffness of structures
d) Weight of tank
e) Damping coefficient
f) Number of cycles to reduce the displacement amplitude to 0.5"



SOLUTION

$$u_1 = \frac{7708}{1000} \text{ inch}$$

$u_1 = 7.708$ inch

After $j = 7$, $u_{j+1} = u_8 = 0.9$ inch

a) Damping ratio = $\zeta$

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$\Rightarrow 7 = \frac{1}{2\pi\zeta} \ln\left(\frac{7.708}{0.9}\right)$$

$\zeta = 0.047 = 4.72\%$

b) $T_n = ?$ Natural period of undamped vibration

7 cycles of vibrations are completed in 3.57 sec.

Time period to complete one cycle $= \frac{3.57}{7}$

$T_D = 0.51$ sec

As we know that

$$\omega_D = \omega_n \sqrt{(1-(\zeta)^2)}$$

$$\frac{2\pi}{\omega_D} = 2\pi / \omega_n \sqrt{1-\zeta^2)}$$

$$\Rightarrow T_D = T_n / \sqrt{1-(\zeta)^2}$$

$$T_n = T_D \sqrt{(1)-(\zeta)^2}$$

By Putting value's we get;

~~$T_n = (9.8 \sqrt{(1)-(0.0488)^2}$~~

~~$T_n = 10.5099$~~

$$T_D = T_n = 0.5 \sqrt{(1)-(0.047)^2}$$

$$T_n = 0.499 \text{ sec}$$

c) Stiffness of Structures = k

As we know that

$$k = \frac{60 \cos 60°}{7.708}$$

$$k = 3.89 \text{ k/in} \Rightarrow ~~29\,889.12~~ \text{ lb/ft}$$

46680 lb/ft

d) Weight of tank $= W =$

As we know that

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{\left(\frac{W}{g}\right)}} = \sqrt{\frac{K.g}{W}}$$

$$\omega_n^2 = \frac{K.g}{W}$$

$$W = \frac{K.g}{\omega_n^2} \quad \text{-----} > (a)$$

Also $\omega_n = \frac{2\pi}{T_n}$

eq(a) $\Rightarrow$

$$W = \frac{K.g}{\frac{4\pi^2}{T_n^2}}$$

$$W = \frac{K.g \times T_n^2}{4\pi^2}$$

By putting value we get

$$W = \frac{46680 \times 32.2 \times (0.499)^2}{4\pi^2}$$

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W = 9480.4 lb

W = 9.4804 K

e) Damping Coefficient = C

As we know that

$$\zeta = \frac{c}{2m\omega_n}$$

$$C = \zeta \times 2m\omega_n = \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$C = \zeta \times 2m\left(\frac{2\pi}{T_n}\right)$$

By putting values.

$$C = \frac{0.047 \times 2 \times 2 \times \pi \times (9480.4/32.2)}{0.499}$$

$$\boxed{C = 348.4 \text{ lb.sec/ft}}$$

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f Number of cycles to reduce the displacement amplitude to 0.5" = j:

As we know that;

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$j = \frac{1}{2\times\pi\times 0.047} \ln\left[\frac{7.708}{0.5}\right]$$

j = 9.26 say 9 cycles