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Section A

Subject Advance fluid mechanics

Q.No 1a

Define Drag with its components write down the equations for friction Drag coeficient both in laminar and turbulant bondary layer.

Ans:- forced on Immersed Bodies.

A body which is whely immersed in a homegenous fluid may be Subjected two kind at forces araising from relation ~~are~~ motion b/w body & fluid there forces are termed are drago & lift depending an forces either parallel or rigid angle to motion.

Drag force on Submerged body can have 2 components.

1) pressure Drag (FP) → It is equal to the integration of component in direction of motion of all pressure force exerted on Surface of body.

$$F_p = C_P \cdot \int \frac{v^2}{2} \cdot A$$

Where (CP) depend on Shape.

2) friction Drag:- It is equal to integration ad component of all Shear Stress along the Surface in direction of motion

$$F_p = C_f \cdot \int \frac{v^2}{2} (BL)$$

" Cf " depend on velocity.

Friction Drag of Boundry layer :

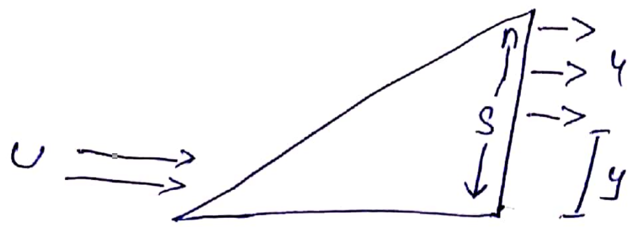
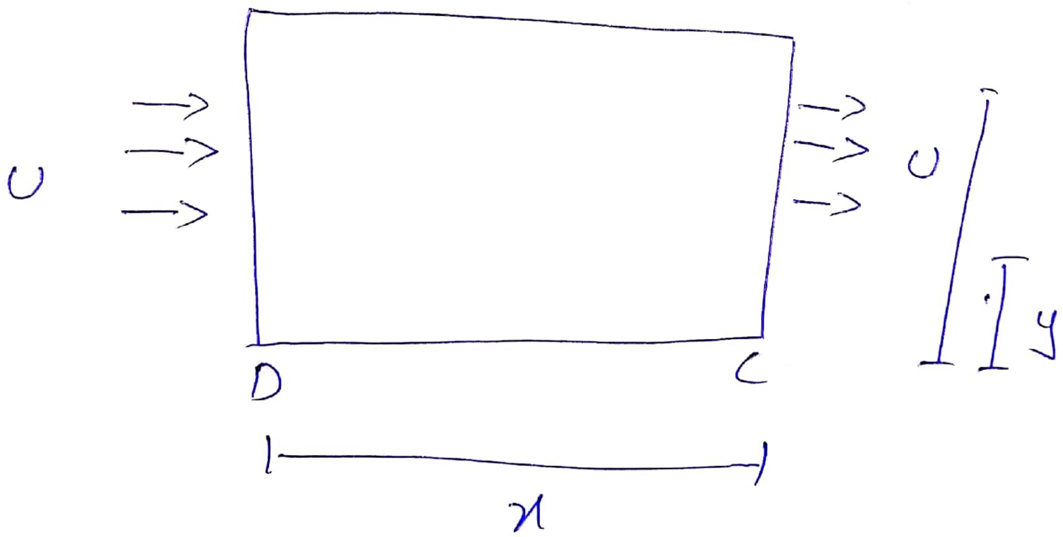


Figure Shows growth of Boundry layer along one Side of Smooth plate in Steady flow of incompressible fluid consider volume where δ is the thickness of boundary layer & U is the undisturbed velocity.



As we have $\Sigma F_x = 0$

where

(3)

$$F_x = \frac{\Delta p}{\Delta t} = \frac{\Delta m v}{\Delta t} \quad \because m = \rho v$$

$$F_x = \frac{\Delta \rho \cdot v_o l \cdot v}{\Delta t} = \Delta \rho Q v$$

$$F_x = \Delta \rho Q v$$

$-F_x$ = rate of change of $BC + AB - AD$

$$AD = \rho U (U \delta B)$$

$$BC = \rho B (U^2 dy)$$

$$AB = \rho U (UB\delta) - B \int_0^{\delta} u\,dy$$

$$F_x = \rho B \int_0^{\delta} u (U - u)\,dy$$

integration on b/s — (1)

$$F_x = \rho B U^2 \delta \alpha$$

Where α is a fuction of boundry layer velocity distribution.

Now to find Shear Stress

$$\tau = \frac{F_x}{A} = \frac{dF_x}{B\,dx} = \frac{dF_x}{B\,dx}$$

$$\bar{\tau} = \rho B U^2 \alpha \frac{d\delta}{B\,dx} = \rho U^2 \alpha \frac{d\delta}{dx}$$

$$\boxed{\bar{\tau}_o = \rho U^2 \alpha \frac{d\delta}{dx}}$$

laminar boundary layer:-

$$\frac{u}{U} = f\left(\frac{y}{\delta}\right) \quad - (1)$$

$$\frac{y}{\delta} = \eta \Rightarrow y = \delta\eta$$

$$dy = \delta d\eta \quad - (2)$$

$$\frac{u}{U} = f(\eta)$$

$$du = U df(\eta) \quad - (3)$$

for laminar flow

$$\tau_0 = \frac{\mu du}{dy} \quad - (4)$$

$$\tau_0 = \mu \frac{U df(\eta)}{\delta d(\eta)}$$

$$\tau_0 = \frac{\mu U B}{\delta} \quad - (5)$$

As we have $\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$

Compare both

$$\rho U^2 \alpha \frac{d\delta}{dx} = \frac{\mu U B}{\delta}$$

5 8

$$\delta d\delta = \frac{\mu B dx}{\rho U a}$$

Int on both side

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U a} x + c$$

$$\delta = \frac{\sqrt{2B}}{d} \cdot \sqrt{\frac{\mu x}{\rho U}}$$

$$R_x = \frac{U x \rho}{\mu}$$

$$B = 1.63, \ d = 0.135$$

$$\delta = \frac{4.91 x}{\sqrt{R_x}} \quad \text{--- (6)}$$

Where (R_x) is Local Reynold number

As we have

$$\tau_o = \frac{\mu U B}{\delta}$$

Put equation (6) in 5

$$\tau_o = 0.332 \frac{\mu U}{x} \sqrt{R_x}$$

Now

$$F_f = B \int_0^L \tau_o dx$$

(6)

Where $\tau_0 = 0.332 \frac{\mu u}{x} \sqrt{R_x}$

$$R_x = \frac{x u \rho}{\mu}$$

then put values

$$F_f = 0.664 \sqrt{\rho \mu L U^3}$$

$$F_f = C_f \frac{\rho v^2}{2} B L$$

equation on b/s

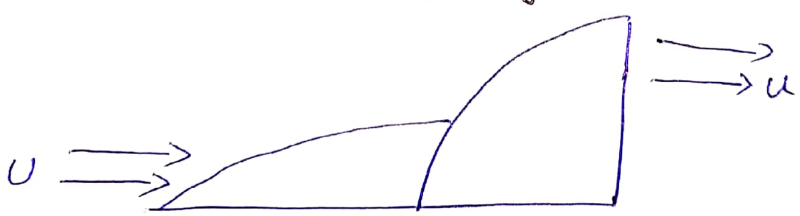
$$C_F = 1.328 \sqrt{\frac{\mu}{\rho L v}}$$

$$\boxed{C_F = \frac{1.328}{\sqrt{R_x}}}$$

for

Laminar $R < 500{,}000$.

Turbulent Boundary layer:-



The figr Shows that velocity distribution of boundary layer which is steeper near walls & flater through out reminder of layer.

The shear stress is greater in turbulent then in laminar.

Thus
$$\tau_0 = \rho f \frac{v^2}{8}$$

Where v is the average velocity to obtain relation b/w average & max we have

$$\frac{V}{U_{max}} = \frac{1}{1+1.33\sqrt{0.023}}$$

$$U = 1.235V$$

$$V = \frac{U}{1.235}$$

&
$$F = \frac{0.31\gamma}{(Rh)^{1/4}} \qquad \because Rh = \left(\frac{Dv}{\nu}\right)$$

$$D = 2\delta$$

$$\tau_0 = f\rho \frac{v^2}{8}$$

$$\tau_0 = \frac{0.316}{\left(\left(\frac{D}{\nu}\right)\left(\frac{U}{1.235}\right)\right)^{1/4}} \cdot \frac{\rho}{8}\left(\frac{V}{1.235}\right)^2$$

$$\tau_0 = \frac{0.023\rho v^2}{\left(\frac{2\delta}{\nu}\right)^{1/4}} \quad \text{— (1)}$$

As we have general eq 4

$$\tau_0 = \rho v^2 \alpha \frac{d\delta}{du} \quad -2$$

eq 4 1 & 2

$u = 0, \; \delta = 0$

$$\delta = \frac{(0.0287)^{4/5}}{\alpha} \left(\frac{v}{ux}\right)^{1/5} \cdot x$$

$\alpha = 0.0972$

$$\boxed{\delta = \frac{0.377}{(Rh)^{1/5}} \cdot x}$$

$$\tau_0 = 0.0587 \rho \frac{v^2}{2} \left(\frac{v}{ux}\right)^{1/5}$$

Now

$$Ff = B \int_0^L \tau_0 \, dx$$

$$Ft = 0.0735 \rho \frac{v^2}{2} \left(\frac{v}{vL}\right)^{1/5} \cdot Bc$$

$$Ff = Cf \cdot \rho \frac{v^2}{2} BL$$

equation b/s

$$CF = \frac{0.0735}{(R)^{1/5}} \quad (500000 < R < 10^7)$$

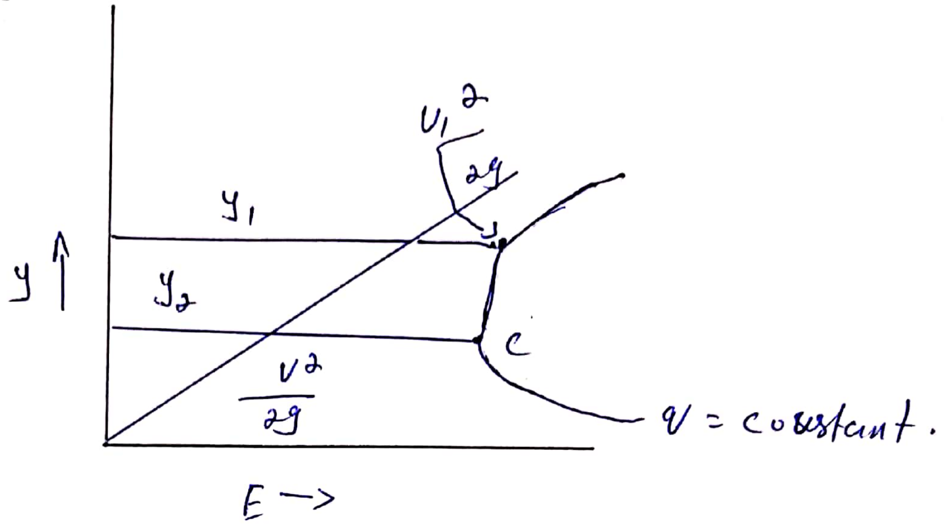
For $R > 10^7$

$$\boxed{CF \; \frac{0.455}{(\log R)^{2.58}}}$$

(9)

Q1 (b)

Derive equation for critical depth critical velocity of rectangular Section of a channel.



This is Specific energy equation :-
for particular q There will be two kind of possible value of y for given E. The equation is cubic with three roots with third being negative giving no values Thus two alternative depth represents two totally different flow regimes - Slow & deep an upper position & fast & Shallow on lower position.
point represent dividing point between two regims of flow Thus for given "q" values of E in minimum & flow at this point is Critical flow Depth of flow at this point is critical depth y_c & velocity at this point is critical velcity.

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The relation of critical depth can be found as

$$E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

for minmum Specific energy.

$$\frac{dE}{dy} = 0$$

$$\frac{dE}{dy} = 1 - \frac{2}{2g}\left(\frac{q^2}{y^3}\right)$$

$$\frac{dE}{dy} = 1 - \frac{q^2}{gy^3}$$

$$1 = \frac{q^2}{gy^3} = q^2 = gy^3$$

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3} \text{ critical depth}$$

As $q = vy$, $v_c^2 = gy^3$

OR $\boxed{v_c = \sqrt{g y_c}}$ critical velocity

$$y_c = \frac{v_c^2}{g}$$

Scanned with CamScanner

(11)

Now

$$\frac{y_c}{2} = \frac{V_c^2}{2g}$$

$$E_{min} = y_c + \frac{V_c^2}{2g} = y_c + \frac{y_c}{2}$$

$\frac{3}{2} y_c$ oR $c_y = \frac{2}{3}$ constant

Depth of flow	subcritical $y > y_c$	critical $y = y_c$	Super critical $y < y_c$
velocity Slop	$V < V_c$ maild Slop $S_o < S_c$	$V = V_c$ critical slop	$V > V_c$

(12)

Q 2

Given Data:-

Depth of Rectangular channel $(d) = ?$

flow rate $(Q) = 3.5 m^3/sec$

Slop of Bed $(S_o) = 0.0008$

$n = 0.0219$

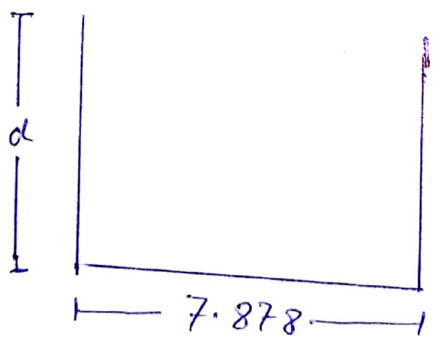
Width of bed = 7878mm

= 7.878m

Critical depth = ?

Flow Subcritical or

Sol:-



Area = $7.878 \times d$

= $7.878d$

parameter = $d + 7.878 + d$

= $7.878 + 2d$

Hydraulic Radius $(R_h) = A/P$

$$A/P = \frac{7.878d}{7.878+2d}$$

By using manning equ.

$$Q = \frac{1}{n} A R_h^{2/3} (S_0)^{1/2} \quad \text{---} \ast$$

put value in $(\ast)$ eq^n

$$3.5 = \frac{0}{0.0219} \times 7.878\,d \times \left(\frac{7.878\,d}{2d + 7.8\cancel{7}8}\right) \times (0.008)^{1/2}$$

$$\boxed{d = 0.55\,m}$$

$$\text{Area} = 7.878(0.55)$$

$$= 4.3329\,m^2$$

$$\text{paramet} = 7.878 + 2(0.55)$$

$$= 8.978\,m$$

$$\text{Hydraulic Radius } (R_h) = \frac{4.37}{8.978}$$

$$= 0.4867\,m$$

(14)

Finding critical depth.

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}$$

As $q = Q/B$

$$= \frac{3.5}{7.878}$$

$$= 0.4442 \ m^2/sec$$

$$y_{cr} = \left(\frac{(0.4442)^2}{9.81}\right)^{1/3}$$

$$= 0.27195$$

$y > y_{cr}$ (So flow is sub critical

$0.55 > 0.27$

$(0.1973)^{0.333}$

(15)

Q3

Given data

Friction Drag (F_D) = ?

Width (B) = 200mm = 0.2m

Length (L) = 800mm = 0.8m

undisturbed velocity (v) = 5m/sec

Specific gravity (S) = 0.89.

kinematic viscosity (ν) = 0.98×10^{-4} m^2/sec.

Sol:.

Check whether flow is laminar or not By Reynold number

$$R = \frac{Dv}{\nu}$$

for Smooth plate

$D = L$; $V = U$

So

$$R = \frac{Lv}{\nu}$$

$$= \frac{0.8 \times 5}{0.98 \times 10^{-4}} = 43010$$

43010 < 500,000 → Laminar.

(16)

By using formula.

$$F_f = C_f \cdot \rho \cdot \frac{v^2}{2} \cdot BL$$

Where

$$C_f = \frac{1.328}{\sqrt{R}} = \frac{1.328}{\sqrt{43010}}$$

$$C_f = 0.0064$$

$$S = \frac{\rho_{oil}}{\rho_{water}} = 0.89 = \frac{\rho_{oil}}{1000}$$

$$\rho_{oil} = 0.89 \times 1000$$

$$\rho_{oil} = 890\, kg/m^3$$

$$F_f = C_f \cdot \rho \cdot \frac{v^2}{2} \cdot BL$$

$$F_f = 0.0064 \times 890 \times \frac{(5)^2}{2} \times 0.2 \times 0.8$$

$$\boxed{F_f = 11.39\,N}$$