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NAME = JAVERIA SHAMS

I.D = 13785

DESCIPLINE = B.S (RADIOLOGY)

SUBJECT = BIO-STATISTICS

SUBMITTED TO = SIR-ANWAR SHAMIM

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QUESTION 1 :-

Part (a) :-

Calculate the correlation coefficient b/w x and y.

Price (x)	3	4	5	6	7	8	9	10	11	13
Demand (Y)	25	24	20	20	19	17	16	13	10	8

ANSWER OR SOLUTION :-

X	Y	XY	X^2	Y^2
3	25	75	9	625
4	24	96	16	576
5	20	100	25	400
6	20	120	36	400
7	19	133	49	361
8	17	136	64	289
9	16	144	81	256
10	13	130	100	169
11	10	110	121	100
13	8	104	169	64
$\Sigma x = 76$	$\Sigma Y = 172$	$\Sigma xY = 1148$	$\Sigma x^2 = 670$	$\Sigma Y^2 = 3240$

$$r = \frac{n\Sigma xY - \Sigma x \, \Sigma Y}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma Y^2 - (\Sigma Y)^2]}}$$

Here

$n = 10, \Sigma xY = 1148, \Sigma x = 76, \Sigma Y = 172$

②

$\Sigma x^2 = 670 \;, \Sigma y^2 = 3240$

$$r = \frac{(10)(1148) - (76)(172)}{\sqrt{[(10)(670) - (76)^2][(10)(3240) - (172)^2]}}$$

$$r = \frac{11480 - 13072}{\sqrt{(6700 - 5776)(32400 - 29584)}}$$

$$r = \frac{-1592}{\sqrt{(924)(2816)}}$$

$$r = \frac{-1592}{\sqrt{2601984}}$$

$$r = \frac{-1592}{1613.06}$$

$$r = \boxed{-0.98}$$

★ Part (b) :-

Given the following set of values

X	20	11	15	10	17	18	21	25	28
Y	5	15	14	17	8	9	12	16	18

(a) Determine the equation of the least squares regression line of Y on X and X on Y.

(b) Find the predicted values of Y for X = 20, 11, 15, 25, 28 and X for Y = 5, 15, 9, 12, 16, 18.

★ (a) :-

★ Solution :-

Regression line Y on X is

$$\hat{Y} = a + bx \quad \text{—— (A)}$$

Now the two normal equations are:

$$\Sigma Y = \Sigma a + b\Sigma x$$

$$\Sigma Y = na + b\Sigma X \quad \text{—— (1)}$$

$$\Sigma XY = a\Sigma X + b\Sigma X^2 \quad \text{—— (2)}$$

eq (1) $\Rightarrow$ $\Sigma Y = na + b\Sigma X$

$$\Sigma Y - b\Sigma X = na$$

$$na = \Sigma Y - b\Sigma X$$

Dividing on both sides by 'n'

$$\frac{\cancel{n}a}{\cancel{n}} = \frac{\Sigma Y}{n} - b\frac{\Sigma Y}{n}$$

$$a = \bar{Y} - b\bar{X} \quad \Rightarrow \text{(B)}$$

R.W

$$\bar{Y} = \frac{\Sigma Y}{n}$$

$$\bar{X} = \frac{\Sigma X}{n}$$

Multiplying eq 1 on ~~bonth~~ both sides by Σx and eq 2 on both sides by "n" and subtracting

$$+\Sigma x \Sigma Y = \cancel{na}\Sigma x + b(\Sigma x)^2$$
$$\pm n\Sigma xY = \pm \cancel{na}\Sigma x \pm bn\Sigma x^2$$

$$\Sigma x\Sigma Y - n\Sigma xY = b(\Sigma x)^2 - bn\Sigma x^2$$

$$\cancel{-}\frac{(n\Sigma xY - \Sigma x\Sigma Y)}{n\Sigma x^2 - (\Sigma x)^2} = \cancel{-}\frac{b[n\Sigma x^2 - (\Sigma x)^2]}{\cancel{n\Sigma x^2 - (\Sigma x)^2}}$$

$$\Rightarrow b = \frac{n\Sigma xY - \Sigma x\Sigma Y}{n\Sigma x^2 - (\Sigma x)^2} \quad \text{--- } C$$

X	Y	X^2	Y^2	XY
20	5	400	25	100
11	15	121	225	165
15	14	225	196	210
10	17	100	289	170
17	8	289	64	136
18	9	324	81	162
21	12	441	144	252
25	16	625	256	400
28	18	784	324	504
$\Sigma x = 165$	$\Sigma Y = 114$	$\Sigma x^2 = 3309$	$\Sigma Y^2 = 1604$	$\Sigma xY = 2099$

Here $n = 9$, $\Sigma xY = 2099$, $\Sigma x = 165$, $\Sigma Y = 114$, $\Sigma x^2 = 3309$

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Now eq C $\Rightarrow$

$$b = \frac{n\sum XY - \sum X \sum Y}{n\sum X^2 - (\sum X)^2}$$

$$b = \frac{(9)(2099) - (165)(114)}{(9)(3309) - (165)^2}$$

$$b = \frac{18891 - 18810}{29781 - 27225}$$

$$b = \frac{81}{2556}$$

$$= \boxed{0.03}$$

R.W

$\bar{Y} = \frac{\sum Y}{n} = \frac{114}{9} = 12.66$

$\bar{X} = \frac{\sum X}{n} = \frac{165}{9} = 18.33$

Now eq B $\Rightarrow$

$a = \bar{Y} - b\bar{X}$

$a = 12.66 - (0.03)(18.33)$

$a = 12.66 - 0.55$

$a = \boxed{12.11}$

Now the fitted regression line Y on X is

$$\hat{Y} = a + bX$$

$$\boxed{\hat{Y} = 12.11 + 0.03X} \quad \text{——} \; (x)$$

Now fitted regressine line X on Y is

$$\hat{X} = a + bY \quad \text{——} \; (D)$$

The two normal equations are

$$\sum X = na + b\sum Y \quad \text{——} \; (3)$$

$$\sum XY = a\sum Y + b\sum Y^2 \quad \text{——} \; (4)$$

eq (3) $\Rightarrow$ $\sum X = na + b\sum Y$

$$na = \Sigma X - b\Sigma Y$$

Dividing on both sides by 'n'

$$\frac{\cancel{n}a}{\cancel{n}} = \frac{\Sigma X}{n} - b\frac{\Sigma Y}{n}$$

$$a = \bar{X} - b\bar{Y} \quad \text{—— (E)}$$

Multiplying eq (3) on both sides by 'ΣY' and eq (4) on both sides by 'n' and subtracting

$$\Sigma X\Sigma Y = na\Sigma Y + b(\Sigma Y)^2$$

$$\pm n\Sigma XY = \cancel{\pm na\Sigma Y} \pm nb\Sigma Y^2$$

$$\Sigma X\Sigma Y - n\Sigma XY = b(\Sigma Y)^2 - nb\Sigma Y^2$$

$$\frac{-(n\Sigma XY - \Sigma X\Sigma Y)}{n\Sigma Y^2 - (\Sigma Y)^2} = \frac{-b[n\Sigma Y^2 - (\Sigma Y)^2]}{n\Sigma Y^2 - (\Sigma Y)^2}$$

$$b = \frac{n\Sigma XY - \Sigma X\Sigma Y}{n\Sigma Y^2 - (\Sigma Y)^2}$$

Now $n = 9$, $\Sigma XY = 2099$, $\Sigma X = 165$

$\Sigma Y = 114$, $\Sigma Y^2 = 1604$

$$b = \frac{(9)(2099) - (165)(114)}{(9)(1604) - (114)^2}$$

$$b = \frac{18891 - 18810}{14436 - 12996}$$

$$b = \frac{81}{1440}$$

$$= \boxed{0.05}$$

R.W

$$\bar{X} = \frac{\Sigma X}{n} = \frac{165}{9} = 18.33$$

$$\bar{Y} = \frac{\Sigma Y}{n} = \frac{114}{9} = 12.66$$

Eq (E) $\Rightarrow a = \bar{X} - b\bar{Y}$

$$a = 18.33 - (0.05)(12.66)$$

$$a = 18.33 - 0.633$$

$$a = \boxed{17.69}$$

Now fitted regression line X on Y is

$$\hat{X} = a + bY$$

$$\boxed{\hat{X} = 17.69 + 0.05Y} \text{ ——— (xx)}$$

★ Part (b):-

Predicted Regression Y on X

X	$\hat{Y} = 12.11 + 0.03X$
20	$\hat{Y} = 12.11 + 0.03(20) = 11.51$
11	$\hat{Y} = 12.11 + 0.03(11) = 11.78$
15	$\hat{Y} = 12.11 + 0.03(15) = 11.66$
25	$\hat{Y} = 12.11 + 0.03(25) = 11.36$
28	$\hat{Y} = 12.11 + 0.03(28) = 11.27$

Predicted Regression X on Y

Y	$\hat{X} = 17.69 + 0.05Y$
5	$\hat{X} = 17.69 + 0.05(5) = 17.94$
15	$\hat{X} = 17.69 + 0.05(15) = 18.44$
9	$\hat{X} = 17.69 + 0.05(9) = 18.14$
12	$\hat{X} = 17.69 + 0.05(12) = 18.29$
16	$\hat{X} = 17.69 + 0.05(16) = 18.49$
18	$\hat{X} = 17.69 + 0.05(18) = 18.59$

* QUES 2 :-

Find the following:

(a) A fair coin is tossed 5 times. Find the probabilities of obtaining various numbers of heads.

(b) A and B play a game in which A,s probability of wining is 2/3. In a series of 10 games, what is the probability that A will win (i) at least 4 games (ii) Exactly equal to 4/10 games (iii) Exactly equals to 11 games (iv) 60 or more games.

* Part (a) :-

* Solution :-

Here $n = 5$

Let P: probability of head

$$P = 1/2$$

$$P + q = 1$$

$$q = 1 - P$$

$$q = 1 - 1/2$$

$$\boxed{q = \frac{1}{2}}$$

Let X = no of head

$$x = 0, 1, 2, 3, 4, 5$$

Now

$$P[X = x] = \binom{n}{x} P^x q^{n-x}, \; x = 0, 1, \ldots n$$

$$P(X = x) = \binom{5}{x} (1/2)^x (1/2)^{5-x}; \; x = 0, 1, 2 \ldots 5$$

→ (1)

Put $x = 0$ in eq 1

$P(X=0) = \binom{5}{0}(1/2)^0(1/2)^{5-0}$

$P(X=0) = (1)(1)(\frac{1}{2})^5 = \boxed{\frac{1}{32}}$

Put $x=1$ in eq 1

$P(X=1) = \binom{5}{1}(1/2)^1(1/2)^{5-1}$

$P(X=1) = (5)(\frac{1}{2})(1/2)^4 = \boxed{5/32}$

Put $x=2$ in eq 1

$P(X=2) = \binom{5}{2}(\frac{1}{2})^2(1/2)^{5-2}$

$P(X=2) = (10)(\frac{1}{2})^2(1/2)^3$

$= \boxed{\frac{10}{32}}$

Put $x=3$ in eq 1

$P(X=3) = \binom{5}{3}(1/2)^3(1/2)^{5-3}$

$P(X=3) = (10)(1/2)^3(1/2)^2$

$= \boxed{\frac{10}{32}}$

Put $x=4$ in eq 1

$P(X=4) = \binom{5}{4}(1/2)^4(1/2)^{5-4}$

$P(X=4) = (5)(1/2)^4(1/2)^1$

$P(X=4) = \boxed{5/32}$

Put $x=5$ in eq 1

$P(X=5) = \binom{5}{5}(\frac{1}{2})^5(1/2)^{5-5}$

$P(X=5) = (1)(\frac{1}{2})^5(\frac{1}{2})^0$

$P(X=5) = (1)(\frac{1}{2})^5(1)$

$P(X=5) = \boxed{\frac{1}{32}}$

In Tabular form

x	0	1	2	3	4	5	Total
$P(X=x)$	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$	1

★ **Part (b):-**

★ **Solution:-**

Let P = Probability of wining players

$P = \boxed{\frac{2}{3}}$

$P = q = 1$

$q = 1 - P$

$q = 1 - \frac{2}{3}$

$q = \boxed{\frac{1}{3}}$

Now

$P(X=x) = \binom{n}{x} p^x q^{n-x}, x = 0, 1, \ldots, n$

$P(X=x) = \binom{10}{x} (\frac{2}{3})^x (\frac{1}{3})^{10-x}, x = 0, 1 \ldots 10$ → 1

(i):

P(At least 4 games) = ?

$P(X \geq 4) = 1 - P(X < 4)$

$= 1 - \sum_{x=0}^{3} \binom{10}{x} (\frac{2}{3})^x (\frac{1}{3})^{10-x}$

$$= 1 - \left[\binom{10}{0} (2/3)^0 (1/3)^{10-0} + \binom{10}{1} (2/3)^1 (1/3)^{10-1} + \binom{10}{2} (2/3)^2 (1/3)^{10-2} + \binom{10}{3} \left(\frac{2}{3}\right)^3 (1/3)^{10-3} \right]$$

$$= 1 - \left[(1/3)^{10} + (10)(2/3)(1/3)^9 + (45)(2/3)^2 (1/3)^8 + (120)(2/3)^3 (1/3)^7 \right]$$

$$= 1 - \left[\frac{1}{59049} + \frac{20}{59049} + \frac{180}{59049} + \frac{960}{59049} \right]$$

$$= 1 - \left[\frac{1 + 20 + 180 + 96}{59049} \right]$$

$$= 1 - \left(\frac{1161}{59049} \right)$$

$$= 1 - 0.019$$

$$P[X \geq 4] = \boxed{0.98}$$

★ **(ii):-**

$P(X = 4/10) = 0$

Because random variable 'X' is binomial distribution takes only one of the integer values 0, 1, 2, ---, n.

★ **(iii):-**

$P(X = 11) = ?$

Here the total number of games of series is 10.

$P(X = 11) = 0$

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★ **(iv):-**

P(6 or more games) = ?

$$P(X \geq 6) = \sum_{x=6}^{10} \binom{10}{x}\left(\frac{2}{3}\right)^x\left(\frac{1}{3}\right)^{10-x}$$

$$P(X \geq 6) = \binom{10}{6}\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^{10-6} + \binom{10}{7}\left(\frac{2}{3}\right)^7\left(\frac{1}{3}\right)^{10-7}$$

$$\binom{10}{8}\left(\frac{2}{3}\right)^8\left(\frac{1}{3}\right)^{10-8} + \binom{10}{9}\left(\frac{2}{3}\right)^9\left(\frac{1}{3}\right)^{10-9} +$$

$$\binom{10}{10}\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^{10-10}$$

$$= (210)\left(\frac{2}{3}\right)^6\left(\frac{1}{3}\right)^4 + (120)\left(\frac{2}{3}\right)^7\left(\frac{1}{3}\right)^3$$

$$+ (45)\left(\frac{2}{3}\right)^8\left(\frac{1}{3}\right)^2 + (10)\left(\frac{2}{3}\right)^9\left(\frac{1}{3}\right)^1 +$$

$$(1)\left(\frac{2}{3}\right)^{10}\left(\frac{1}{3}\right)^0$$

$$P(X \geq 6) = \frac{13440}{59049} + \frac{15360}{59049} + \frac{11520}{59049} +$$

$$\frac{5120}{59049} + \frac{1024}{59049}$$

$$P(X \geq 6) = \frac{13440 + 15360 + 11520 + 5120 + 1024}{59049}$$

$$P(X \geq 6) = \frac{46464}{59049}$$

$$= \boxed{0.78}$$

★ QUES 3 :-

The following figures give the number of children born to 50 women.

2	6	1	5	4	3	3	8	10	1
4	3	3	0	5	2	1	4	10	3
5	3	3	6	3	3	2	2	7	4
1	4	2	4	4	4	6	8	10	7
7	5	6	5	3	2	3	9	2	2

(a) Construct the ungrouped frequency distribution of these data.

(b) Construct the grouped frequency distribution of these data.

★ Part (a) :-

★ Solution :-

Arrange the data in ascending order of magnitude

0, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 7, 7, 7, 8, 8, 9, 10, 10, 10

Let X: no of children born to 50 womens

X	Tellybar	f
0	I	1
1	IIII	4
2	~~IIII~~ III	8
3	~~IIII~~ ~~IIII~~ I	11
4	~~IIII~~ III	8
5	~~IIII~~	5
6	IIII	4
7	III	3
8	II	2
9	I	1
10	III	3
		Total = 50

★ **Part (b):-**

★ **Step 01:-**

Arrange the data in ascending order of magnitude

0, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 7, 7, 7, 8, 8, 9, 10, 10, 10

★ **Step 02:-**

★ **Range:-**

Range = Max: − Min:

Here Max: = 10, Min: = 0

Range 10 − 0 = $\boxed{10}$

★ **Step 03:-**

★ **No of classes (k):**

Put 2^k

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k	2^k
1	2
2	4
3	8
4	16
5	32
$k =$ 6	64
7	128
⋮	⋮

Here n = 50
Inspect until we find the value that exceeds our sample size.
Here our sample size is 50, ~~which~~ 64 exceeds the sample size 50 therefore k = 6.

* Step 04 :-

* **Class Interval (h) :-**

$$h = \frac{\text{Range} + 1}{k}$$

Here Range = 10 , k = 6

$$h = \frac{10+1}{6}$$

$$h = \frac{11}{6}$$

$$h = 1.83 \cong \boxed{2}$$

Now

No of classes = k = 6

Class interval = h = 2

Minimum value = 0

No of children born	Telly bar	f
0 – 1	卌	5
2 – 3	卌 卌 卌 ||||	19
4 – 5	卌 卌 |||	13
6 – 7	卌 ||	7
8 – 9	|||	3
10 – 11	|||	3
		Total = 50