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Paper # Digital Signal Processing

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Question #1.

part (a).

Answer:-

$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$.

(i) Ans: According to Sampling Theorem

$F_1 = 100\ Hz$, $F_2 = 200\ Hz$

$F_s \geq 7max$ $\quad f = \frac{\omega}{2\pi}$

So f_2 is max (greater than F_1)

$f_s > 2 \times 100$

$f_s = 200\ Hz$

(ii) Ans:- Solution:-

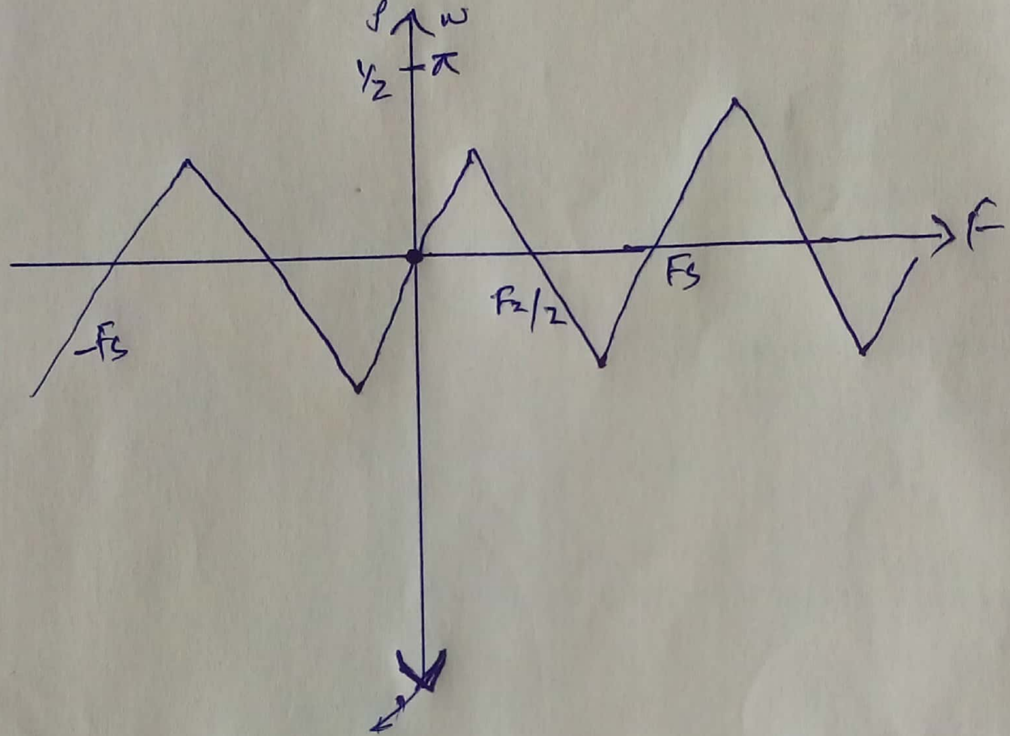
$f_s = 100\ Hz$

$f = \frac{100}{2} = 50\ Hz$

This is the max frequency that can be represented uniquely by the sampled signal

As $x_a[n] = 3\cos 2\pi\left(\frac{50}{100}\right)n + 4\sin 2\pi\left(\frac{100}{100}\right)n$

$$= 3\cos\pi\left(\frac{5}{10}\right)n + 4\sin 2\pi n$$



The effect of sampling rate on the newly generated discrete time signal is that There will be no display phenomenan mean there will be not present unwanted comprent in Re reconstruct of the signals.

(iii) **Solution** :-

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folding frequency $= \frac{f_1}{2} = \frac{100}{2}$

$f_1 = 50\,Hz$, $f_2 = 100\,Hz$

$= 50\,HZ$

frequency are either equal or greater the, folding frequency.

Hence for ideal interpolation we can construct the original signal

$$x_a(t) = 3\cos 100\pi + 4\sin 200\pi t$$

Since only the frequency compounds at 100Hz are present on the sampled signals. The analog signal we can remove or reconstruct is

$$y_a(t) = 3\cos 100\pi t \quad \text{Ans.}$$

X ——————— X ———————

Question # 1.
Part (b).
Answer:-

(A)

$$x(n) = \begin{cases} 0.5^n & , n \geq 0 \\ 0 & , n < 0 \end{cases}$$

(i) Ans:-

$$f_s = \frac{1}{T} = T = 1/f_s$$

$$= \frac{1}{2} = 0.5 \text{ sec}$$

x_n	0.5^n
0	1
0.5	0.707
1	0.5
1.5	0.353

(ii) Ans:-

$$L = 2^n$$

$$n = \text{bits} = 3$$

$$L = 2^3 = 8 \text{ level}$$

$$\text{Resolution} = \frac{X_{max} - X_{min}}{L}$$

$$= \frac{1-0}{8} = 0.125$$

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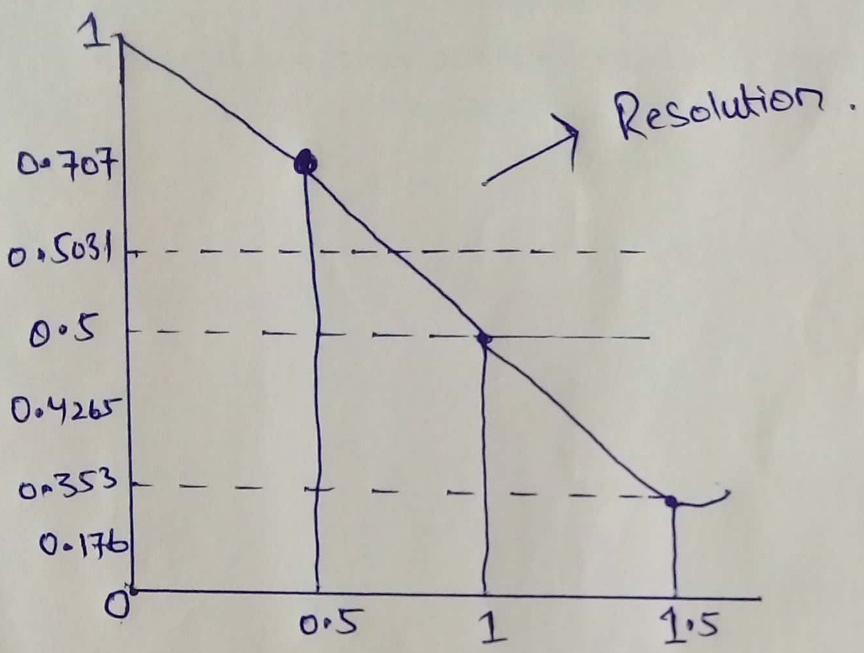


(iii) Ans:-

	Distr signal	increation	Reading	Error
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.2	0.1	-0.1

X ——— X ———

Question # 2.

Answer:-

*) Compute the convolution $y(n)$

$$x(n) = \begin{cases} a^{n+1} & -3 \leq n \leq 5 \\ 0 & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 2^{n} & 0 \leq n \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

Solution:-

We have.

$$x(n) = x(k) = \{\alpha-2, \alpha-1, 1, \alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6, 0, a, ---\}$$

$$h(n) = h(k) = \{0, 1, 2, 4, 8, 16, 0, -----\}$$

to find $y(n)$

$$y(n) = \sum_{K=\infty}^{\infty} x(k)\, h(n-k)$$

for $n=0$ first to find $h(n-k) = h(0-k)$

so by inverting $h(k)$ we get $h(-k)$

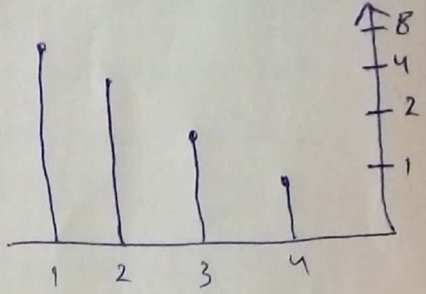
$$\Rightarrow n(-k) = \{16, 8, 4, 2, 1\} \quad \text{—— (2)}$$

So $y(0) = \sum_{K=-\infty}^{\infty} x(K)\, x\, h(-K)$

$y(0) = (\alpha^{-2}\times 8) + (\alpha^{-1}\times 4) + (1\times 2) + (\alpha \times 1)$

$y(0) = 8\alpha^{-2} + 4\alpha^{-1} + \alpha + 2$

$= 9-2+4\alpha^{2}+4\alpha^{-2}$

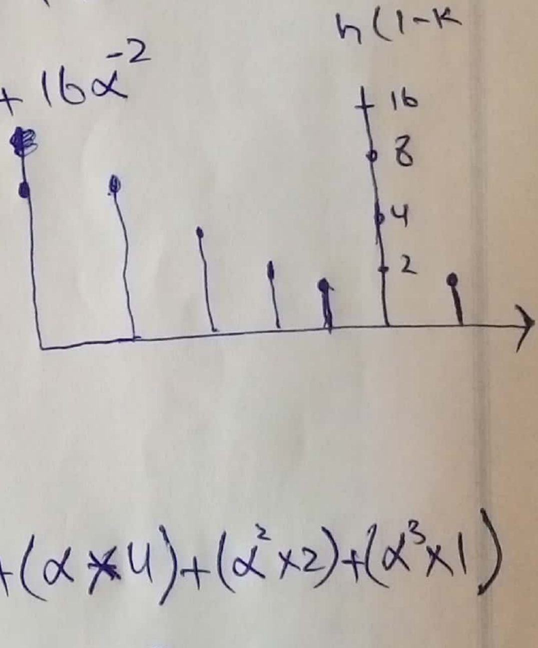


for $n=1$ $h(1-K) = \{16, 8, 4, 2, 1\}$.

so $y(n)\ (\alpha^{-2}\times 16) + \alpha^{-2}\times 8) + (1\times 4) + (\alpha\times 2) + (\alpha^{2}\times 1)$

$y(1) = 16\alpha^{-2} + 8\alpha^{-1} + 4 + \alpha\alpha + \alpha^{2}$

$= \alpha^{2} + 2a + 4 + 8\alpha^{-1} + 16\alpha^{-2}$

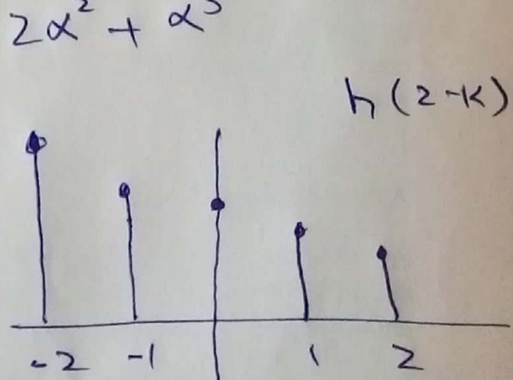


for $n = 2$

$n(2-K) = \{16, 8, 4, 2, 1\}$.

$y(2) = \{(\alpha^{-1}\times 16) + (1\times 8) + (\alpha\times 4) + (\alpha^{2}\times 2) + (\alpha^{3}\times 1)$

$= 16\alpha^{-1} + 8 + 4\alpha + 2\alpha^{2} + \alpha^{3}$



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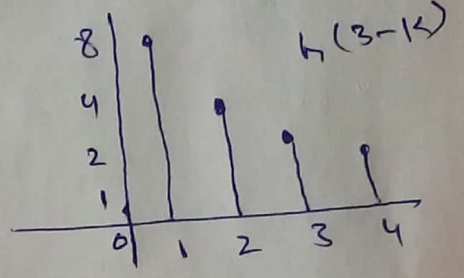
for $n=3$

$$h(3-k) = \{16, 8, 4, 2, 1\}$$

$$y(3) = (1\times16)+(\alpha\times8)+(\alpha^2\times4)+(\alpha^3\times2)+(\alpha^4\times1)$$

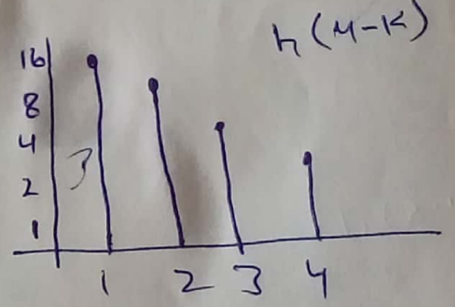
$$= 16+8\alpha+4\alpha^2+2\alpha^3+\alpha^4$$



$\Rightarrow$ Now $h(4-k) = \{16, 8, 4, 2, 1\}$

$$y(4) = (\alpha^1\times16)+(\alpha^2\times8)+(\alpha^3\times4)+(\alpha^4\times2)+(\alpha^5\times1)$$

$$= 16\alpha+8\alpha^2+4\alpha^3+2\alpha^4+1\alpha^5$$

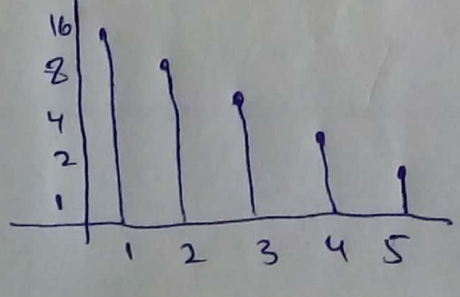


$\Rightarrow$ $h(5-k) = \{0, 16, 8, 4, 2, 1\}$

$$y(5) = (\alpha\times0)+(\alpha^2\times16)+(\alpha^3 * 8)+(\alpha^4\times4)+(\alpha^5\times2)+(\alpha^6\times1)$$

$$= 16\alpha^2+8\alpha^3+4\alpha^4+2\alpha^5\times\alpha^6$$

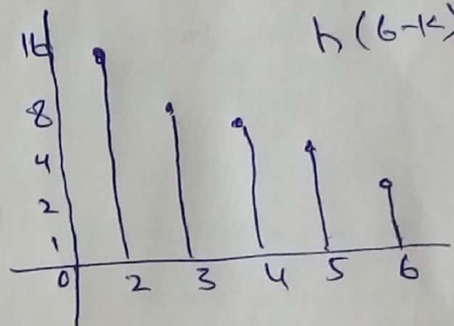
h(5-k)



Similarly if we calculate for rest of the values of n uptill These are any common values we get.

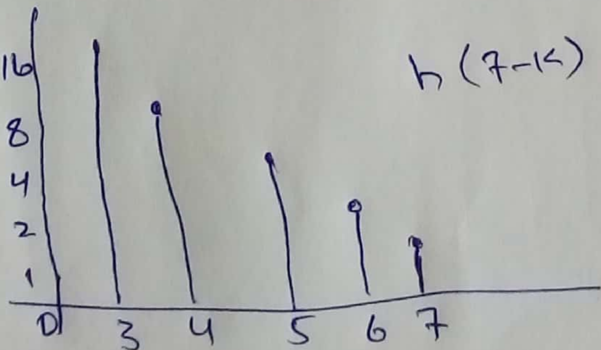
$$y(6) = 0+0+16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$$

$$= 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$$



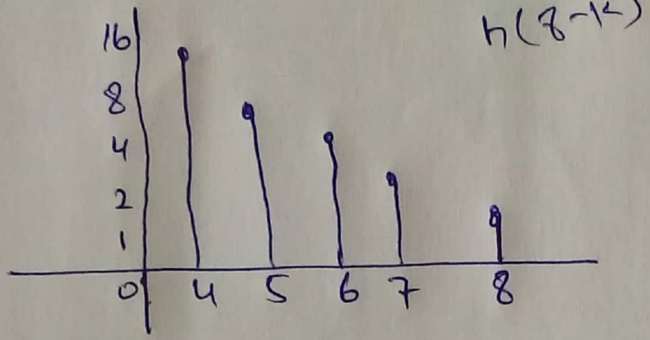
$$y(7) = 0+0+0+16\alpha^4 + 8\alpha^5 + 4\alpha^6$$

$$= 16\alpha^4 + 8\alpha^5 + 4\alpha^6$$



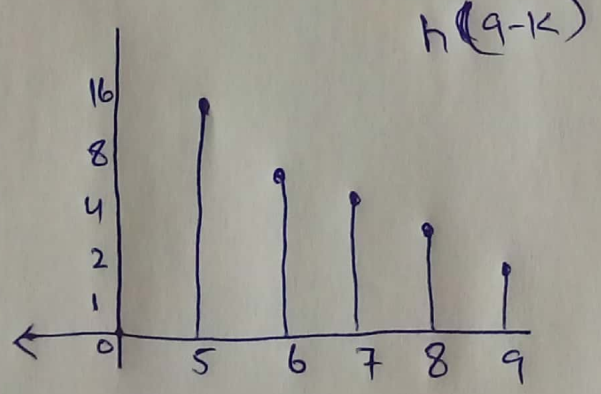
$$y(8) = 0+0+0+0+16\alpha^5 + 8\alpha^6$$

$$\Rightarrow 16\alpha^5 + 8\alpha^6$$



$$y(9) = 0+0+0+0+0+16\alpha^6$$

$$= 16\alpha^6$$



Question # 3

Answer :-

(i) $x(n) = \begin{cases} (1/4)^n, & n > 0 \\ (1/3)^n, & n < 0 \end{cases}$

Solution:- As we know that

Z - transform

$$x(Z) = \sum_{n=0}^{\infty} (1/4)^n_z + \sum_{n=-\infty}^{0} (1/3)^n_{z^n} - 1$$

Using geomatric series.

$$\Rightarrow \frac{1}{1 - 1/4\, z^{-1}} + \sum_{n=0}^{\infty} (1/3)^n z^n - 1$$

$$\Rightarrow \frac{1}{1 - \frac{1}{4} z^{-1}} + \frac{1}{1 - \frac{1}{3}} - 2$$

$$= \frac{1 - \frac{1}{\frac{1}{4} z^{-1}} + 1 - \frac{1}{3} - 1}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z^{-3}\right)^{-1}}$$

$$= \frac{1 - \frac{1}{3} z + 1 - \frac{1}{4} z^{1} - \left(1 - \frac{1}{4} z^{-2}\right)\left(1 - \frac{1}{3} z\right)}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z\right)}$$

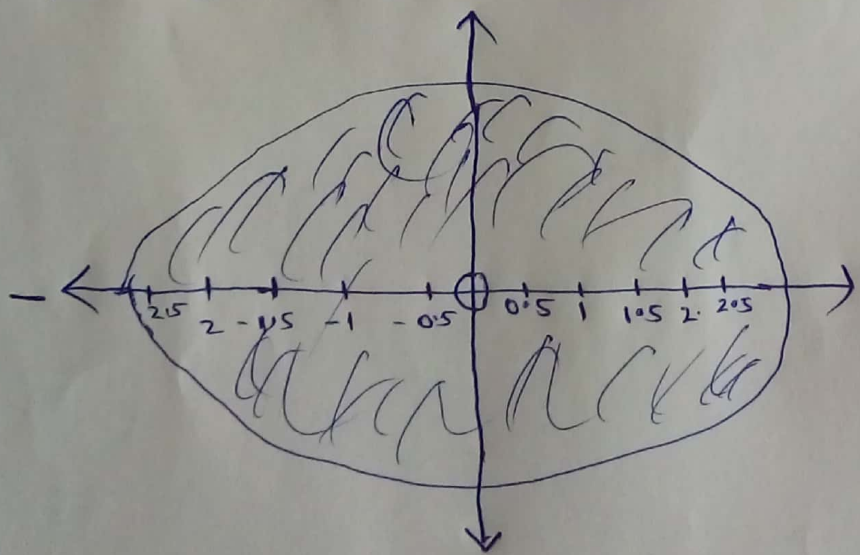
$$= \frac{1 - \cancel{\tfrac{1}{3}z} + 1 - \cancel{\tfrac{1}{4}z^{-1}} - 1 + \cancel{\tfrac{1}{3}} + \cancel{\tfrac{1}{4}z} + \tfrac{1}{12}}{(1 - \tfrac{1}{4}z^{-1})(1 - \tfrac{1}{3}z)}$$

$$= \frac{1 - \frac{1}{12}}{(1 - \frac{1}{4}z^{-1})(1 - \frac{1}{3}z)}$$

$$= \frac{\frac{13}{12}}{(1 - \frac{1}{4}z^{-1})(1 - \frac{1}{3}z)}$$

Hence the ROC is $\frac{1}{4} < |z| < 3$.

The sketch is under.



* " (ii) "

$$x(n) = \begin{cases} (1/2)^n - 3^n & n \geq 0 \\ 0 & \text{elsewhere.} \end{cases}$$

Solution :- using the z transform for eq.

i.e $x(n) = \alpha^n u(n) \leftrightarrow x(z) = \frac{1}{1-\alpha z^{-1}}$ → eq (B)

By putting values.

$$X_2(z) = \sum_{n=0}^{\infty} (1/2) z^n - n - \sum_{n=0}^{\infty} 2^n z^{-n}$$

$$= \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-2z^{-1}}$$

$$= \frac{-\frac{5}{2} z^{-1}}{(1-\frac{1}{2}z^{-1})(1-2z^{-1})}$$

A seen The ROC use $|z| > 2$

The sketch are.