Introduction To Structural Dynamics And Earthquake Engineering

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SECTION A

Submitted to

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Q: No: 1

Given Data:-

$E = 29{,}000 ksi$

$I = 150 in^4$

δ_{st} = Deflection due to 7683 lb static load.

Beam is pulled $\frac{1}{2}$" downwards.

Required Data:-

⇒ Natural time period of system develope and slove the equation.

⇒ Draw graphs to show the variation of displacement with time and the variation of equlivent static forces with time.

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Solution:-

General EOM for SDOF system is,

$$ku + cu + m\ddot{u} = p(t)$$

Since system is undamped ($c=0$) undergoing free vibration $p(t)=0$

Hence general EOM becomes,

$$ku + mu = 0 \rightarrow (1)$$

$$k = \frac{3EI}{L^3}$$

$$= 3 \times 29000 \; k/in^2 \times 150$$

$$k = 7.55208 \; k/in$$

$\Rightarrow$ In order to eliminate chances of mistake during calculation, it is more appropriate to use fundamental units like lb, ft.sec or kg, m sec.

$k = 7.55208\ k/in$

$k = 90625\ lb/ft$

$$m = \frac{w}{g} = \frac{7683}{32.2}$$

$m = 238.6024\ slug$

so,

$$\omega_n = \sqrt{\frac{k}{m}}$$

$$\omega_n = \sqrt{\frac{90625}{238.6024}}$$

$\omega_n = 19.4889\ rad/sec$

and,

$$T_n = \frac{2\pi}{\omega_n}$$

$$T_n = \frac{2\pi}{19.4889}$$

$T_n = 0.3224\ sec$

put m and k in eq (1)

$90625 u + 238.6024 \ddot{u} = 0$

where k is in lb/ft and is in lb sec/ft²

⇒ General solution to EOM for undamped free vibration is,

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n \sin(\omega_n t)}$$

$$u(0) = \frac{1}{2''} = \frac{1}{24} ft \; \$ \; \dot{u}(0) = 0$$

$$u(t) = \left(\frac{1}{24}\right) \times \cos(19.4889) + 0$$

$$= \left(\frac{1}{24}\right) \times \cos(19.4889t)$$

Equivalent static force at anytime 't' is

$$f_s(t) = k \cdot u(t)$$

$$= \frac{90625 \times \cos(19.4889t)}{24}$$

$$= 3776 \cos(19.4889t)$$

⇒ Amplitude of dynamic displacement u_o for undamped free vibration is,

$$u_o = \sqrt{u(0)^2 + \left(\frac{v(0)}{\omega_n}\right)^2}$$

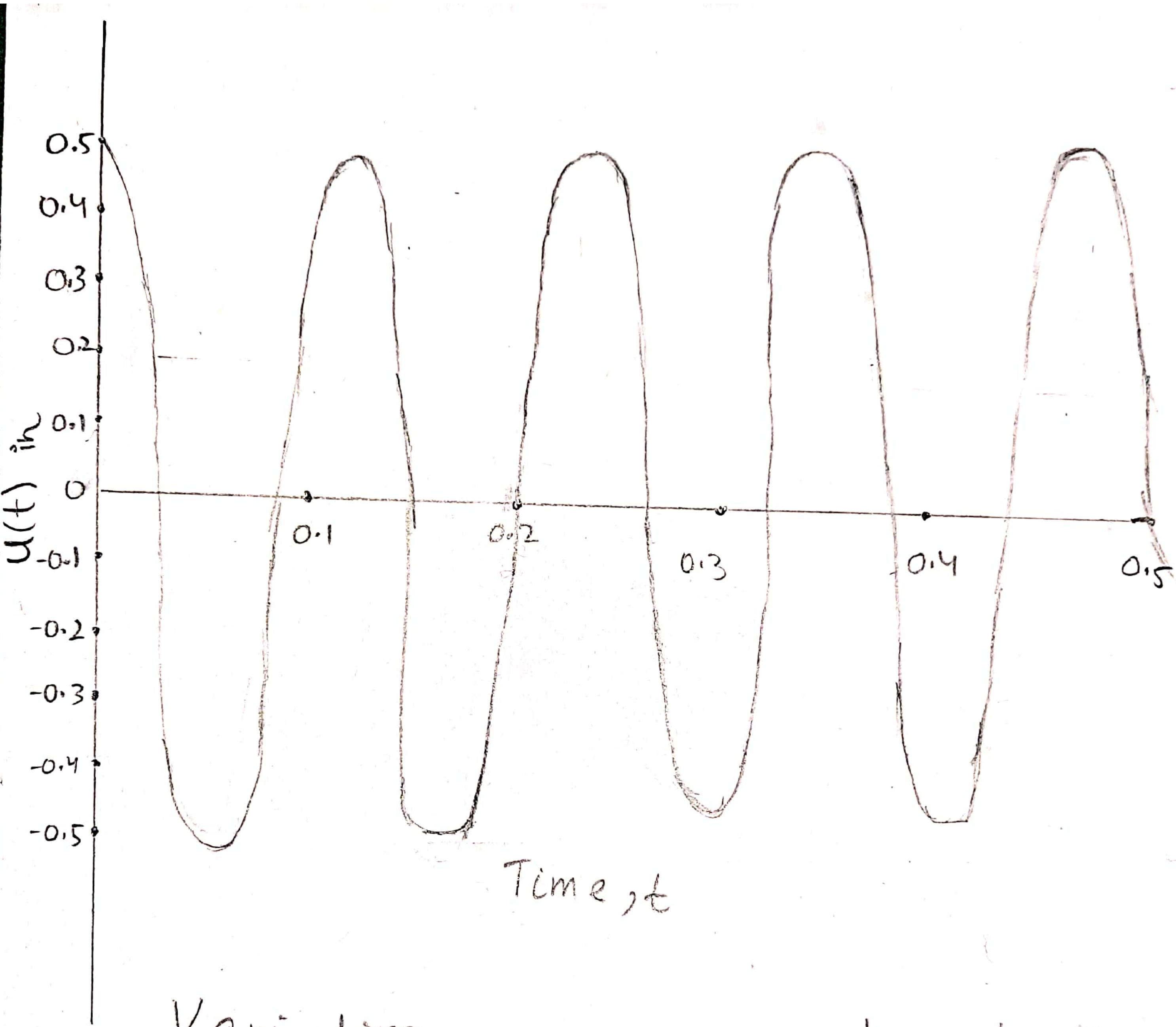
$$u_o = \sqrt{\left(\frac{1}{24}\right)^2 + 0}$$

$$u_o = \frac{1}{24}\ ft$$

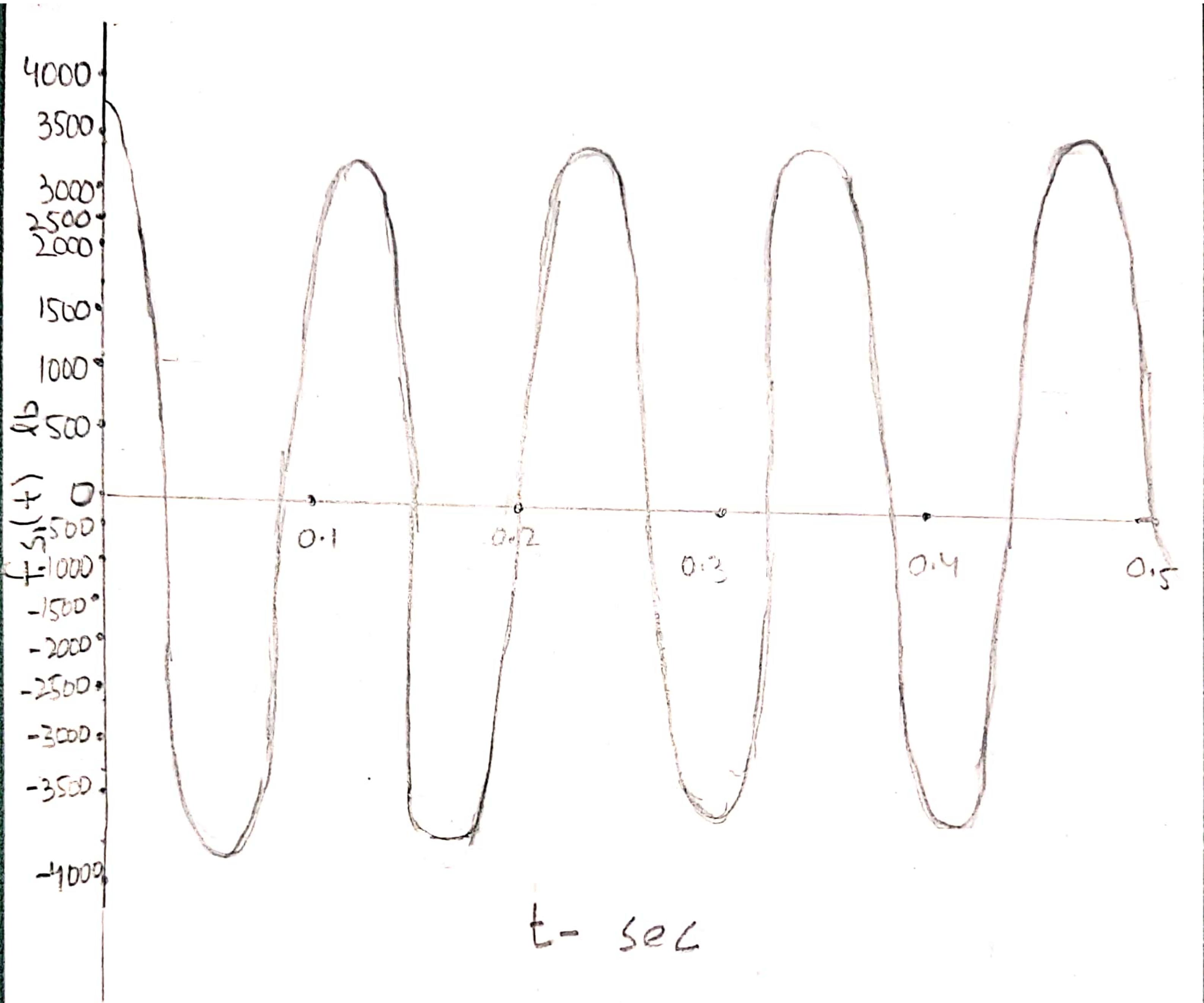
Amplitude of equilvent static force, f_{so}

$$ku_o = 90625 \times \frac{1}{24}$$

$$ku_o = 3776.$$



Variation of Displacement with time



Variation of Equivalent Static Forces with Time

Q: No: 2

Given Data:-

ξ (Damping ratio) of Reinforced Concrete with considered cracking = 3-5% = 3%

Using data of beam given in Question No: 1.

Required Data:-

Develope and solve the equation showing in equivalent static force with time.

Solution:-

EOM for damped free vibration is,

$$KU + C\dot{U} + m\ddot{u} = 0 \rightarrow (1)$$

From Question #1

$K = 90625\ lb/ft$

$m = 238.6024\ lb - \frac{sec^2}{ft}$

$\omega_n = 19.4889$ rad/sec

$C = \zeta \times 2 m \omega_n$

$= 0.03 \times 2 \times 238.6024 \times 19.4889$

$= 279.02$ lb.sec/ft

Put these values in eq (1)

$90625 u + 279.02 u + 238.6024 u = 0$

Solution to the EOM for damped free vibration is,

$$u(t) = e^{-\zeta \omega_n t}\left[u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left[\ddot{u}(0) + u(0)\zeta\omega_n\right] \times \sin \omega_D t\right]$$

$\omega_D = 19.4889$ rad/sec

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$$u(t) = e^{-0.03\times 19.4889t}\left[\frac{1}{24}\times \cos(19.4889) + \frac{1}{19.4889}\times\left[0 + \frac{1}{24}\times 0.03\times 19.4889\times \sin(19.4889t)\right]\right.$$

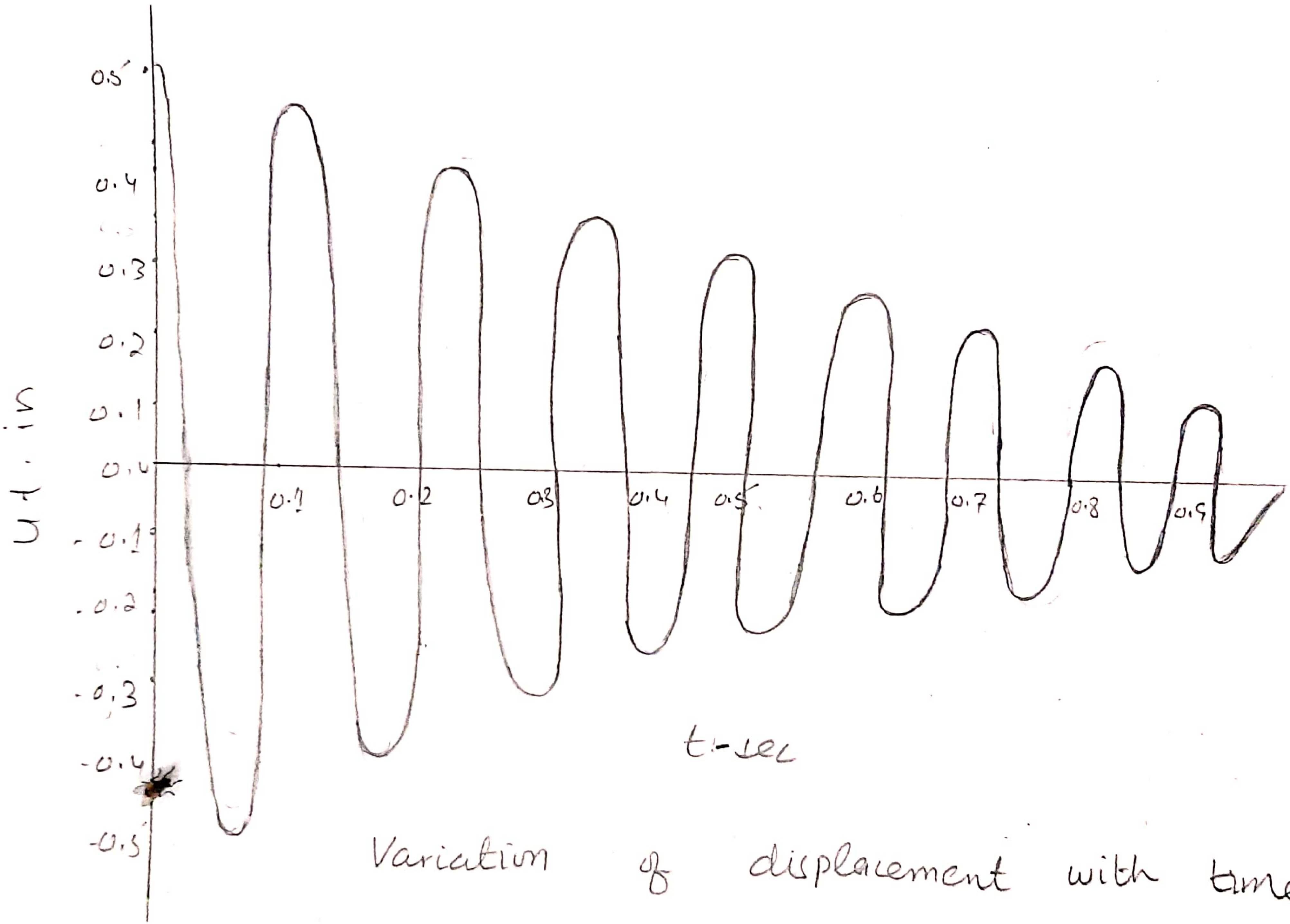
$$u(t) = e^{-0.03\times 19.4889t}\left[0.041\times \cos(19.4889t) + 0.00125\times \sin(19.4889t)\right.$$

$$f_s(t) = K\cdot u(t) \Rightarrow 90625\times u(t)$$

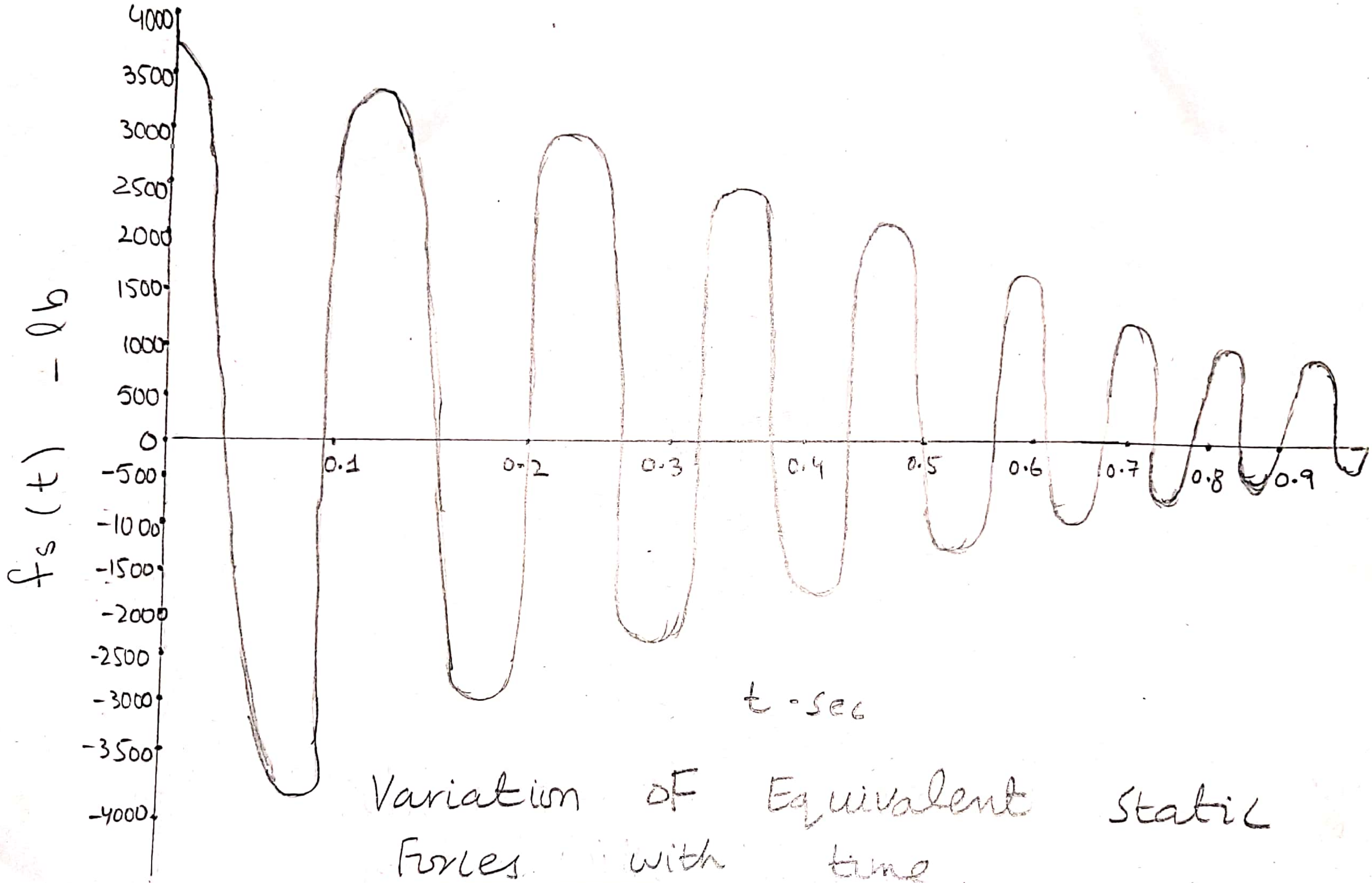
$$f_s(t) = e^{-0.585}\left[(90625\times 0.041)\cos(19.4889t) + 90625\times 0.00125\times \sin(19.4889t)\right]$$

$$f_s(t) = e^{-0.585}\left[3715.62\cos(19.4889t) + 113.28\sin(19.4889t)\right.$$

ANS.



Variation of displacement with time



Variation of Equivalent Static Forces with time

Q: No: 03

Given Data:-

- Force = 60 kips
- Displacement of tank $= \left(\frac{ID}{1000}\right)''$

$$= \left(\frac{7683}{1000}\right)'' = 7.683''$$

- Time taken to complete 7 cycles = 3.57 sec.
- Amplitude of displacement = 2.286 cm

$$= 0.9''$$

Required Data:-

(a) Damping ratios, ζ

(b) Natural period of un-damped vibration

(c) Stiffness of structures

(d) Weight of tank

(e) Damping co-efficient

(f) Number of cycles to reduce the displacement amplitude to 0.5''

Solution:-

→ Displacement of tank, $u_1 = 7.683''$

→ After 7 cycles, i.e After $j = 7$,

$u_{j+1} = u_8 = 0.9''$

(a) Damping ratio, $\zeta = ?$

Damping ratio, is find as,

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{1+1}}\right]$$

$$7 = \frac{1}{2\pi\zeta} \ln\left[\frac{7.683}{0.9}\right]$$

$$\boxed{\zeta = 0.0488 = 4.88\%}$$

(b) Natural Period of undamped Vibration, $T_n = ?$

As the 7 cycles of vibrations are completed in 3.57 sec

⇒ Time required to complete one cycle, $T_D = \frac{3.57}{7} = 0.51$ sec

Now,

$$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{(\omega_n \sqrt{1-\zeta^2})}$$

$$\Rightarrow T_D = \frac{T_n}{(1-\zeta)^2}$$

$$\Rightarrow T_n = T_D \times \sqrt{1-\zeta^2}$$

$$T_n = 0.51 \times \sqrt{1-(0.0488)^2}$$

$$T_n = 0.5094 = 0.51 \text{ sec}$$

$$\boxed{T_n = 0.51 \text{ sec}}$$

So, the Natural period of undamped vibration, $T_n = 0.51$ sec

(c) Stiffness of structure, k = ?

$$K = \frac{60 \times \cos 60^\circ}{7.683} = 3.91 \text{ k/in}$$

$$\boxed{K = 3.91 \text{ k/in} = 46920 \text{ lb/ft}}$$

(d) Weight of tank:-

Weight of tank, W is find as,

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{\frac{W}{g}}} = \sqrt{\frac{k \cdot g}{W}}$$

$$\Rightarrow \omega_n^2 = \frac{k \times g}{W}$$

$$W = \frac{k \times g}{\omega_n^2}$$

Also $\omega_n = \frac{2\pi}{T_n}$

$$W = \frac{k \cdot g}{\left(\frac{4\pi^2}{T_n^2}\right)}$$

$$W = k \cdot g \times \frac{T_n^2}{4\pi^2}$$

$$W = \left[\frac{46920\ lb}{ft} \times \frac{32.2\ ft}{sec^2}\right] \times \frac{(0.51\ sec)^2}{4\pi^2}$$

$$\boxed{W = 9953.93\ lb = 9.95\ K}$$

(e) Damping Co-efficient, c = ?

It is known that,

$$\zeta = \frac{c}{2m\omega_n}$$

$$\Rightarrow c = \zeta \times 2m \times \omega_n$$

$$= \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$c = \frac{(0.0488) \times 4 \times \pi \times \left(\frac{9953.93}{32.2}\right)}{0.51}$$

$$\boxed{c = 371.71 \text{ lb.sec/ft}}$$

(f) Number of cycles to reduce the displacement amplitude to 0.5", j

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$j = \frac{1}{2 \times \pi \times 0.0488} \ln\left[\frac{7.683}{0.5}\right]$$

$$\boxed{j = 8.91 \text{ or } 9 \text{ cycles}}$$