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Q(1) Find PQ where P is the point in three dimensional space with cordinates (4, 1, 3) and the Point Q with coordinates (1, 2, 4) Find the distance b/w P and Q. Further, Find the Position vector of the point dividing PQ in the ratio 1:3.

Solution:- Coordinate of P = (4, 1, 3)

$$OP = 4i + 1j + 3k$$

(OR) $$OQ = \vec{OQ} - \vec{OP}$$

$$= (i + 2j + 4k) - (4i + 1j + 3k)$$

$$= -3i + 1j + 1k \rightarrow ①$$

Now distance b/w P and Q = |PQ|

$$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$= \sqrt{11} \rightarrow ②$$

Let M be the Point which divided PQ in ratio 1:3, then by ratio theorem.

Position vector of M = $\vec{OM}$

$$= \frac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1 + 3}$$

$$= \frac{12j + 3j + 9k + i + 2j + 4k}{4}$$

$$= \frac{13j + 5j + 13k}{4} \rightarrow 3$$

$\Rightarrow$ Hense eq 1 2 3 are the required solution.

$\longleftrightarrow$

Q2 Evaluate: $\int \frac{4x^3+10x+4}{2x^2+x}$

Solution:-

$$\begin{array}{r|l} & 2x-1 \\ \hline 2x^2+x & 4x^3+10x+4 \\ & \underline{\pm 4x^3 \quad \pm 2x^2} \\ & -2x^2+10x+4 \\ & \underline{\mp 2x^2 \mp x} \\ & 11x+4 \end{array}$$

So $2x-1+\frac{11x+4}{2x^2+x} = \frac{4x^3+10x+4}{2x^2+x}$

$$\Rightarrow \int \frac{4x^3+10x+4}{2x^2+x} = \int 2x-1+\int \frac{11x+4}{2x^2+x} \rightarrow (1)$$

$$= 2\int x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$= \frac{2x^2}{2} - x + \int \frac{11x+4}{x(2x+1)}\,dx \rightarrow (2)$$

Now find

$$= \int \frac{11x+4}{x(2x+1)}\,dx = ?$$

$$= \frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \rightarrow A$$

$$= \frac{11x+4}{\cancel{x(2x+1)}} = \frac{A(2x+1)+Bx}{\cancel{x(2x+1)}}$$

$$= 11x+4 = A(2x+1)+Bx \rightarrow 3$$

Put $x = 0$ in 3

$$\boxed{4 = A}$$

Now Put $x = -1/2$ in 3

$$= 11(-1/2) + 4 = B(-1/2)$$

$$= \frac{-11}{2} + 4 = \frac{-B}{2}$$

$$= \frac{-11+8}{\cancel{2}} = \frac{-B}{\cancel{2}}$$

$$= -3 = -B \Rightarrow \boxed{B=3}$$

Now Putting the values of A and B in A

$$= \frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Taking integral on both sides

$$\int \frac{11x+4}{x(2x+1)}\,dx = \int \frac{4\,dx}{x} + \int \frac{3}{2x+1}\,dx$$

$$= 4\int \frac{1}{x}\,dx + 3\int \frac{1}{2x+1}\,dx$$

$$= 4\ln|x| + \tfrac{3}{2}\ln|2x+1|$$

Putting these values in 2

$$= x^2 - x + 4\ln|x| + \tfrac{3}{2}\ln(2x+1)$$

Now put these value in 1

$$= \boxed{\int \frac{4x^3+10x+4}{2x^2+x}\,dx = x^2 - x + 4\ln|x| + \tfrac{3}{2}\ln(2x+1) + C}$$

Q(3)
(a) $\int_0^2 x^2 e^x \, dx$

<u>Solution:-</u> First find Integration.

$= \int x^2 e^x \, dx$

$= x^2 \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x^2 \right) dx$

$= x^2 e^x - \int e^x (2x) \, dx$

$= x^2 e^x - 2 \int x e^x \, dx$

$= x^2 e^x - 2 \left\{ x \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x \right) dx \right]$

$= x^2 e^x - 2 \{ x e^x - \int e^x dx]$

$= x^2 e^x - 2x e^x + 2e^x$

Now put limits.

$= \left| x^2 e^x - 2x e^x + 2e^x \right|_0^2$

$= (2^2 e^2 - 2(2) e^2 + 2e^2 - (0 - 0 + 2e^0))$

$= (\cancel{4e^2} - \cancel{4e^2} + 2e^2 - 2)$

$= \boxed{2e^2 - 2}$ <u>Ans.</u>

Q3 (7)

(b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

Solution:- First find Integration.

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = ? \rightarrow (1)$$

let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$\boxed{2dy = \frac{1}{\sqrt{x}}\,dx}$$ Put in (1)

$$\int \sin(y)(2dy) = 2\int \sin(y)\,dy$$

$$= 2(-\cos y)$$

$$= -2\cos y$$

Put $y = \sqrt{x}$

$$= -2\cos\sqrt{x}$$

Put limits.

$$= -2\left|\cos\sqrt{x}\right|_1^2 = -2(\cos\sqrt{2} - \cos 1)$$

$$= \boxed{-2\cos\sqrt{2} + 2\cos(1)}$$ Ans.

Q(4) Verify that: $U(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$

Satisfies that three-dimensional laplace" equation.

Solution:-

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \rightarrow (A)$$

$$\text{So } U(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

$$= U(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

$$= \frac{\partial u}{\partial x} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$= \frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{-3/2}$$

$$= \frac{\partial^2 u}{\partial x^2} = -\left[x\left(-\frac{3}{\cancel{2}}\right)(x^2+y^2+z^2)^{-5/2}(\cancel{2}x) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$= \frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} (x^2+y^2+z^2)^{-3/2} \rightarrow (1)$$

Now

$$\frac{\partial u}{\partial y} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}y)$$

$$= \frac{\partial u}{\partial y} = -y\left(x^2+y^2+z^2\right)^{-3/2}$$

$$= \frac{\partial^2 u}{\partial y^2} = -\left[y\left(-\frac{3}{\cancel{2}}\right)\left(x^2+y^2+z^2\right)^{-5/2}(\cancel{2}y) + \left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$= \boxed{\frac{\partial^2 u}{\partial y^2} = 3y^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2}} \rightarrow (2)$$

$$= \frac{\partial u}{\partial z} = -\frac{1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-3/2}(\cancel{2}z)$$

$$= \frac{\partial u}{\partial z} = -z\left(x^2+y^2+z^2\right)^{-3/2}$$

$$= \boxed{\frac{\partial^2 u}{\partial z^2} = 3z^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2}} \rightarrow (3)$$

Putting (1) (2) and (3) in eq (A).

$$= 3x^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2} + 3y^2\left(x^2+y^2+z^2\right)^{-5/2}$$
$$- \left(x^2+y^2+z^2\right)^{-3/2} + 3z^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2}$$

$$= \left(x^2+y^2+z^2\right)^{-5/2}\left[3x^2 - \left(x^2+y^2+z^2\right) + 3y^2 - \left(x^2+y^2+z^2\right) + 3z^2 - \left(x^2+y^2+z^2\right)\right]$$

$$= \left(x^2+y^2+z^2\right)^{-5/2}\left[3x^2 - \cancel{x^2} - \cancel{y^2} - \cancel{z^2} + 3y - \cancel{x^2} - \cancel{y^2} - \cancel{z^2} + 3z^2 - \cancel{x^2} - \cancel{y^2} - \cancel{z^2}\right]$$

$= (x^2+y^2+z^2)^{-5/2}\,(0) = 0$

So the given $u(x,y,z)$ is solution of Laplace equation.

$\longleftrightarrow$