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Sec :- 4C⁴

Subject :- Applied Calculas

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Q(1) Find PQ where P is the point in three dimensional space with coordinates (4, 1, 3) and the point Q with coordinates (1, 2, 4). Find the distance blw P and Q. Further, Find the position vector of the point dividing PQ in the ratio 1:3. ①

Solution:- Coordinate of P = (4, 1, 3)

$$OP = 4i + 1j + 3k$$

$$\begin{aligned} \text{(OR)} \quad OQ &= \vec{OQ} - \vec{OP} \\ &= (i + 2j + 4k) - (4i + 1j + 3k) \\ &= -3i + 1j + 1k \rightarrow \text{①} \end{aligned}$$

Now distance blw P and Q = |PQ|

$$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$= \sqrt{11} \rightarrow \text{②}$$

Let M be the point which divided PQ in ratio 1:3, then by ratio theorem.

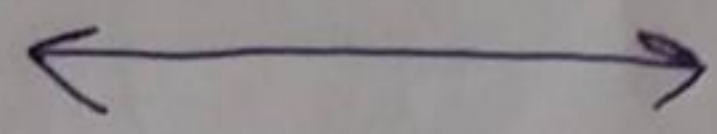
Position vector of M = \vec{OM}

$$= \frac{3(4i + 1j + 3k) + (1)(i + 2j + 4k)}{1 + 3}$$

$$= \frac{12j + 3j + 9k + i + 2j + 4k}{4}$$

$$= \frac{13j + 5j + 13k}{4} \rightarrow (3)$$

⇒ Hence eq (1) (2) (3) are the required solution.



Q2 Evaluate: $\int \frac{4x^3 + 10x + 4}{2x^2 + x}$

Solution:-

$$\begin{array}{r}
 2x - 1 \\
 \hline
 2x^2 + x \overline{) 4x^3 + 10x + 4} \\
 \underline{+ 4x^3} \qquad \qquad \underline{+ 2x^2} \\
 -2x^2 + 10x + 4 \\
 \underline{+ 2x^2} \quad \underline{+ x} \\
 11x + 4
 \end{array}$$

So $2x - 1 + \frac{11x + 4}{2x^2 + x} = \frac{4x^3 + 10x + 4}{2x^2 + x}$

$\Rightarrow \int \frac{4x^3 + 10x + 4}{2x^2 + x} = \int 2x - 1 + \int \frac{11x + 4}{2x^2 + x} \rightarrow (1)$

$= 2 \int x dx - \int 1 dx + \int \frac{11x + 4}{2x^2 + x} dx$

$= \frac{2x^2}{2} - x + \int \frac{11x + 4}{x(2x + 1)} dx \rightarrow (2)$

Now find

$= \int \frac{11x + 4}{x(2x + 1)} dx = ?$

$$= \frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{(2x+1)} \rightarrow \textcircled{A}$$

$$= \frac{11x+4}{\cancel{x(2x+1)}} = \frac{A(2x+1) + Bx}{\cancel{x(2x+1)}}$$

$$= 11x+4 = A(2x+1) + Bx \rightarrow \textcircled{B}$$

Put $x=0$ in \textcircled{B}

$$\boxed{4 = A}$$

Now Put $x = -\frac{1}{2}$ in \textcircled{B}

$$= 11\left(-\frac{1}{2}\right) + 4 = B\left(-\frac{1}{2}\right)$$

$$= -\frac{11}{2} + 4 = -\frac{B}{2}$$

$$= \frac{-11+8}{2} = -\frac{B}{2}$$

$$= -\frac{3}{2} = -\frac{B}{2} \Rightarrow \boxed{B=3}$$

Now Putting the values of A
and B in \textcircled{A}

$$= \frac{11x+4}{x(2x+1)} = \frac{4}{x} + \frac{3}{2x+1}$$

Taking integral on both sides

$$\int \frac{11x+4}{x(2x+1)} dx = \int \frac{4}{x} dx + \int \frac{3}{2x+1} dx$$

$$= 4 \int \frac{1}{x} dx + 3 \int \frac{1}{2x+1} dx$$

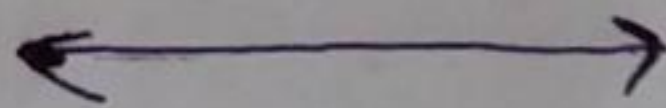
$$= 4 \ln|x| + \frac{3}{2} \ln|2x+1|$$

Putting these values in (2)

$$= x^2 - x + 4 \ln|x| + \frac{3}{2} \ln(2x+1)$$

Now put these value in (1)

$$\int \frac{4x^3 + 10x + 4}{2x^2 + x} dx = x^2 - x + 4 \ln|x| + \frac{3}{2} \ln(2x+1) + C$$



Q(3)

(a)

$$\int_0^2 x^2 e^x dx$$

(6)

Solution: - First find integration.

$$= \int x^2 e^x dx$$

$$= x^2 \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x^2 \right) dx$$

$$= x^2 e^x - \int e^x (2x) dx$$

$$= x^2 e^x - 2 \int x e^x dx$$

$$= x^2 e^x - 2 \left[x \int e^x dx - \int \left(\int e^x dx \frac{d}{dx} x \right) dx \right]$$

$$= x^2 e^x - 2 \left[x e^x - \int e^x dx \right]$$

$$= x^2 e^x - 2x e^x + 2e^x$$

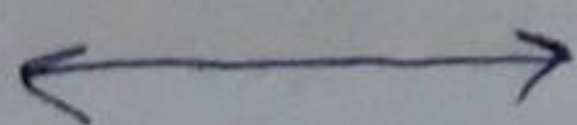
Now put limits.

$$= \left[x^2 e^x - 2x e^x + 2e^x \right]_0^2$$

$$= (2^2 e^2 - 2(2)e^2 + 2e^2 - (0 - 0 + 2e^0))$$

$$= (4e^2 - 4e^2 + 2e^2 - 2)$$

$$= \boxed{2e^2 - 2} \text{ Ans.}$$



Q3
(b)

$$\int_1^2 \frac{\sin \sqrt{x}}{\sqrt{x}} dx$$

Solution:- First find Integration.

$$\int \frac{\sin \sqrt{x}}{\sqrt{x}} dx = ? \rightarrow (1)$$

Let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$2dy = \frac{1}{\sqrt{x}} dx$ Put in (1)

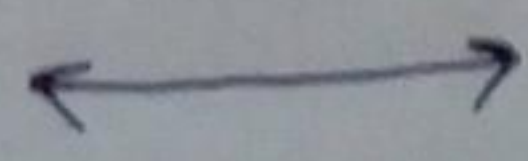
$$\begin{aligned} \int \sin(y) (2dy) &= 2 \int \sin(y) dy \\ &= 2(-\cos y) \\ &= -2 \cos y \end{aligned}$$

Put $y = \sqrt{x}$
 $= -2 \cos \sqrt{x}$

Put limits.

$$= -2 |\cos \sqrt{x}|_1^2 = -2 (\cos \sqrt{2} - \cos 1)$$

$= \boxed{-2 \cos \sqrt{2} + 2 \cos(1)}$ Ans.



Q4) Verify that:

$$u(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

satisfies that three-dimensional Laplace's equation.

Solution:-

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \rightarrow \text{A}$$

$$\text{So } u(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$$

$$= u(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$$

$$= \frac{\partial u}{\partial x} = -\frac{1}{2} (x^2 + y^2 + z^2)^{-3/2} (2x)$$

$$= \frac{\partial u}{\partial x} = -x (x^2 + y^2 + z^2)^{-3/2}$$

$$= \frac{\partial^2 u}{\partial x^2} = - \left[x (-3/2) (x^2 + y^2 + z^2)^{-5/2} (2x) + (x^2 + y^2 + z^2)^{-3/2} \right]$$

$$= \frac{\partial^2 u}{\partial x^2} = 3x^2 (x^2 + y^2 + z^2)^{-5/2} - (x^2 + y^2 + z^2)^{-3/2} \rightarrow \text{B}$$

Now

$$\frac{\partial u}{\partial y} = -\frac{1}{2} (x^2 + y^2 + z^2)^{-3/2} (2y)$$

$$= \frac{\partial u}{\partial y} = -y(x^2 + y^2 + z^2)^{-3/2}$$

$$= \frac{\partial^2 u}{\partial y^2} = - \left[y(-3/2)(x^2 + y^2 + z^2)^{-5/2} (2y) + (x^2 + y^2 + z^2)^{-3/2} \right]$$

$$= \frac{\partial^2 u}{\partial y^2} = 3y^2(x^2 + y^2 + z^2)^{-5/2} - (x^2 + y^2 + z^2)^{-3/2} \rightarrow \textcircled{2}$$

$$= \frac{\partial u}{\partial z} = -\frac{1}{2}(x^2 + y^2 + z^2)^{-3/2} (2z)$$

$$= \frac{\partial u}{\partial z} = -z(x^2 + y^2 + z^2)^{-3/2}$$

$$= \frac{\partial^2 u}{\partial z^2} = -3z^2(x^2 + y^2 + z^2)^{-5/2} - (x^2 + y^2 + z^2)^{-3/2} \rightarrow \textcircled{3}$$

Putting ① ② and ③ in eq (A).

$$= \cancel{3x^2} 3x^2(x^2 + y^2 + z^2)^{-5/2} - (x^2 + y^2 + z^2)^{-3/2} + 3y^2(x^2 + y^2 + z^2)^{-5/2} - (x^2 + y^2 + z^2)^{-3/2} + 3z^2(x^2 + y^2 + z^2)^{-5/2} - (x^2 + y^2 + z^2)^{-3/2}$$

$$= (x^2 + y^2 + z^2)^{-5/2} \left[3x^2 - (x^2 + y^2 + z^2) + 3y^2 - (x^2 + y^2 + z^2) + 3z^2 - (x^2 + y^2 + z^2) \right]$$

$$= (x^2 + y^2 + z^2)^{-5/2} \left[3x^2 - x^2 - y^2 - z^2 + 3y^2 - x^2 - y^2 - z^2 + 3z^2 - x^2 - y^2 - z^2 \right]$$

$$= (x^2 + y^2 + z^2)^{-5/2} (0) = 0$$

(10)

So the given $u(x, y, z)$ is solution
of Laplace equation.

