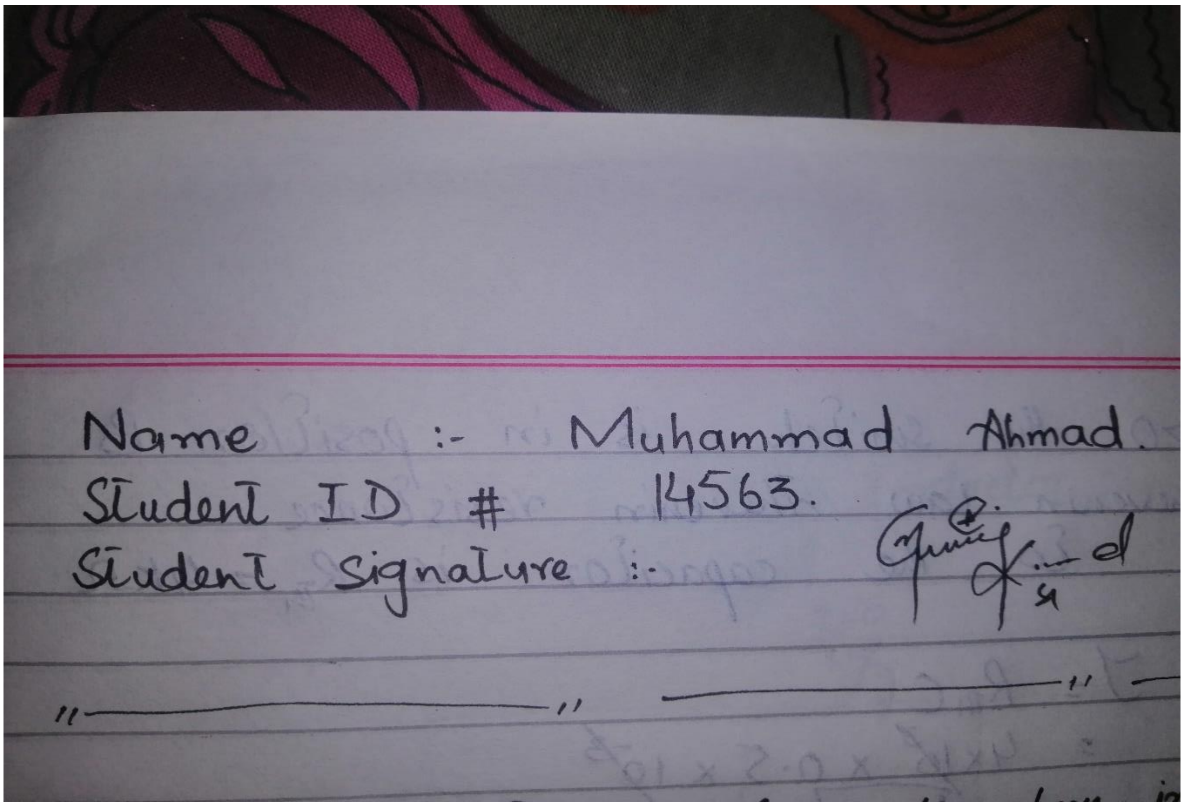
Department of Electrical Engineering

Course Title: Electrical Network Analysis

Module: 4th

Student Detail



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QNo1:- The switch in fig 1 has been in position A for a long time. At $t = 0$ the switch moves to B. Determine $v(t)$ for $t > 0$ & calculate its value at $t = 2s$ & $8s$.

Solution:-

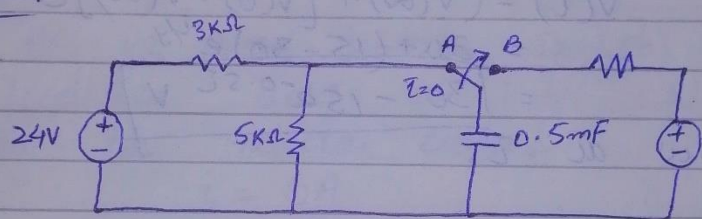


fig 1.

The capacitor acts as open ckt to dc when it at position $t<0$. The voltage across the capacitor just before $t=0$ is obtained by voltage division.

$$V(0^-) = \frac{5}{(5+3)K\Omega}(24)$$
$$= 15V$$

Capacitor voltage cannot change instantaneously because of storing quality.

$$V(0) = V(0^-)$$
$$V(0^+) = 15V.$$

When $t>0$ the switch is in position B
As per Thevenin Law Thevenin resistance connected to the capacitor is $R_{Th} = 4k\Omega$

$$\tau = R_{Th}C$$
$$= 4\times 10^{3} \times 0.5 \times 10^{-3}$$
$$= \boxed{2s}$$

As we disscus capacitor act as open ckt to dc
$$V(\infty) = 30V.$$

Now
$$V(t) = (V(\infty) + [V(0) - V(\infty)]e^{-t/\tau})$$
$$= 30 + (15-30)e^{-t/\tau}$$
$$= \boxed{30 - 15e^{-0.5t} \ V}$$

Now at t.

When $t = 2s$
$$V(2) = 30 - 15e^{-t/\tau}$$
$$= 30 - 15e^{-2/2}$$
$$= 30 - 15e^{-1}$$
$$= 30 - 15(0.3678)$$
$$= 30 - 5.517$$
$$= \boxed{24.483V}$$

When $t = 8s$.
$$V(8) = 30 - 15e^{-t/\tau}$$
$$= 30 - 15e^{-8/2}$$
$$= 30 - 15(0.0183)$$
$$= 30 - 0.2745$$
$$= \boxed{29.7255V}$$

2

QNo2:- Determine the Inductor current for both $t>0$ & $t<0$ for the ckt.

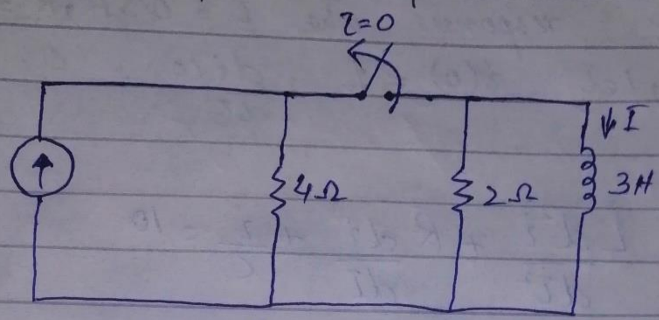


fig 2.

Solution:-

As we know $t<0$ the switch is closed and Inductor acts as short ckt.

Thus the inductor current

$$i = 6A.$$

For $t>0$ the switch is opend & Time constant $\tau = \frac{L}{R}$

$$\tau = \frac{3}{2}.$$

Now the inductor current $i(t) = 6e^{-t/\tau}$

$$i(t) = 6e^{-t/3/2}$$

$$i(t) = 6e^{-\frac{2t}{3}} \, A$$

(3)

Answer $i(t) = 6e^{-\frac{2t}{3}} u(t)A$

QNO3:- A series RLC ckt is describe by

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 10.$$

find the response whe $L = 0.5H, R = 4\Omega$ & $C = 0.2F$, Let $i(0) = 1$, $\frac{di}{dt}(0) = 0$.

Solution:-

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 10$$

Divided by L

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = \frac{10}{L}.$$

Multiple $\frac{C}{C}$ on R.H.S.

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = \frac{10C}{LC}$$

And $C = 0.2F$.

$$\frac{d^2i}{dt} + \frac{R}{L}\cdot\frac{di}{dt} + \frac{i}{LC} = \frac{10(0.2)}{LC}$$

Substitute

$$\frac{d^2i}{dt^2} + 8\frac{di}{dt} + 10i = 20 \rightarrow ①.$$

The general eq for Source Free series RLC ckt is given by

$$\frac{d^2i}{d^2t^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = \frac{I_s}{LC} \rightarrow ②.$$

compare ① and ②.

$\frac{R}{L} = 8 \rightarrow ③$

$\frac{1}{LC} = 10 \rightarrow ④$

$\frac{I_s}{LC} = 20 \rightarrow ⑤$.

from ③

$\alpha = \frac{R}{2L} = \frac{8}{2} = 4 \text{ rad/s} \rightarrow ⑥$

The natural frequency ω_0 is given by

$\omega_0 = \sqrt{\frac{1}{LC}}$

from (4)

$\omega_0 = \sqrt{10} \text{ rad/s} \rightarrow ⑦$.

from 6 and 7

$\alpha > \omega_0$.

∴ The ckt is over damped.

The root of the characteristic equation are give by:

$S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$

$= -4 + \sqrt{4^2 - \sqrt{10}^2}$

$= -4 + \sqrt{6} \text{ rad/s}$

$S_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$

$= -4 - \sqrt{4^2 - \sqrt{10}^2}$

$= -4 - \sqrt{6} \text{ rad/s}$.

From 5, The steady state I is given

$I_s = 20 \times LC = 20 \times 0.5 \times 0.2$

$= 2 A \rightarrow ⑧$.

The current in over damped case is given

$$i(t) = I_s + A_1 e^{s_1 t} + A_2 e^{s_2 t} \quad t>0 \rightarrow (9)$$

Substitute $t = 0$,

$$i(0) = I_s + A_1 + A_2.$$

$$1 = 2 + A_1 + A_2.$$

Thus $$A_1 + A_2 = -1 \rightarrow (10).$$

From (9) Find $\frac{di(t)}{dt}$.

$$\frac{di}{dt}(t) = A_1 s_1 e^{s_1 t} + A_2 e^{s_2 t} s_2.$$

$t = 0$

$$\frac{di}{dt}(0) = A_1 s_1 + A_2 s_2.$$

Putting value.

$$(-4+\sqrt{6})A_1 + (-4-\sqrt{6})A_2 = 0 \rightarrow (11).$$

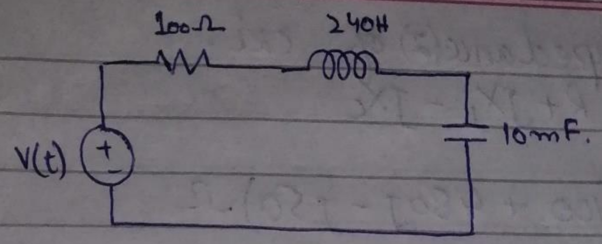
Solve (10) and (11).

$$A_1 = -1.316.$$

$$A_2 = 0.316.$$

$$\boxed{i(t) = 2 - 1.316 e^{(-4+\sqrt{6})t} + 0.316 e^{(-4-\sqrt{6})t} A}$$

QNo4:- A series RLC ckt has $R = 100\Omega$, $L = 240H$ and $C = 10mF$. If the input voltage is $V(t) = 10\cos 2t$, Find the current Flowing through the ckt.



Solution:-

$V(t) = 10 \cos 2t \, V$.

Here

The amplitude $V_m = 10V$.

The angular frequency $\omega = 2 \, rad/s$.

The phase angle $\phi = 0°$.

∴ The phasor is a complex number that represents the amplitude & phase of a sinusoid

$V(t) = 10 \angle 0° \, V$.

Now Inductive ~~Resistance~~ Reactance of the ckt.

$X_L = \omega L$.

$X_L = (2 \, rad/s)(240H)$.

$= 480 \, \Omega$.

Now the capacitive Reactance of ckt.

$$X_C = \frac{1}{\omega C}$$

$$= \frac{1}{(2 \, rad/s)(10 \times 10^{-3})}$$

$$= \frac{1}{20 \times 10^{-3}} \, \Omega$$

$$= \frac{\cancel{1000}^{50}}{\cancel{20}}$$

50Ω.

⑦

Now the impedance(Z) of ckt.

$$Z = R + JX_L - JX_C$$

$$Z = (100 + 480J - J50)\,\Omega$$

$$(100 + J430)\,\Omega$$

(Z) Impedance in Phasor Form.

$$Z = (100 - J430)\,\Omega$$

$$\sqrt{(100)^2 + (430)^2} \angle \tan^{-1}\left[\frac{430}{100}\right]$$

$$\sqrt{10000 + 184900} \angle \tan^{-1} 4.3$$

$$\sqrt{194900} \angle (76.9081)$$

$$= 441.47 \angle 76.9081^\circ\,\Omega$$

Current (I) for the ckt.

$$i = \frac{V(t)}{Z}$$

substitute $10\angle 0^\circ V$ for $V(t)$

$$i = \frac{10\angle 0^\circ V}{441.47\angle 76.9081^\circ\,\Omega}$$

$$= \frac{10}{441.47} \angle\left[0 - (76.9081^\circ)\right] A$$

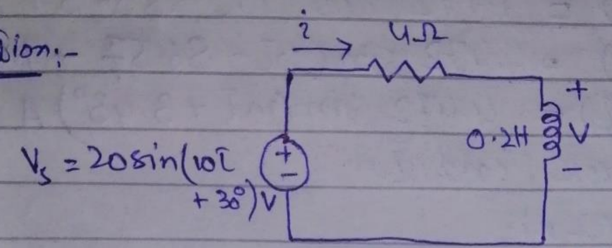
$$= 22.65 \angle -76.90^\circ \text{ mAmp}$$

$$i = 22.65 \cos\left[2t - 76.90^\circ\right] mA$$

$$i = 22.65\cos(2t - 76.90^\circ)\, mA$$

Q No 5 Find $V(t)$ and $i(t)$ in the ckt shown in fig 3.

Solution:-



$V_s = 20\sin(10t + 30^\circ)V$.

$V_s = 20\cos(10t + 30^\circ - 90^\circ)V$.

$V_s = 20\cos(10t - 60^\circ)V$.

$V_s = 20\angle -60^\circ V$.

$\omega = 10$ rad/sec.

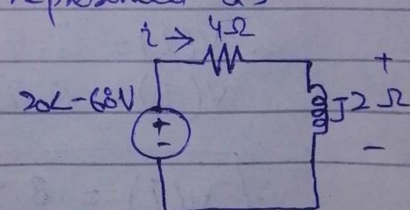
$X_L = j\omega L$.

$0.2H = j \times 10 \times 0.2$.

$0.2H = j2\Omega$.

Given ckt can be represented as.

From the represented ckt →



$Z = 4 + j2\Omega$.

Hence the current is

$$I = \frac{20\angle -60^\circ}{\sqrt{4^2+2^2}\angle \tan^{-1}\left(\frac{x}{y}\right)}$$

$$I = \frac{20\angle -60^\circ}{4.472\angle \tan^{-1}(0.5)}.$$

$$I = \frac{20\angle -60^\circ}{4.472\angle 26.57^\circ}.$$

$I = 4.472\angle -86.57°$.

Now $i(t) = 4.472 \cos(10t - 86.57°)$.

$i(t) = 4.472 \sin(10t - 86.57 + 90)$.

$i(t) = 4.472 \sin(10t + 3.43°)$ A

Voltage Accross inductor

$V = J2 \times i$.

$V = J2 \times (4.472\angle -86.57°)$.

Converting polar form to Rectangular form.

$V = J2 \times (0.26756 - J4.464)$.

$V = 8.928 + J0.53512)$.

$V = \left(\sqrt{(8.926)^2 + (0.53512)^2}\right) \angle \tan^{-1}\left(\frac{0.5312}{8.928}\right)$

$V = 8.944\angle 3.4V$.

$V(t) = 8.944 \cos(10t + 3.4°)$

$V(t) = 8.944 \sin(10t + 3.4° + 90°)$.

$$\boxed{V(t) = 8.944 \sin(10t + 3.4° + 90°)\,V}$$