Final paper.

Name :- Tauseef Ali

I.D :- 7701

Section :- B

Subject :- differential equation

Submitted to:- maam Shumaila mazhar

TVQ#1: find the fourier series representation of: $f(t) = 1+t, \ -\pi \le t \le \pi$.

Solution

$$F(t) = 1+t, \ -\pi \le t \le n.$$

Here we use the formula

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t \longrightarrow \text{equation } 1$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\, dt$$

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\, dt$$

$$a_0 = \frac{1}{2\pi} \left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(\frac{-\pi}{2}\right)^2\right)$$

$$a_0 = \frac{1}{2\pi} \left(2\pi + \frac{2\pi^2}{2}\right)$$

$$\boxed{a_0 = \frac{1}{\pi} (2\pi + \pi^2)}$$

②

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\,(\cos t)\,dt$$

$$a_n = \frac{1}{\pi}\left(\left((1+t)\frac{\sin nt}{n}\right|_{-\pi}^{\pi} - \int\left(\frac{\sin nt}{n}\frac{d}{dt}(1+t)\right)\right)$$

$$a_n = \frac{1}{\pi}\left(\left((1+t)\cancel{\frac{\sin nt}{n}}^{0} - \frac{\cos nt}{n^2}\right|_{-\pi}^{\pi}\right.$$

$$a_n = \frac{-1}{n^2\pi}\left(\cos n\pi - \cos n(-n)\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(-1-(-1)\right)$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\sin nt\,dt$$

$$b_n = \frac{1}{\pi}\int(1+t)\int_{-\pi}^{\pi}\sin nt - \int\left(\int \sin nt - \frac{d}{dt}(1+t)dt\right)$$

$$b_n = \frac{1}{\pi}\left(\left.\frac{(1+t)(-\cos nt)}{n}\right|_{-\pi}^{\pi} - \int\left(\frac{-\cos nt}{n}(1)\right)\right)$$

(3)

$$bn=\frac{1}{\pi}\left(\frac{-(1+t)\cos nt}{n}\Big|_{-\pi}^{\pi}+\left(\frac{\sin n\cancel{x}}{\cancel{n^2}}\Big|_{-\pi}^{\pi}\right)\right)$$

$$bn=\frac{-1}{n\pi}\left(\left(1+\pi\right)(\cos n\pi)-\left(1+(\pi\right)\left(\cos n(\pi)\right)\right)$$

$$bn=\frac{-1}{n\pi}\left(\cos n\pi+\pi\cos n\pi-\cos n\pi+\pi\cos n\pi\right)$$

$$bn=\frac{-1}{n\pi}\left(2\pi\cos n\pi\right)$$

Here $\cos n\pi=\frac{(-1)^{n+1}}{n}$

$$bn=\frac{2}{n}(-1)^{n+1}$$

So equation become

$$\boxed{f(x)=\frac{1}{2\pi}(2\pi+\pi^2)+0+\sum_{n=1}^{\infty}\frac{2(-1)^{n+1}}{n}\sin t}$$

QNo#2

Calculate the characteristics equation the eigen value of the system where A is given by

$$\begin{matrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{matrix}$$

Solution

$$\begin{matrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{matrix}$$

Solution

$\boxed{\text{Step\#1}}$

We have

A = given matrix

$$(A - \lambda I)x = 0$$

$\boxed{\text{Step\#2}}$

we have, The characteristic equation

$$|A - \lambda T| = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(5)

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

$\boxed{\text{Step\#3}}$

$$\lambda^3 - \left|\begin{matrix}\text{sum of} \\ \text{diagonal elem}\end{matrix}\right| \lambda^2 + \left|\begin{matrix}\text{Sum of} \\ \text{diagonal} \\ \text{minor}\end{matrix}\right| \lambda - |A| = 0 \quad \text{—(B)}$$

Sum of diagonal element $= 1+1+2 = 4$

Sum of diagonal minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= (-6) + (2) - (1)$

$= -6 + 2 + 1$

$= -3$

By putting value in equation (B)

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \quad \text{—(C)}$$

$$|A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$= 1(2-8) - 0 + 1(6-0)$

$= -6 + 6$

$\boxed{= 0}$

by putting value in (c)s

$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$

$\lambda^3 - 4\lambda^2 - 3\lambda = 0$

$\lambda(\lambda^2 - 4\lambda - 3) = 0$

$\lambda = 0$

$\lambda^2 - 4\lambda - 3 = 0$

using quadratic formula

$a = 1$
$b = -4$
$c = -3$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4 + \sqrt{28}}{2}, \quad \lambda = \frac{4 - \sqrt{28}}{2}$$

We have eigen values,

$$\lambda = \left(0, \frac{4 + \sqrt{28}}{2}, \frac{4 - \sqrt{28}}{2}\right)$$

required Solution.

Q NO#3

$5x + 4z + 2m = 3$

$x - y + 2z + m = 1$

$4x + y + 2z \cancel{+m} = 1$

$x + y + z + m = 0$

Solution

writing In matrix from.

$$\begin{bmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & +1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ m \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

$$= \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & -1 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & +6/5 & +4/5 & 3/5 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] -\tfrac{1}{5} \times R_3$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad C_2 \times -5$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/2 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad 5/4 \times R_1$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$= \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 8/5 & 1/5 \end{array}\right] \quad 5\times R_3 \text{ and } 5\times R_4$$

$$= \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 5R_3 \text{ and } 5R_4$$

$$= \left[\begin{array}{cccc|} 1 & 0 & 4/5 & 2/5 \\ 1 & -1 & 2 & 1 \\ 0 & -5 & 6 & 4 \\ 0 & 0 & 7 & 8 \end{array}\right] \quad 1/5 \times R_1$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_2\times 5$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_3 - R_2$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_3 - R_2$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/7 \end{array}\right] \quad \begin{array}{l} R_3 - R_4 \\ 1/7 \times R_3 \\ 1/3 \times R_4 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 0 & 1 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$(x, y, z, m) = \left(\tfrac{3}{7}, \tfrac{31}{21}, \tfrac{-11}{21}, \tfrac{1}{3}\right)$$

$$x = 3/4$$
$$y = 31/21$$
$$z = -11/21$$
$$m = 1/3$$

Q#4

Question # 4

Verify that
$u(x,t) = \sin(x+2t)$
is a Solution of the one-dimensional equation

Solution

given data

$$u(x,t) = \sin(x+2t)$$

differential w.r.t x partially

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\sin(x+2t)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)\frac{\partial}{\partial x}(x+2t)$$

$$\frac{\partial u}{\partial x} = \cos(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\cos(x+2t$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)\frac{\partial}{2x}(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)(1+0)$$

$$\boxed{\frac{\partial^2 y}{\partial x^2} = -\sin(x+2t)}$$

and

$u(x,t) = \sin(x+2t)$

differentiate w.r.t "t"

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\sin(x+2t)$$

$$\frac{\partial u}{\partial t} = \cos(x+2t)\ (0+2)$$

$$\frac{\partial u}{\partial t} = 2\cos(x+2t)$$

$$\frac{\partial^2 u}{\partial t^2} = (2) - \sin(x+2t)\ (0+2)$$

$$\boxed{\frac{\partial^2 u}{\partial t^2} = -4\sin(x+2t)}$$

As we know that one-dimensional equation is

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$-4\sin(x+2t) = c^2[-\sin(x+2t)]$$

$$-4\sin(x+2t) = -c^2\sin(x+2t)$$

$$-4\sin(x+2t + c^2\sin(x+2t) = 0$$

for the orbitrary constant $c = \pm 2$

$$-4\sin(x+2t) + (\pm 2)^2 \sin(x+2t) = 0$$

$$-4\sin(x+2t) + 4\sin(x+2t) = 0$$

$$0 = 0$$

then it will be verified for the orbitrary constant

$$\boxed{c = 2}$$