

Discrete Structure

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P#1

Course Title # Discrete Structure

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QNO 1 (a)

Answer The following each sentence expressed in symbolic notation.

(a) $P \wedge (Q \wedge X)$

(b) $P \wedge \sim Q$

(c) $P \wedge (\sim Q \vee \sim X)$

(d) $(\sim P \wedge Q) \wedge \sim X$

(e) $\sim P \vee (Q \wedge X)$

QNO 1 (b)

Answer In the below ^{table} shown the $P \vee Q \rightarrow X$ and $(P \rightarrow X) \wedge (Q \rightarrow X)$ has the equal truth value for each combination of truth values P, Q and X .

Hence in the below table two statements are logically equal

P	Q	X	$P \vee Q$	$P \rightarrow X$	$Q \rightarrow X$	$P \vee Q \rightarrow X$	$(P \rightarrow X) \wedge (Q \rightarrow X)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

\uparrow $P \vee Q \rightarrow X = (P \rightarrow X) \wedge (Q \rightarrow X)$
 has the equal truth values
 Hence they are logically equal.

P#2

Q NO 2 (A) ... Converse, Inverse, Contrapositive

Answer Converse:

(a) If Howard can swim to the island then Howard can swim across the lake

(b) If tomorrow is Monday then today is Easter.

Inverse:

(a) If Howard cannot swim across the lake, then Howard cannot swim to the island

(b) If today is not Easter, then tomorrow is not Monday.

Contrapositive:

(a) If Howard cannot swim to the island, then Howard cannot swim across the lake.

(b) If tomorrow is not Monday, then today is not Easter.

Q NO 2 (B)

(a) P
 $P \rightarrow q$
 $\sim q \vee r$
 $\therefore r$

P	q	r	$\sim q$	$P \rightarrow q$	$\sim q \vee r$	r
T	T	T	F	T	T	T
T	T	F	F	T	F	F
T	F	T	T	F	T	T
T	F	F	T	F	T	F
F	T	T	F	T	T	T
F	T	F	F	T	F	F
F	F	T	T	T	T	T
F	F	F	T	T	T	F

Premises Conclusion

P#3

This is invalid. Rows 1, 5, 7, 8 the hypothesis premises are the only rows that are true and the same rows in the Conclusion are T, T, T, F - if the hypothesis premises were valid (true) - then all four of the Conclusion would have been valid (true).
but the truth table shows false on row number 8. Therefore this is invalid.

Q. NO 2 (B) ^{in the below}

(b) This is invalid. Rows 2, 3, 4 the hypothesis premises are the rows, which are true and the equals rows of the Conclusion are T, F, T - if the hypothesis premises were valid (true) then all three of the Conclusion would have been valid (true) but the truth table shows false in row number 3. Therefore this is invalid.

$P \wedge Q \rightarrow \sim R$
 $P \vee \sim Q$
 $\sim Q \rightarrow P$
 $\therefore \sim R$

P	Q	R	$\sim Q$	$\sim R$	$P \wedge Q$	$P \wedge Q \rightarrow \sim R$	$P \vee \sim Q$	$\sim Q \rightarrow P$	R
T	T	T	F	F	T	F	T	T	F
T	T	F	F	T	T	T	T	T	T
T	F	T	T	F	F	T	T	T	F
T	F	F	T	T	F	T	T	T	T
F	T	T	F	F	F	T	F	T	F
F	T	F	F	T	F	T	F	T	T
F	F	T	T	F	F	F	T	F	F
F	F	F	T	T	F	T	T	F	T

↓ Premises
↓ Conclusion

P#4

Q.NO 3:-

SOLUTION: The treasure is buried under the flagpole.

Let

- $P =$ This house is next to a lake.
 $q =$ The treasure is in the kitchen.
 $r =$ The tree in the front yard is an elm.
 $s =$ The treasure is buried under the flagpole.
 $t =$ The tree in the back yard is an oak.
 $u =$ The treasure is in the garage.

The Premises are:-

- (a) $P \rightarrow \sim q$
 (b) $r \rightarrow q$
 (c) P
 (d) $\sim r \vee s$
 (e) $t \rightarrow u$

Step:

- | | |
|----------------------------|--------------------------------|
| (i) $P \rightarrow \sim q$ | Premise |
| (ii) $r \rightarrow q$ | Premise |
| (iii) P | Premise |
| (iv) $\sim r \vee s$ | Premise |
| (v) $t \rightarrow u$ | Premise |
| (vi) $\sim q$ | Modus ponens of (i) and (iii) |
| (vii) $\sim r$ | Modus tollens of (ii) and (vi) |
| (viii) s | Elimination of (i) and (7) |

s is true and thus treasure is buried under the flagpole.