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SECTION: "B"

DEPT.: BE Civil

SUBJECT: Differential equation.

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QUES # NO 1:

The wave equation:

We generally visit beach ....

one dimensional equation.

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

where w is the wave height, x is ....

.... partial derivatives.

i). $w = \sin(x + ct) + \cos(2x + 2ct)$.

ii). $w = \tan(2x + ct)$.

Solution:

$$w = \sin(x + ct) + \cos(2x + 2ct)$$

$$\frac{\partial w}{\partial t} = [\cos(x+ct).c] + [-\sin(2x+2ct)].2c$$

$$= c\cos(x+ct) - 2c\sin(2x+2ct)$$

$$\frac{\partial^2 w}{\partial t^2} = c - \sin(x+ct).c - 2c\cos(2x+2ct).2c$$

$$\boxed{\frac{\partial^2 w}{\partial t^2} = -c^2\sin(x+ct) - 4c^2\cos(2x+2ct)} \rightarrow (i)$$

$$\frac{\partial w}{\partial x} = \cos(x+ct) - \sin(2x+2ct)\cdot 2$$

$$\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 2\cos(2x+2ct)\cdot 2$$

$$\boxed{\frac{\partial^2 w}{\partial x^2} = -\sin(x+ct) - 4\cos(2x+2ct)}$$

From equation

$$\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$

$$\Rightarrow -c^2 \sin(x+ct) - 4c^2 \cos(2x+2ct) =$$

$$-c^2 \sin(x+ct) - 4c^2 \cos(2x+2ct).$$

Hence Proved.

ii) $W = \tan(2x+ct)$.

Solution:

$$\frac{\partial W}{\partial t} = \sec^2(2x+ct)\frac{\partial}{\partial t}(2x+ct).$$

$$= c\sec^2(2x+ct).$$

$$\frac{\partial^2 w}{\partial t^2} = c^2 \sec(2x+ct)\ \partial/\partial x \sec(2x+ct)$$

$$= c^2 \sec(2x+ct)\ \sec(2x+ct)\tan(2x+ct)$$

$$\boxed{\frac{\partial^2 w}{\partial t^2} = 2c^2 \sec^2(2x+ct)\tan(2x+ct).}$$

$$\frac{\partial w}{\partial x} = \sec^2(2x+ct).2$$

$$= 2\sec^2(2x+ct).$$

$$\frac{\partial^2 w}{\partial^2 x} = 2.\ 2\sec(2x+ct).\sec(2x+ct$$

$$.\tan(2x+ct).2$$

$$= 8\sec^2(2x+ct)\ \tan(2x+ct).2$$

$$= 8\sec^2(2x+ct)\ \tan(\cancel{\tan}\ 2x+ct).$$

$$= 2c^2\sec^2(2x+ct)\tan(2x+ct) \neq$$

$$c^2\, 8\sec^2(2x+ct).\tan(2x+ct).$$

From. eq, $\frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial^2 x}$ **Not satisfy.**

QUES # NO.2

=> Expand the following function in a Fourier series.

$$F(x) = x, \quad -\pi < x \leq 0$$
$$= 2x, \quad 0 \leq x \leq \pi$$

Solution:

We have to find fourier series co-effecient, a_0, a_n and b_n.

Now,

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} F(x)\,dx = \frac{1}{\pi}\int_{-\pi}^{0} x\,dx + 1/\pi\int_{0}^{\pi} 2x\,dx$$

$$= 1/\pi\left[\frac{x^2}{2}\right]_{-\pi}^{0} + \frac{2}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi}$$

$$= 1/\pi\left[0 - \frac{\pi^2}{2}\right] + \frac{2}{\pi}\left[\frac{\pi^2}{2} - 0\right]$$

$$\boxed{a_0 = \frac{-\pi}{2} + \pi = \frac{\pi}{2}}$$

$$a_n = 1/\pi \int_{-\pi}^{\pi} f(x) \cos nx \, dx$$

$$= \frac{1}{\pi} \int_{-\pi}^{0} (x \cos nx \, dx) + 1/\pi \int_{0}^{\pi} (2x \cos nx) \, dx$$

$$= \frac{1}{\pi} \left[x \left(\frac{\sin.nx}{n} \right) - \left(- \frac{\cos nx}{n^2} \right) \right]_{-\pi}^{0} +$$

$$\frac{2}{\pi} \left[x \left(\frac{\sin.nx}{n} \right) - \left(- \frac{\cos n.x}{n^2} \right) \right]_{0}^{\pi}$$

$$a_n = \frac{1}{\pi} \left[\frac{\cos(0)}{n^2} - \frac{\cos nx}{n^2} + \frac{2}{\pi} \right] \left[\frac{\cos nx}{n^2} - \frac{\cos(0)}{n^2} \right]$$

$$= \frac{1}{\pi} \left[\frac{1 - (-1)^n + 2(-1)^n - 2}{n^2} \right]$$

$$= \frac{(-1)^n - 1}{\pi n^2}$$

So,

$$\boxed{a_n = \begin{cases} \dfrac{-2}{\pi n^2} & ; \text{ if } n \text{ is odd.} \\ 0 & ; \text{ if } n \text{ is even} \end{cases}} \longrightarrow \text{ii}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin x\,dx = \frac{1}{\pi}\int_{\pi}^{0} x\sin nx\,dx +$$

$$\frac{2}{\pi}\int_{0}^{\pi} x\sin nx.$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{-\pi}^{0} +$$

$$\frac{2}{\pi}\left[x\left(-\frac{\cos nx}{n}\right) - \left(-\frac{\sin nx}{n^2}\right)\right]_{0}^{\pi}$$

$$b_n = \frac{1}{\pi}\left[\frac{-\pi\cos n\pi}{n}\right] + \frac{2}{\pi}\left[-\frac{\pi\cos n\pi}{n}\right]$$

$$= -\frac{3\cos n\pi}{n} = \frac{3(-1)^{n+1}}{n}$$

So the required fourier

$$f(x) = \frac{A_0}{2} + \sum_{n=1}^{\infty}\left(a_n\cos nx + b_n\sin nx\right)$$

$$= \frac{\pi}{4} - \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\cos(2x-1)}{(2x-1)^2} + 3\sum_{n=1}(-1)^{n+1}\frac{\sin nx}{n}$$

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QUES # NO.3

=> Solve the following initial value problem.

$y'' - 4y' + 13y = 8\sin 3x$, $y(0) = 1$ & $y'(0) = 2$

Solution:

$y'' - 4y' + 13y = 8\sin 3x$, $y(0) = 1$
and $y'(0) = 2$.

Homogeneous equation is

$$y'' - 4y' + 13y = 0 \quad \text{———} \quad (2)$$

$$8\sin 3x \quad \text{———} \quad (1)$$

change (2) into Auxilary equation

put $y = m$ in (2)

$$m^2 - 4m + 13 = 0$$

Now quadratic formula:

$$ax^2 + bx + c = 0$$

Here; $a = 1$, $b = -4$ and $c = 13$

$$m = \frac{-b \pm \sqrt{b^2-4ac}}{2a}.$$

$$m = \frac{-(4) \pm \sqrt{(-4)^2-4(1)(13)}}{2(1)}$$

$$m = \frac{4 \pm \sqrt{-36}}{2} = \frac{4 \pm \sqrt{36}\, i}{2}$$

$$= \frac{4 \pm 6i}{2}$$

$$= 2 \pm 3i$$

$$m_1 = 2 + 3i$$

$$m_2 = 2 - 3i$$

$$y_c = e^{2x}\left(c_1 \cos 3x + c_2 \sin 3x\right) \longrightarrow \text{A}$$

Diff w.r.t "x"

$$y_p' = -3A \sin 3x + 3B \cos 3x$$

Again differentiating w.r.t x

$$y_p'' = -9A \cos 3x - 9B \sin 3x$$

put in eq 1

$= (-9A\cos 3x - 9B\sin 3x) - 4(-3A\sin 3x + 3B\cos^3 x) + B(A\cos 3x + B\sin 3x) - B\sin 3x$

$= -9A\cos 3x - 12B\cos 3x + A13\cos 3x - 9B\sin 3x + 12A\sin 3x + 13B\sin 3x - 8\sin 3x$

$= (-9A - 12B + 13A)\cos 3x + (-9B + 12A + 13B)\sin 3x = 8\sin 3x$

comparing co-effecient.

$\sin 3x = 4B + 12A = 8$

$4A - 12B = 0 \Rightarrow 4A - 12B$

$\boxed{A = 3B}$

put (b) in (a)

$4B + 12(3B) = 8$

$40B = 8$

$\boxed{B = 1/5}$ —— 1

put ① in ⓑ

$$\boxed{A = 3/5} \text{ — } ⓓ$$

put c & d in (x)

$$y_p = 3/5 \cos 3x + 1/5 \sin 3x \text{ ——— } Ⓑ$$

the general solution is;

$$y = y_c + y_p.$$

$$y = e^{2x}(c_1 \cos 3x + c_2 \sin 3x) + 3/5 \cos 3x$$
$$+ 1/5 \sin 3x \text{ ——— } Ⓒ$$

Now we need to find the values of c_1 & c_2 for this

put $x = 0$ and $y = 1$ in Ⓒ

$$1 = e^{x(0)} = (c_1 \cos 3(0)) + (2 \sin 3(0)) + 3/5$$
$$\cos 3(0) + 1/5 \sin 3(0).$$

$$1 = (c_1 (1) + (2(0) + 3/5 (1) + 1/5 (0)$$

$$1 = c_1 + 3/5.$$

$$c_1 = 2/5 \text{ ——— } (xx)$$

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put $y' = 2$, $x = 0$ in (D)

$y' = C_1 (2e^{2x} \cos 3x - 3\cancel{x}^{2x} \sin 3x) +$

$C_2 (2e^{2x} \sin 3x + 3e^{2x} \cos 3x - 6/5$

$\sin 3x + 3/5 \cos 3x$

put $y' = 2 x = 0$

$2 = C_1 (2e^{2(0)} \cos 3(0) - 3e^{2(0)} \sin 3(0)) +$

$C_2 (2e^{2(0)} \sin(0) + 3e^{2(0)} \cos 3(0))$

$2 = 2C_1 + 3C_2 + 3/5.$

put $C_1 = 2/5.$

$2 = 4/5 + 3C_2 + 3/5$

$3C_2 = 3/5$

$\boxed{C_2 = 3/15.}$ (xxx)

put (xx) & (xxx) in C

$y = e^{3x}\left(2/5 \cos 3x + 3/15 \sin 3x\right) + 3/5$

$\cos 3x + 1/5 \sin 3x.$

$$\boxed{y = 2/5\, e^{2x} \cos 3x + 3/15\, e^{2x} \sin 3x + 3/5 \cos 3x + 1/5 \sin 3x.}$$

Required G.E.

QUES # NO. 04.

Solution:-

$(D^2 - DD')z = \cos x \cos 2y$

the given PDE can be rewrite as $D(D-D')z = \cos x \cos 2y$ in

CF is given by

$$CF = \phi_1(y) + \phi_2(y+x).$$

while its PI is given by

$$PI = \frac{1}{(D^2-DD')}, \frac{1}{2}\left[\cos(x+2y) + \cos(x-2y)\right]$$

$$= 1/2 \left[\frac{1}{(-1+2)} \cos(x+2y + \frac{1}{(-1-2)} \cos(x-2y))\right]$$

$$= 1/2 \left[\frac{1}{(-1+2)} \cos(x+2y) + \frac{1}{(-1-2)} \cos(x-2y)\right.$$

$= 1/2 \cos(x+2y) - 1/6 \cos(x-2y)$

Hence the complete solution of the PDE is

$$\boxed{Z = \phi_1(y) + \phi_2(y+x) + 1/2 \cos(x+2y) - 1/16 \cos(x-y).}$$