

FINAL EXAM

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Sec : B

Subject : Numerical Analysis

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Q No 1:

$$\frac{dy}{dx} = 2x ; y(0) = 1$$

$$h = 0.1$$

Sol:

$$F(x, y) = 2x$$

$$x_0 = 0, y_0 = 1$$

$$h = 0.1$$

$$x_{n+1} = x_n + h$$

put $n=0$

$$x_1 = x_0 + h$$

$$= 0 + 0.1$$

$$x_2 = x_1 + h$$

$$= 0.1 + 0.1$$

$$= 0.2$$

$$x_3 = 0.3$$

$$x_4 = 0.4$$

$$x_5 = 0.5$$

1st iteration.

Euler's Formula.

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$n = 0.$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + 0.1 [(0) + (1)]$$

$$= 1.1$$

Modified Euler's Formula.

$$y_{n+1} = y_n + \frac{h}{2} [F(x_n, y_n) + F(x_{n+1}, y_{n+1}^*)]$$

$$y_1 = y_0 + \frac{h}{2} [F(x_0, y_0) + F(x_1, y_1^*)]$$

$$y_1 = 1 + \frac{0.1}{2} [(0) + (1) + (0.1) + (1.1)]$$

$$y_1 = 1 + 0.05 [1 + 1.1]$$

$$= 1 + 0.05(2.2)$$

$$= 1.11$$

2nd iteration:

Euler formula.

$$y_2^* = y_1 + hf(x_1, y_1)$$

$$y_2^* = 1.11 + 0.1 \left[(0.1)^* (1.11) \right]$$

$$y_2^* = 1.11 + 0.1(1.21)$$

$$= 1.11 + 0.121$$

$$= 1.231$$

Modified Euler formula

$$y_2 = y_1 + \frac{0.1}{2} \left[f(x_1, y_1) + f(x_2, y_2^*) \right]$$

$$= 1.11 + 0.05 \left[(0.1)^* (1.11) + (0.2)^* (1.231) \right]$$

$$= 1.11 + 0.05 \left[1.21 + 1.431 \right]$$

$$= 1.11 + 0.05 (2.641)$$

$$= 1.242$$

3rd iteration.

$$y_3^* = y_2 + hf(x_2, y_2)$$

$$y_3^* = 1.242 + 0.1 (0.2)^+ (1.242)$$

$$= 1.242 + 0.1442$$

$$= 1.3862$$

Modified Euler's Method.

$$y_3 = y_2 + \frac{0.1}{2} [f(x_2, y_2) + f(x_3, y_3^*)]$$

$$= 1.242 + 0.05 [(0.2) + (1.242)] + (0.3) + (1.3862)$$

$$= 1.242 + 0.05 [1.442 + 1.6862]$$

$$= 1.398$$

4th iteration

$$y_4^* = y_3 + hf(x_3, y_3)$$

$$y_4^* = 1.398 + 0.05(0.3 + 1.398)$$

$$= 1.483$$

Modified Euler Method.

$$y_4 = y_3 + 0.05 [f(x_3, y_3) + f(x_4, y_4^*)]$$

$$= 1.398 + 0.05 [0.3 + 1.398] + [0.4 + 1.483]$$

$$= 1.398 + 0.05(1.698 + 1.883)$$

$$= 1.577$$

5th iteration

$$y_5^* = y_4 + hf(x_4, y_4)$$

$$= 1.577 + 0.05(0.4 + 1.577)$$

$$= 1.675$$

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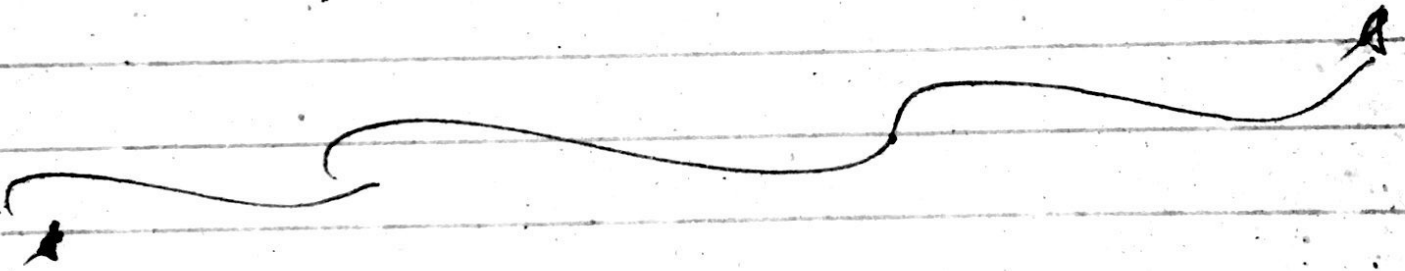
Modified Euler's Method

$$y_5 = y_4 + 0.05 [f(x_4, y_4) + f(x_5, y_5^*)]$$

$$= 1.577 + 0.05 [0.4 + 1.577] + [0.5 + 1.675]$$

$$= 1.577 + 0.05 [9.152]$$

$$= 1.785$$



Q No 2:

Given Data:

$$y=0, \quad n=0, \quad h=0.2 \quad 0 \leq n \leq 6$$

$$y_{n+1} = y_n + k$$

1st iteration:

$$n=0$$

$$y_1 = y_0 + k, \quad k = \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = hf(x_n, y_n)$$

$$k_1 = h(x_0^2 - x_0 - y_0)$$

$$k_1 = 0.2(0^2 - 0 - 0)$$

$$k_1 = 0$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}\right)$$

$$= 0.2 f\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}\right)$$

$$0.2 f\left(\frac{0+0.2}{2}, \frac{0+0.2}{2}\right)$$

$$0.2 (0.1^2 + \cancel{0.1} - \cancel{0.1})$$

$$k_2 = 0.0020$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_3 = 0.2 f\left(\frac{0+0.2}{2}, \frac{0+0.0020}{2}\right)$$

$$k_3 = 0.2 f(0.1, 0.001)$$

$$k_3 = 0.2 (0.1^2 + 0.1 - 0.001)$$

$$k_3 = 0.0218$$

$$k_4 = hf(x_n + h, y_n + k_3)$$

$$= 0.2 f(0+0.2, 0+0.0218)$$

$$= 0.2 (0.2^2 + 0.2 - 0.0218)$$

$$K_u = 0.0436$$

$$K = \frac{1}{6} (0 + 2(0.002) + 2(0.218) + 0.0436)$$

$$K = 0.0152$$

$$y_1 = 0 + 0.0152$$

$$y_1 = 0.0152$$

Q3:

Given Data:-

$$a = 0, \quad b = 10, \quad n = 10$$

$$h = \frac{b-a}{n} = \frac{10-0}{10} = 1$$

Sol:-

x	0	1	2	3	4	5	6	7	8	9	10
$f(x)$	10.1	17.2	24.4	29.2	34.2	41.2	50.9	57.8	60.3	61.2	62.1

Using formula:-

$$f(x)dx = \frac{h}{2} \left[f(x_0) + 2(f(x_1) + f(x_2) + f(x_3) + \dots + f(x_{n-1})) + f(x_n) \right]$$

$$= \frac{1}{2} \left[10.1 + 2(17.2 + 24.4 + 29.2 + 34.6 + 41.2 +$$

$$50.9 + 57.8 + 60.3 + 61.2) + 62.1 \right]$$

$$= \boxed{412.9} \text{ Ans.}$$

Q4:

$$\int_2^3 \ln(x^3+1) dx$$

Use 10 strips:

Sol:.

$$n = 10$$

$$h = \frac{3-2}{10} = 0.1$$

x	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
$f(x)$	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
	0.693	0.846	1.063	1.162	1.320	1.476	1.628	1.777	1.922	2.062

Now Using formula

$$\int_a^b f(x) dx = \frac{h}{3} \left[f(x_0) + 4(f(x_1) + f(x_3) + \dots) + 2[f(x_2) + \dots + f(x_n)] \right]$$

$$= \frac{0.1}{3} \left[0.693 + 4(0.846 + 1.162 + 1.476 + 1.777) + 2(1.063 + 1.320 + 1.628 + 1.922) + 2.062 \right]$$

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$$\# = \boxed{1.784}$$

Ans

