Department of Electrical Engineering
Assignment
Date:13/04/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:	**Pir Meher Ali Shah**	**Total Marks:**	30

Student Details

Name: **IQBAL HUSSAIN** **Student ID:** **13690**

Q1.	(a)	Consider the following analog signal $$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate F_s = 100Hz.What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal y_a(t) we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
	(b)	Consider a discrete time signal which is given by $$x(n) = \begin{cases} 0.5n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$ This is signal is sampled at the rate F_s= 200Hz. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1
Q2.	(a)	Determine the response of the system to the following input signal with given impulse response $$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\} \quad , h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$	Marks 5 CLO 2

	(b)	Compute the convolution y(n) of the following signal $$x(n)=\begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & elsewhere \end{cases}$$ $$h(n)=\begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & elsewhere \end{cases}$$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i) $$X(n)\begin{cases} (\frac{1}{4}^{n}) & , n \ge 0 \\ (\frac{1}{3})^{-n} & , n < 0 \end{cases}$$ ii) $$X(n)=\begin{cases} (\frac{1}{2})^{n} - 3^{n} & , \quad n \ge 0 \\ 0 & , \quad elsewhere \end{cases}$$	Marks 10 CLO 2

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Q:1 (a) Consider the following analog signal. $x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$.

(i) Determine the minimum Sampling rate required to avoid aliasing.

Solution:

$$f_s \geq 2 f_{max}$$

$$f = \frac{\omega}{2\pi} \qquad \omega_1 = 100$$

$$f_1 = \frac{100}{2\pi}$$

$$f_1 = 50HZ$$

$$\omega_2 = 200$$

$$f_2 = \frac{200}{2\pi} \Rightarrow f_2 = 100HZ$$

f_2 is max than f_1.

$$f_s \geq 2 \times 100HZ$$

Sample frequency to avoid aliasing.

Q:1 (a) (ii) Suppose that the signal is Sampled at the rate $F_s = 100HZ$ What is the discreate-time signal obtained after sampling? also explain the effect of this Sampling rate on the newly generated

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discrete time signal.

Solution:-

$Fs = 100 HZ$

f_1 becomes

$f_1' = \frac{f_1}{F_3} = \frac{150}{100} = 0.5 HZ$

And f_2 becomes

$f_2 = \frac{f_2}{100}$

$f_2 = \frac{100}{100} = 1 HZ$

So $\omega_1' = 2\pi f_1'$

$\omega_1' = 2\pi \times 0.5$

$\omega_1' = 1\pi$

$\omega_2' = 2\pi f_2'$

$\omega_2' = 2\pi \times 1$

$\omega_2' = 2\pi$

$x[n] = 3\cos 100\pi n + 4\sin 200\pi n$

the signals are

$x[n] = 3\cos\pi n + 4\sin 2\pi n$

The effect of this sampling rate on the newly generated discrete time signal is that. there will be no Aliasing. Phenomena. Means there will not present unwanted components in the reconstruction of the signal & we can reconstruct

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the original signal.

Q:1 a (iii)

Solution:

folding frequency of the sampled signal is

$$\text{folding frequency} = F_s/2 = \frac{100}{2} = 50$$

$= 50\,HZ$

we have frequency of the original signals

$f_1 = 50\,Hz,\ f_2 = 100\,HZ$

Both the frequency are either equal or greater the ~~following~~ folding frequency.

Hence for ideal interpolation we can construct the original Signal.

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

The original signal is Constructed because we use Sampling frequency at Nyquist rate.

we can also reconstruct the signal for sampling frequency above the Nyquist rate.

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Q: 1 (b) Consider a discreate time signal which is given by:

$$x(n) = \begin{cases} 0.5^{n}, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

this is signal is sampled at the rate Fs = 2 HZ.

(i)

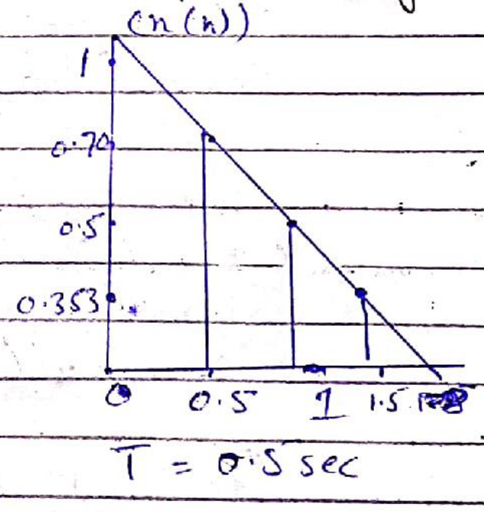
Solution:

$$Fs = \frac{1}{T}$$

$$T = \frac{1}{Fs} = \frac{1}{2} = 0.5 \text{ sec}$$

(i) Draw the sampled signal.

x_n	0.5^n
0	1
0.5	0.7071
1	0.5
1.5	0.353



Q: 1 (b) (ii)

Solution: $L = 2^n$

$n = 3 = \text{bits}$

$L = 2^3 = 8$ levels

$$\text{Resolution} = \frac{x_{max} - x_{min}}{L}$$

$$= \frac{1 - 0}{8}$$

$$R = 0.125$$

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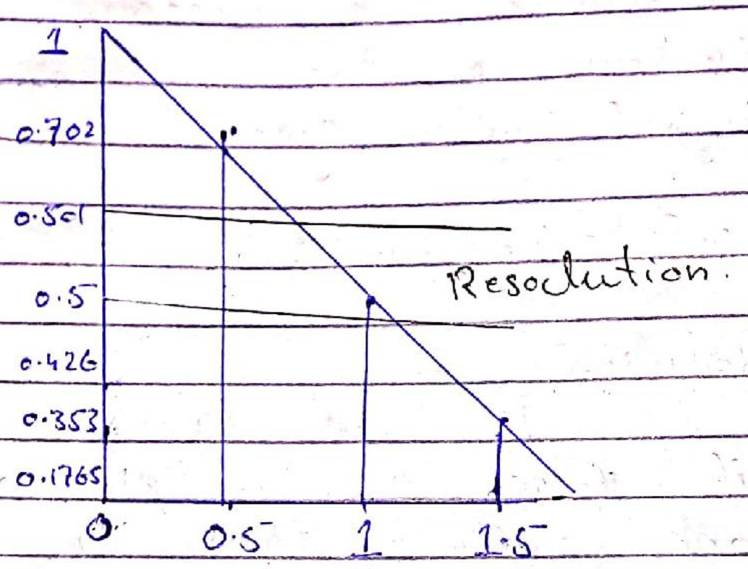

Q:1 (b) (iii) Solution:

	Discrete signal	Trunction	Reading	error
0	1	1.0	1.0	0.1
1	0.8535	0.8	0.9	-0.1
2	0.707	0.7	0.7	0.0
3	0.6035	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.426	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.2	-0.1

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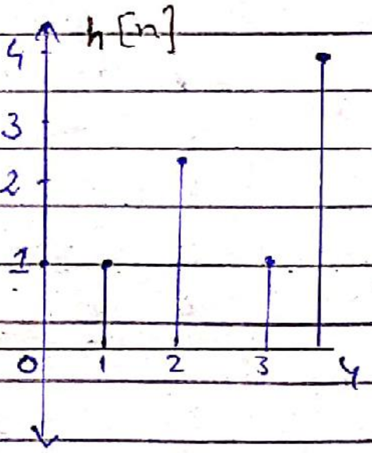
Q:2 (a) Determine the response of the System to the following input Signal with given impulse response

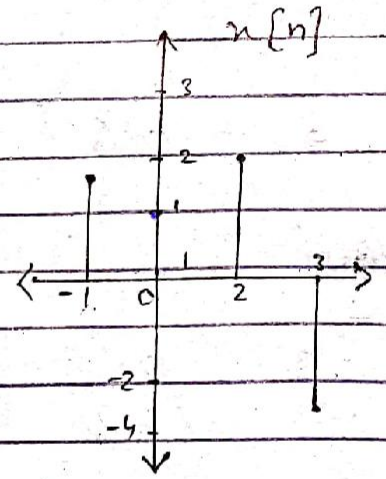
$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$, $h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$

Solution:

As we know that if there is multiplication in one domain then in the other domain there is Convolution.

To find $y[n]$, we convole $x[n]$ and $h[n]$.





$y[n] = x[n] * h[n]$

$y[n] = \sum_{n=-\infty}^{\infty} x[n]\, h[n]$

Replace n with K

$= \sum_{K=-\infty}^{\infty} x[K]\, h[-K]$

Now, we introduce shifting of

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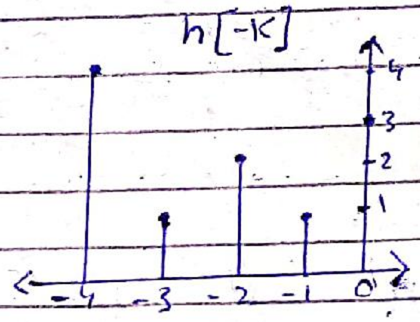
n_o in $h[-k]$

$$y[n_o] = \sum_{k=-\infty}^{\infty} x[k]\,h[n_o-k]$$

For $n_o = 0$

$y[0] = 2+3 = 5$



for $n_o = -1$

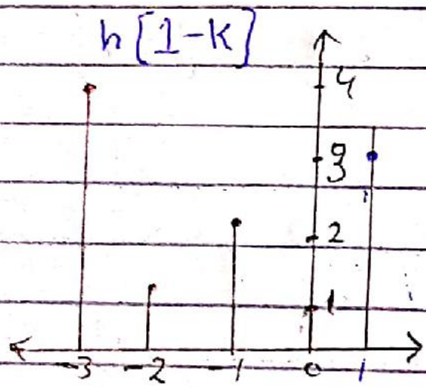
$y[-1] = 6$

For $n_o \le -2$

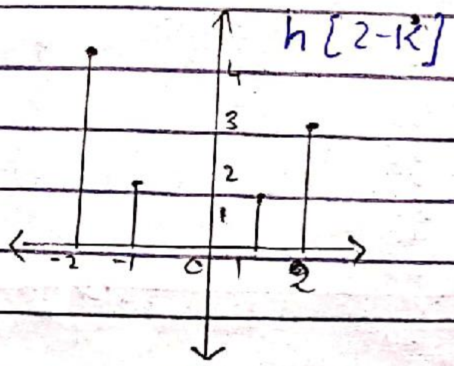
$y[n] = 0$

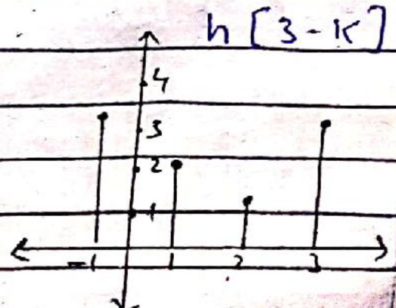
for $n_o = 1$

$y[1] = -6+1+4 = -1$



$y[2] = 2+2-2+9 = 11$





$y[3] = 8+1-4+3-12 = -4$

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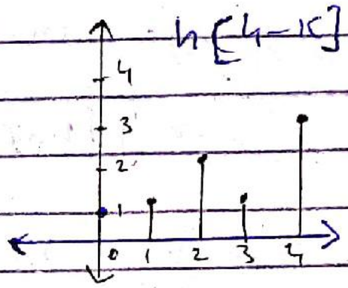
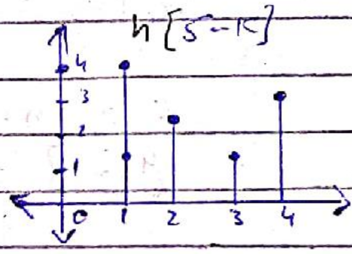
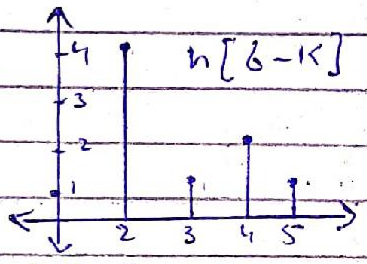
$y[4] = 4+6-4-2 = 4$



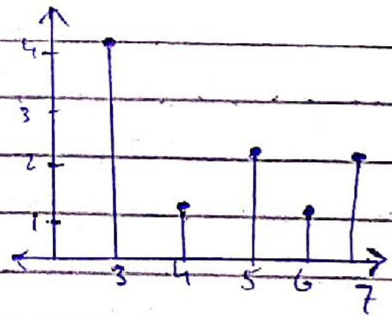
$y[5] = -8+3-8 = -13$



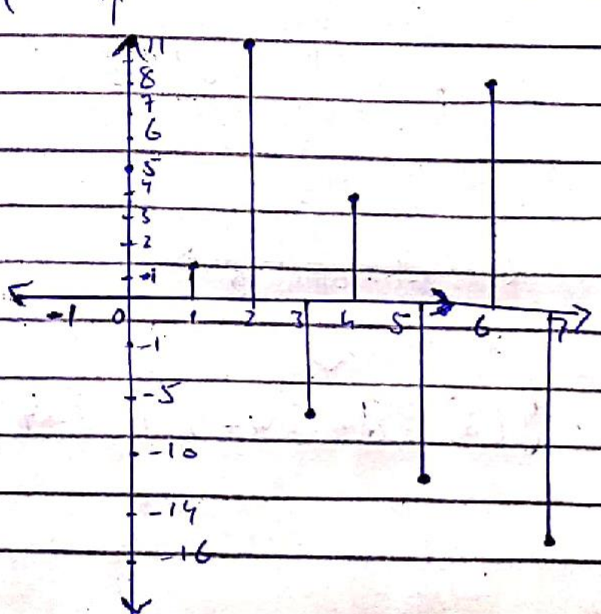
$y[6] = 12-4 = 8$



$y[7] = -16$



$y[n] = \{ \underset{\uparrow}{6}, 5, 1, 11, -4, 4, -13, 8, -16 \}$



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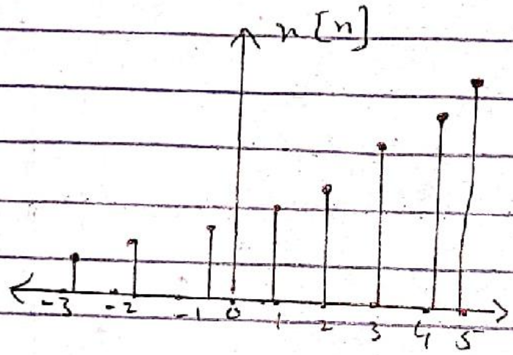
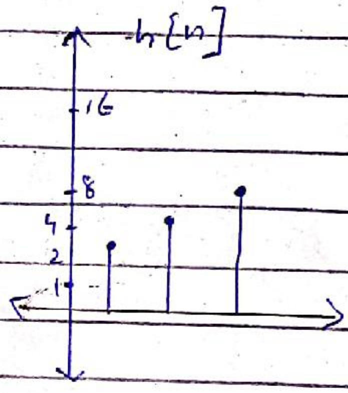
Q: 2 (b)

Solution:

$$x[n] = \begin{cases} \alpha^{n+1} & , -3 \le n \le 5 \\ 0 & , \text{elsewhere} \end{cases}$$

$$h[n] = \begin{cases} 2^{n} & , 0 \le n \le 4 \\ 0 & , \text{elsewhere} \end{cases}$$

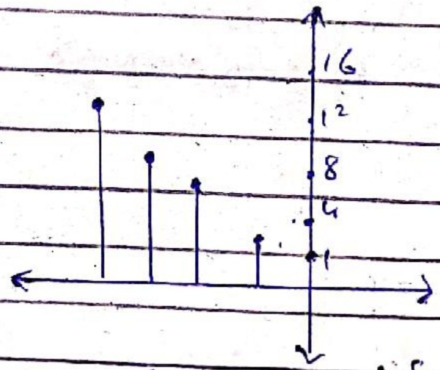




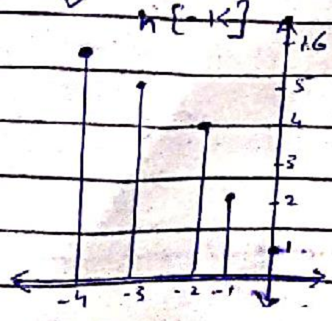
$$x[n] = \{\alpha^{-2}, \alpha^{-1}, 1, \underset{\uparrow}{\alpha}, \alpha^{2}, \alpha^{3}, \alpha^{4}, \alpha^{5}, \alpha^{6}\}$$

$$h[n] = \{\underset{\uparrow}{1}, 2, 4, 8, 8, 16\}$$

$$y[n_0] = \sum_{K=-\alpha}^{\alpha} x[K]\, h[n_0 - K]$$



$$y[0] = \alpha - 2 + 4\alpha^{-1} + 8\alpha^{-2}$$



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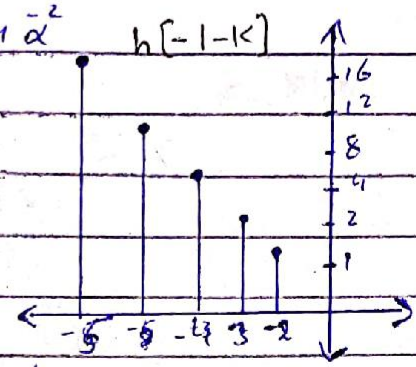
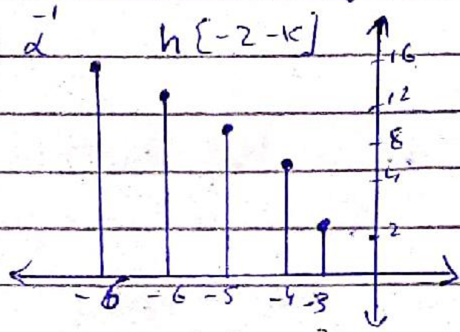
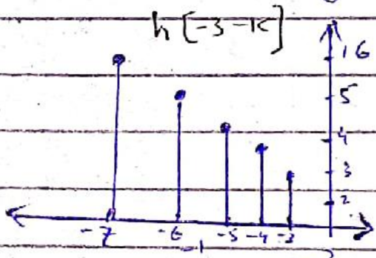
$y[-1] = 1 + 2\alpha^{-1} + 4\alpha^{-2}$



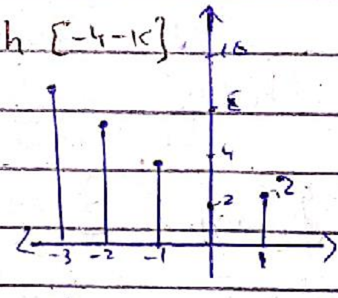
$y[-2] = 2\alpha^{-2} + \alpha^{-1}$



$y[-3] = \alpha^{-2}$

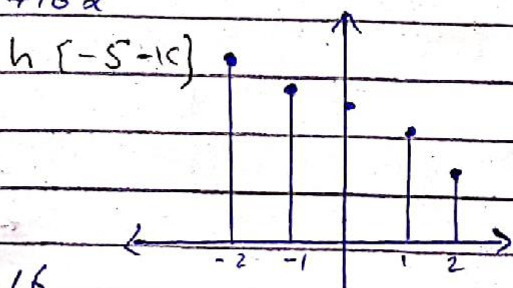


$y[1] = \alpha^{2} + 2\alpha + 4 + 8\alpha^{-1} + 16\alpha^{-2}$



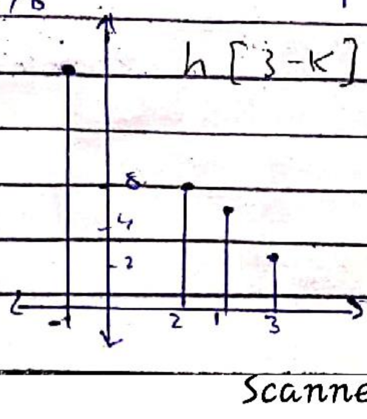
$y[2] =$

$\alpha^{3} + 2\alpha^{2} + 4\alpha + 8 + 16\alpha^{-1}$



$y[3] =$

$\alpha^{4} + 2\alpha^{3} + 4\alpha^{2} + 8\alpha + 16$



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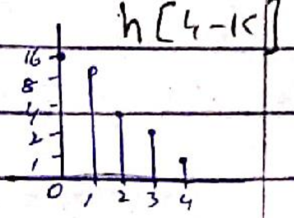
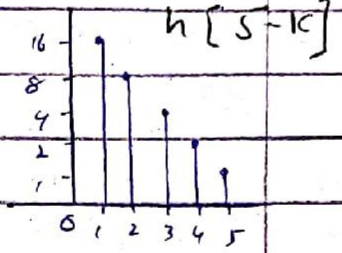
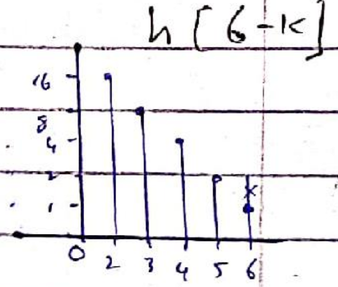
$y[4] = \alpha^5 + 2\alpha^4 + 4\alpha^3 + 8\alpha^2 + 16\alpha$



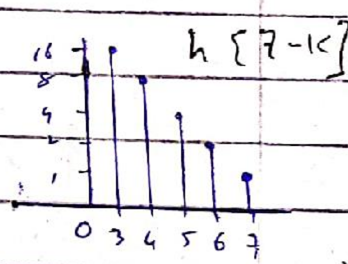
$y[5] = 16\alpha^2 + 8\alpha^3 + 4\alpha^4 + 2\alpha^5 + \alpha^6$



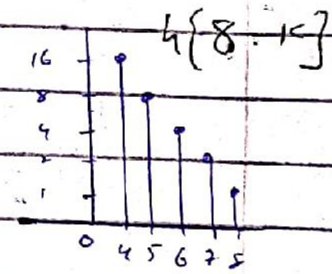
$y[6] = 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$



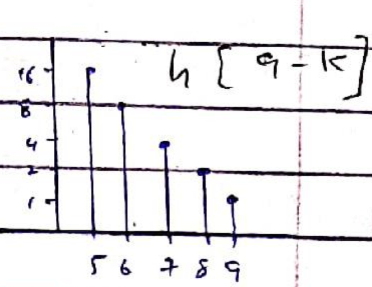
$y[7] = 16\alpha^4 + 8\alpha^5 + 4\alpha^6$



$y[8] = 16\alpha^5 + 8\alpha^6$



$y[9] = 16\alpha^6$



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Q:3 (i) Determine the region of Convergence and Z-transform of the following signal:-

<u>Solution:-</u>

$$x(n) = \begin{cases} \left(\frac{1}{4}\right)^n, & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n}, & n < 0 \end{cases}$$

$$x(n) = \left(\frac{1}{4}\right)^n$$

Applying Z-transform

$$X_1(Z) = \sum_{n=-\alpha}^{\alpha} x[n] Z^{-n}$$

$$X_1[Z] = \sum_{n=-\alpha}^{\alpha} \left[\frac{1}{4}\right]^n Z^{-n}$$

$$|ROC| = |Z| > \frac{1}{4}$$

$$X_1[Z] = \frac{1}{1-\frac{1}{4}Z^{-1}}$$

$$X_2(n) = \left(\frac{1}{3}\right)^{-n}$$

$$X_2(Z) = \sum_{n=-\alpha}^{\alpha} \left(\frac{1}{3}\right) Z^{-n}$$

$, n < 0$

$$X_2[Z] = \frac{1}{1-\frac{1}{3}Z} \qquad ROC = |Z| > 3$$

$$X[Z] = X_1(Z) + X_2(Z)$$

$$X[Z] = \frac{Z}{\left(Z-\frac{1}{4}\right)} + \frac{1}{\left(1-\frac{1}{3}Z\right)} - 1$$

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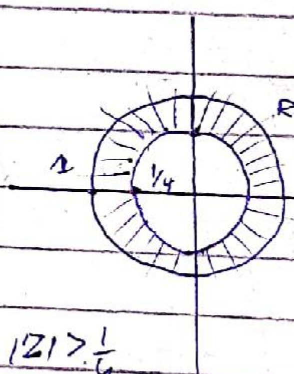
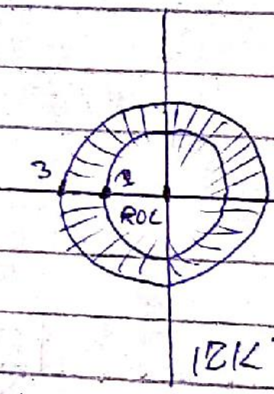
$$= \frac{z - \frac{1}{3}z^2 + z - \frac{1}{4} - z + \frac{1}{3}z^2 + \frac{1}{4} - \frac{1}{12}z}{\left(z - \frac{1}{4}\right)\left(1 - \frac{1}{3}z\right)}$$

$$X[z] = \frac{z - \frac{1}{12}z}{\left(z - \frac{1}{4}\right)\left(1 - \frac{1}{3}z\right)}$$

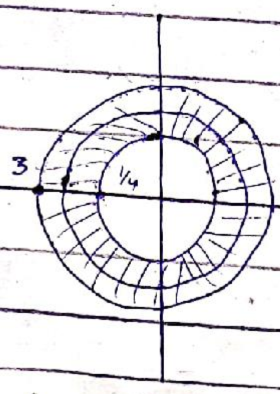
$$ROC = \frac{1}{4} < |z| < 3$$



$|z| > \frac{1}{4}$



$|z| < 3$



$\frac{1}{4} < |z| < 3$

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Ans (3) (ii) $x(n) = \begin{cases} (\frac{1}{2})^n - 3^n, & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$

Solution:

$$X(Z) = \sum_{n=-\infty}^{\infty} x(n) Z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \left[(\tfrac{1}{2})^n - 3^n \right] Z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} (\tfrac{1}{2})^n Z^{-n} - \sum_{n=-\infty}^{\infty} 3^n Z^{-n}$$

$$= \frac{Z}{Z - \frac{1}{2}} - \frac{Z}{Z-3}$$

$$= \frac{Z^2 - 3Z - Z^2 + \frac{1}{2}Z}{(Z - \frac{1}{2})(Z-3)}$$

$$= \frac{-5/2 \, Z}{(Z - \frac{1}{2})(Z-3)}$$

$$ROC = |Z| > 3$$