

(5)

$$= \int \frac{2v}{1+v^2} dv = \int \frac{1}{x} dx + c$$

$$= \ln |1+v^2| = \ln x + \ln c$$

= Take "c" on both sides--

$$= e^{\ln |1+v^2|} = e^{\ln |xc|}$$

$$= 1+v^2 = xc$$

$$= 1+v^2 = xc$$

$$= \text{put } v = y/x$$

$$= 1 + (y/x)^2 = xc$$

$$= \frac{x^2 + y^2}{x^2} = xc$$

$$= x^2 + y^2 = x^3 c \rightarrow (A)$$

= put  $x=2$ ,  $y=6$  in eq (A)

$$= (4) + (36) = 8c$$

$$= c = \frac{40}{8}$$

=  $c=5 \rightarrow$  put it in eq (A)

$$= \text{so, } x^2 + y^2 = 5x^3$$

$$= y^2 = 5x^3 - x^2$$

$$= y^2 = x^2(5x-1)$$

= Taking  $\sqrt{\quad}$  on both sides--

$$= y = +x\sqrt{5x-1}, y = -x\sqrt{5x-1} \text{ or --}$$

$$= \boxed{y = \pm x\sqrt{5x-1}} \rightarrow \text{Ans}$$

(13)

Question No. 03

$$= (x^2 + 3y^2) dx - 2xy dy = 0$$

where,  $x=2$ ,  $y=6$

Sol:

$$= (x^2 + 3y^2) dx - 2xy dy = 0$$

$$= (x^2 + 3y^2) dx = 2xy dy$$

= Divide both side by  $2xy dx$

= there for we got

$$= \frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$

$$= \frac{dy}{dx} = \frac{x^2}{2xy} + \frac{3y^2}{2xy}$$

$$= \frac{dy}{dx} = \frac{1}{2} \left[ \frac{x}{y} + \frac{3y}{x} \right] \rightarrow (i)$$

= let  $y = vx$

= differentiate, by

$$= dy = v dx + x dv$$

= By the division of  $dx$

(14)

= put a in equation  $\rightarrow$  (1)

$$= v + \frac{x \, dv}{dx} = \frac{1}{2} \left[ \frac{x}{xv} + 3 \frac{vx}{x} \right]$$

$$= v + \frac{x \, dv}{dx} = \frac{1}{2} \left[ \frac{1}{v} + 3v \right]$$

= multiply both sides with "2"

$$= 2v + 2x \frac{dv}{dx} = \frac{1}{v} + 3v$$

$$= 2x \frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$= 2x \frac{dv}{dx} = \frac{1}{v} + v$$

$$= 2x \frac{dv}{dx} = \frac{1+v^2}{v}$$

= multiplying both sides by  $\frac{dx}{dv}$  we get

$$= 2x \, dv = \frac{1+v^2}{v} \, dx$$

= multiplying both sides by  $\frac{v}{x(1+v^2)}$

$$= \text{then, } \frac{v}{1+v^2} \, dv = \frac{1}{x} \, dx$$

= Take  $\int$  on both sides...

①

$$= -\lambda^2 - 5\lambda - 5 - 3 + 1$$

$$= -\lambda^2 + 6\lambda - 8 \rightarrow (b)$$

$$= -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

= Expanding by column one...

$$= - \left[ -1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0 \right]$$

$$= - \left[ -(-2 + \lambda - 1) + 1(6 - 3\lambda - 2\lambda + \lambda^2 - 1) \right]$$

$$= -(3 - \lambda + \lambda^2 - 5\lambda + 5)$$

$$= -\lambda^2 + 5\lambda - 5 - 3 + 1$$

$$= -\lambda^2 + 6\lambda - 8 \rightarrow (c)$$

= Put a, b, c in equation (B) ...

$$= (2-\lambda) [-\lambda^3 + 8\lambda^2 - 36\lambda + 16 + 1^4 - 8\lambda^3 + 18\lambda^2 - 8\lambda]$$

$$= -\lambda^2 + 6\lambda - 8 - \lambda^2 + 16\lambda - 8$$

$$= 1^4 - 2\lambda^3 - 8\lambda^2 + 16\lambda^2 + 16\lambda^2 - \lambda^3 - \lambda^2$$

$$= -36\lambda - 8\lambda + 6\lambda + 6\lambda + 16 - 16$$

$$= 1^4 - 10\lambda^2 + 32\lambda^2 - 32\lambda = 0$$

= synthetic division we got

$$= \lambda(\lambda-2)(\lambda^2 - 3\lambda + 16) = 0$$

②

$$= (\lambda = 0)$$

$$= \lambda - 2 = 0 \Rightarrow \boxed{\lambda = 2}$$

$$= \lambda^2 - 8\lambda + 16 = 0$$

= By factorization method...

$$= \lambda^2 - 4\lambda - 4\lambda + 16 = 0$$

$$= \lambda(\lambda - 4) - 4(\lambda - 4) = 0$$

$$= (\lambda - 4)(\lambda - 4)$$

$$= \lambda = 4, \lambda = 4$$

$$= \lambda_1 = 0, \lambda_2 = 2, \lambda_3 = 4, \lambda_4 = 4$$

$$= \text{again, } \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$

= Expansion by Row one-

$$= 3-\lambda \begin{vmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1(-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix}$$

$$= -1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda) [(3-\lambda)(2-\lambda) - (-1)(-1)] + 1(-1)(2-\lambda)$$

$$= -1(-1)(-1) - 1[(-1)(-1) - (-1)(3-\lambda)]$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + (-2+\lambda-1) - (1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5) + (-3+\lambda) - (4-\lambda)$$

$$= 3\lambda^2-15\lambda+15-13+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$= -\lambda^3+8\lambda^2-18\lambda+8 \rightarrow \textcircled{0}$$

$$= +1 \begin{vmatrix} -2 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

= Expanding by column one...

$$= -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$= -1(6-3\lambda-2\lambda+\lambda^2-1) + 1(-2+\lambda-1)$$

B

Question #02 Part "A"

⇒ Find the determinant?

$$= \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

= as, the product of factor which are linear in a, b, c.

= ~~sol~~

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

= Expansion by R1

$$= a \begin{vmatrix} b^2 & c^2 \\ a^2 & c^3 \end{vmatrix} - b \begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c \begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) - a^3b^2c$$

= common a, b, c

$$= abc(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$$

$$= abc[bc(c-b) - ac(c+a) + ab(b-a)] \Rightarrow \text{complete}$$

Question #02 part B

$$= \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol:

$$= \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

= By the equation  $|A-\lambda I| = 0$

$$= |A-\lambda I| = 0$$

$$= \begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 2-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix}$$

= Expansion of by R1...

$$= 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$= -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} = 0 \rightarrow R$$



Q#010

[7]

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \text{ is a ?}$$

Soll:

ANS#10

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

= Expansion of Row ONE:

$$\Rightarrow |A| = +1 \begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - a \begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2 \begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$$

$$= 1(b^2c - b^2c) - a(c^2 - b^2) + a^2(c - b)$$

$$= |A| = b^2c - b^2c - ac^2 + ab^2 + a^2c - a^2b$$

Q#06

4  
solution, of  $\frac{dy}{dx} + 2xy = y$  ?

A#06

$$\frac{dy}{dx} + 2xy = y$$

$$= \frac{dy}{dx} = y - 2xy$$

$$= \frac{dy}{dx} = y(1-2x)$$

$$= \frac{1}{y} dy = (1-2x) dx$$

= Taking integral of Both sides

$$= \int \frac{1}{y} dy = \int (1-2x) dx$$

$$= \ln y = x - \frac{2x^2}{2} + C$$

$$= \ln y = x - x^2 + C$$

Q#07

The order of <sup>⑤</sup> degree of a differential equation?

Ans#07

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \text{ is ?}$$

= The order of differential equation is the order of highest derivatives known as differential coefficient and degree is the power of highest derivatives so, <sup>The</sup>

$$= \text{order} = 2$$

$$= \text{Degree} = 1$$

Q#08 The order and degree of differential equation?

A#08

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) \text{ is ?}$$

$$= \text{order} = 2$$

$$= \text{Degree} = 1$$

Q#09

6

The differential equation, is

$$\frac{2dy}{dx} + x^2y = 2x + 3, \quad y(0) = 5?$$

Sol:

ANS#09

$$\frac{2dy}{dx} + x^2y = 2x + 3$$

$$= \int 2dy = \int (2x + 3 - x^2y) dx$$

$$= 2y = \frac{2x^2}{2} + 3x - y \frac{x^3}{3} + C$$

$$= y = \frac{2x^2}{2 \times 2} + \frac{3x}{2} - \frac{yx^3}{3 \times 2} + C$$

$$= y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3y}{6} + C$$

$$= \text{Put } x=0, \quad y=5$$

$$= 5 = 0 + 0 - 0 + C$$

$$= 5 = C$$

= Then

$$y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3y}{6} + 5$$

Q#02 The number of <sup>2</sup> non-zero rows in  
Echelon form?

Ans#02 IS ONE

Q#03 If  $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$  then,  $|A| = ?$

Sol

• A#03  $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$  ;

$$|A| = \det \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$$

$$= (2i)(-i) - (i)(i)$$

$$= -2i^2 - i^2$$

$$= \text{then, } i^2 = -1$$

$$= -2(-1) - (-1)$$

$$= 2 + 1$$

$$\boxed{|A| = 3}$$

Q #04 If  $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$  is a singular matrix then  $a = ?$  ③

ANS #04 For singular matrix  $|B| = 0$   
then,

$$= |B| = 1 \times a - 4 \times 2 = 0$$

$$= a - 8 = 0$$

So, then the value of  $a = 8$

Q #05 The matrix  $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$  is ?

ANS #05 If every element of a principal diagonal of a matrix is some non-zero scalar and all other elements are zero then it's a scalar matrix

So,

$A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$  is a scalar matrix...

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Paper = Differential equation

ANS:

Section = "B" civil

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## Objective Questions

Q. The order of matrix A is  $m \times p$  and the order of B matrix is  $p \times n$ , then the order of matrix AB is ?

ANS:

Sol:  
→ The order of matrices is equal to the number of the row, multiplying by number of matrices of column,

So,  
→  $A = m \times p$  has "m" no. of rows and p number of columns and p no. of column, then,  $B = p \times n$ ,

then, it's p, the number of column in A is equal to the no. of rows in B, so matrices are valid for multiplication

→ And its order are

$$AB = m \times n$$