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SECTION A

SEMESTER 4th

SUBJECT Differential Equation

QUIZ 1st

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SOLUIONa

1:2:1, 2:1:1, 2:0:2.

Let u, y & z be the lost kgs Pak, Egyptain, American cotton respectively, then according to the given conditions:

$$\left.\begin{aligned} \frac{1}{4}u + \frac{2y}{4} + \frac{1}{4}z &= 40 \\ \frac{2}{4}u + \frac{1}{4}y + \frac{1}{4}z &= 50 \\ \frac{2}{4}u + \frac{2}{4}z &= 60 \end{aligned}\right\} A$$

$$\left.\begin{aligned} 1u + 2y + 1z &= 160 \\ 2u + 1y + 1z &= 200 \\ 1u + 1z &= 120 \end{aligned}\right\} B$$

In matrix form, we can write as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $X = \begin{bmatrix} u \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$

$$\Rightarrow A_1 = \begin{bmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{bmatrix}, \quad A_2 \begin{bmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{bmatrix}$$

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$$A_3 = \begin{bmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{bmatrix}$$ (2)

First $|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$ Expand by R_1

$$= 1\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= 1(1\times 1 - 1\times 0) - 2(2\times 1 - 1\times 1) + 1(2\times 0 - 1\times 1)$$

$$= -2$$

Now

$|A_1| = \begin{vmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{vmatrix}$ Expand by R_1

$$= 160\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} + 1\begin{vmatrix} 200 & 1 \\ 120 & 0 \end{vmatrix}$$

$|A_1| = -120$

Similarly

$|A_2| = \begin{vmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{vmatrix}$ Expand by R_1

$$= 1\begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} - 160\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix}$$

$|A_2| = -40$

$|A_3| = \begin{vmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{vmatrix}$ Expand by R_1

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$$= 1\begin{vmatrix} 1 & 200 \\ 0 & 120 \end{vmatrix} - 2\begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix} + 160\begin{vmatrix} ? & ! \\ ? & 0 \end{vmatrix}$$

$$= 1(120-0) - 2(240-200) + 160(0-1)$$

$$|A_3| = -120$$

Now according to Cramer rule

$$x = \frac{|A_1|}{|A|} = \frac{-120}{-2} = 60$$

$$y = \frac{|A_2|}{|A|} = \frac{+40}{+2} = 20$$

$$z = \frac{|A_3|}{|A|} = \frac{+120}{+2} = 60$$

$$(x, y, z) = (60, 20, 60)$$

Pakistani = 60

Egyptian = 20

American = 60

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