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paper: Digital signal process:

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(1)

Question 1(a):

Solution:

Consider the difference equation

$$y(n)-4y(n-1)+4y(n-2)=x(n)-x(n-1)$$

The homogenous equation of the system is $y(n)-4y(n-1)+4y(n-2)=0$

The characteristic equation system is

$$\lambda-4\lambda^{-1}+\lambda^{-2}=0$$

$$\lambda^{2}-4\lambda+4=0$$

Determine the root of the characteristic equation.

$$\lambda^{-2}-2\lambda-2\lambda+4=0$$

$$\lambda(\lambda-2)-2(\lambda-2)=0$$

$$(\lambda-2)(\lambda-2)$$

$\lambda = 2, 2$. Hence

$y_h(n) = c_1 2^n + c_2 n 2^n$

The particular solution is

$y_p(n) = K(-1)^n u(n)$

By substituting this solution into difference equation we obtain

$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4(K(-1)^{n-2} u(x-2) = (-1) u(n) - (-1)^{n-1} u(x-1)$

For $n = 2$

$K(1+4+4) = 2$

$K = \frac{2}{9}$

So $y(n) = [c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n](2)$

from the initial condition we obtain

$y(0) = 1, \quad y(1) = 2$

Then

$$c_1 + \frac{2}{9} = 1$$

$$\Rightarrow c_1 = \frac{7}{9}$$

$$2c_1 + 2c_2 - \frac{2}{9} = 2$$

$$c = 1/3$$

Q1 (b)

Solution:

consider the difference equation

$$y(n) - 0.7y(n-1) + 0.14(n-2) = 2x(n) - x(x-2)$$

Homogenous equation

$$x(n) = 0$$

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 0$$

Determine the solution obtained in the homogenous equation

$$\lambda^n - 0.7\lambda^{n-1} + 0.\phi\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 0.7\lambda + 0.1) = 0$$

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

Therefore, the roots are.

$(\lambda-0.5)(\lambda-0.2)=0$

Then The roots are

$\lambda=\frac{1}{2}, \frac{1}{5}$ hence

General form of the solution

$y_h(n)=c_1(\lambda_1)^n+c_2(\lambda_2)^n$

$y(n)=c_1(0.2)^n+c_2(0.5)^4 \longrightarrow (1)$

$\lambda=\frac{1}{2}, \lambda=\frac{1}{5}$ Then

$y_h(n)=c+\frac{1}{2}^n+c_2\frac{14}{5}$

with $x(n)=\delta(n)$ we have

$y(0)=2$

$y(1)-0.7y(0)=0$

$y(1)=1.4$

Hence $c_1+c_2=2$

$\frac{1}{2}c_1+\frac{1}{5}=1.4=\frac{7}{5}$

$$\Rightarrow c_1 + \frac{2}{5} c_2 = \frac{14}{5}$$

These equation yield

$c_1 = 10/3$, $c_2 = -4/3$

$$h(n) = [10/3 (1/2)^n - 4/3 (1/5)^n] u[n]$$

Now step response is

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$= 10/3 \sum_{k=0}^{n} (1/2)^{n-k} - 4/3 \sum_{k=0}^{n} (1/5)^{n-k}$$

$$= 10/3 \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - 4/3 (1/5)^n \sum_{k=0}^{n} 5k$$

$$s(n) = \frac{10}{3} \left(\frac{1}{2}\right)^n (2^{n+1} - 1) u(x) - 1/3 (1/3)^n$$

$$(5^{n+1} - 1)\, u(x)$$

Q2(a) Determine the Causal Signal $x(n)$ (6)
having the Z-Transform

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Solution:

Z-Transform is

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

The expression is written as.

$$Y(z) = \frac{1}{(1-\frac{2}{z})(1-\frac{1}{z})^2}$$

$$= \frac{1}{(\frac{z-2}{z})(\frac{z-1}{z})^2}$$

$$= \frac{1}{\frac{(z-2)(z-1)^2}{z^3}}$$

$$= \frac{z^3}{(z-2)(z-1)^2} \quad \text{—— (1)}$$

$X(Z)$ has a simple pole at $p_1=2$ and a double $p_2=p_3=1$ In Such a case The appropiate partial fraction expansion

$$X(Z)=\frac{Z^3}{(Z-2)(Z-1)^2}=\frac{A_1}{(Z-2)}+\frac{A_2}{(Z-1)}+\frac{A_3}{(Z-1)^2}$$

Problem is to determine the coeffiont A_1, A_2, A_3

we proceed as the Case ~~of~~ distint pole To determine A_1 we multiply both side of by $(Z-2)$ and evaluate the result $Z=-2$

$$(Z-2)\,X(Z)=A_1+\frac{Z-2}{Z-1}A_2+\frac{Z-2}{(Z-1)^2}A_3.$$

Evaluate at $Z=2$

$$A_1=\frac{(Z-2)X(Z)}{Z}\Big|_{Z=2}$$

$A_1=4$

$$A_2=A_1+\frac{Z-2}{Z-1}$$

∴- Hence $x[n]=[4[2]^n-3-n]\,u(n)$

$A_2=-3$

$$A_3=A_1+\frac{Z-2}{Z-1}A_2=7-1$$

$x(n)$:

Solution:

We Have

$$x(n) = \frac{1}{2\pi j} \oint \frac{z^{n-1}}{1-az^{-1}}\, dz = \frac{1}{2\pi j} \oint \frac{z^n\, dz}{z-a}$$

where C is a circle at radius greater than $|a|$. We shall evaluate this integral using with $f(z) = z^n$. We distinguish two cases:

1) If $n \geq 0$, $f(z)$ has only zeros and hence no poles inside C. The only pole inside C is $z = a$ Hence

$$x(n) = f(z_0) = a^n, \quad n \geq 0.$$

2) If $n < 0$, $f(z) = z^n$ has an nth-order pole at $z = 0$, which is also inside C. Thus there are contribution from both poles for $n = -1$ we have

$$x(-1) = \frac{1}{2\pi j} \oint_c \frac{1}{z(z-a)}\, dz = \frac{1}{z-a}\Big|_{z=0} + \frac{1}{z}\Big|_{z=a} = 0$$

If $n=-2$ we have

$$x(-2) = \frac{1}{2\pi j} = \frac{1}{2\pi j} \oint_c \frac{1}{z^2(z-a)} dz = \frac{d}{dz}\left(\frac{1}{z-a}\right)\Big|_{z=0}$$

$$= \frac{1}{z^2}\Big|_{z=0} = 0$$

By containing the same way we can show that

$$x(n) = 0 \text{ for } n < 0.$$

Thus

$$x(n) = a^n u(n)$$

Q NO 3(a) :⇒

Solution :⇒

At $w =$ we have

$$H(0) = \frac{b^0}{(1-p)^2} = 1$$

hence

$$b_0 = (1-p)^2$$

At $w = \pi/4$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-p)^2}{(1-p^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1-p\cos(\pi/4)+jp\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{(1-p/\sqrt{2}+jp\sqrt{2})^2}$$

Hence,

$$= \frac{(1-p)^4}{[(1-p/\sqrt{2})^2+p^2/2]} = 1/2.$$

or equivalently

$$\sqrt{2}\,(1-p)^2 = 1-p^2-\sqrt{2}p$$

The value of $p=0.32$ Satisfies this equation consequently the System function for the desired filter is 11

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}.$$

Q 3(b):

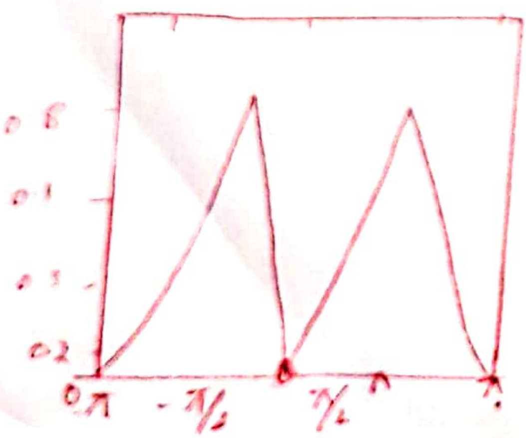
Answer:

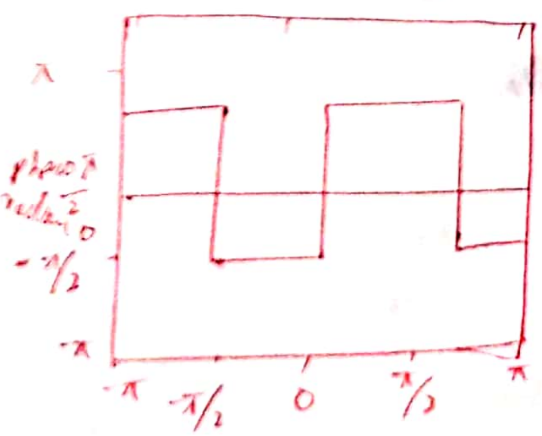
Clearly, the filter must have poles at $P_{1,2} = re^{\pm j\pi/2}$

and zeros at $z=1$ and $z=-1$, consequently the System function is.

$$H(z) = \frac{h(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2+r^2}$$





The gain factor is determined by evaluating the frequency response $H(w)$ of the filter at $w = \pi/2$ Thus we have

$$H\left(\frac{\pi}{2}\right) = G\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(w) = 4\pi/9$. Thus we have.

$$|H\left(\frac{4\pi}{9}\right)|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(r\pi/9)} = \frac{1}{2}.$$

or equivalently.

$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

The value of $r^2 = 0.7$ Satisfies this equation Therefore, the system function for the desired filter is

$$H(z) = 0.15\frac{1-z^{-2}}{1+0.7z^{-2}}$$

Q: 4(a)

Solution:

The fourier Transform of this sequence is

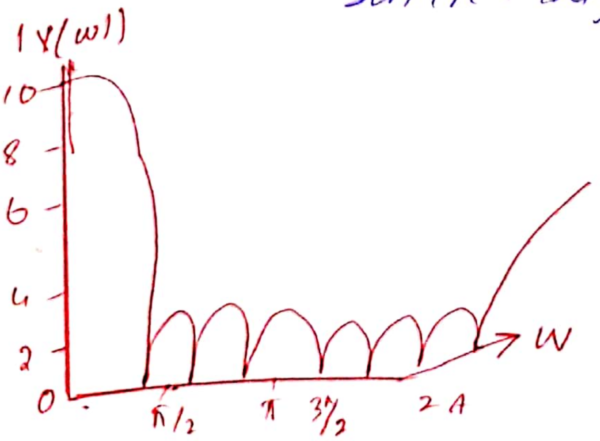
$$X(w) = \sum_{n=0}^{l-1} x(n) e^{-jwn}$$

$$= \sum_{n=0}^{l-1} e^{-jwn} = \frac{1-e^{-jwL}}{1-e^{-jw}} = \frac{\sin(wL/2)\, e^{-jw(l-1/2)}}{\sin(w/2)}$$

The magnitude and phase of $X(w)$ are illustrain for $L = 10$ The N-point DFT of $x(n)$ is simply $X(w)$ evaluated at the set of N equally speed frequencies $w_k = 2\pi k/N$, $k = 0, 1 \cdots N-1$ Hence

$$X(k) = \frac{1-e^{-j2\pi kL/N}}{1-e^{-j2\pi k/N}}, \quad k = 0, 1, \ldots, N-1$$

$$= \frac{\sin(\pi kL/N)\, e^{-jk(l-1)N}}{\sin(\pi k/N)}$$



If N is selected such that N = L, then the DFT become

$$X(k) = \begin{cases} C, 1 & k = 0 \\ 0, & k = 1, 2, \ldots, L-1 \end{cases}$$

Thus there is only one non zero valve in DFT, This is apparent from observation of $x(\omega)$ since $X(\omega) = 0$ at the frequencies $\omega_k = 2\pi k / L$ $k \neq 0$ The reader should verify that $(x)(n)$ can be recovered from $X(k)$ by performing an L-point DFT.

1(b):

Solution:

$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$

$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

→ Each sequence consists of four nonzero points
→ for the purpose of illustrating the operation
→ involved in circular convolution it is desirable
→ to graph each sequence as point on a circle
→ Now, $x_3(m)$ is obtained by circularly convolving
→ $x_1(n)$ with $x_2(n)$ as specified by beginning with
→ $M=0$ we have

$$\longrightarrow x_3(0) = \sum_{n=0}^{3} x_1(n)\, x_2((-n))_N.$$

→ $x(-n)$ is simply the sequence $x_1(n)$ folded
→ and graphed on a circle as illustrated.
→ finally, we sum the value in the product
sequence to obtain

$$x_3(0) = 14.$$

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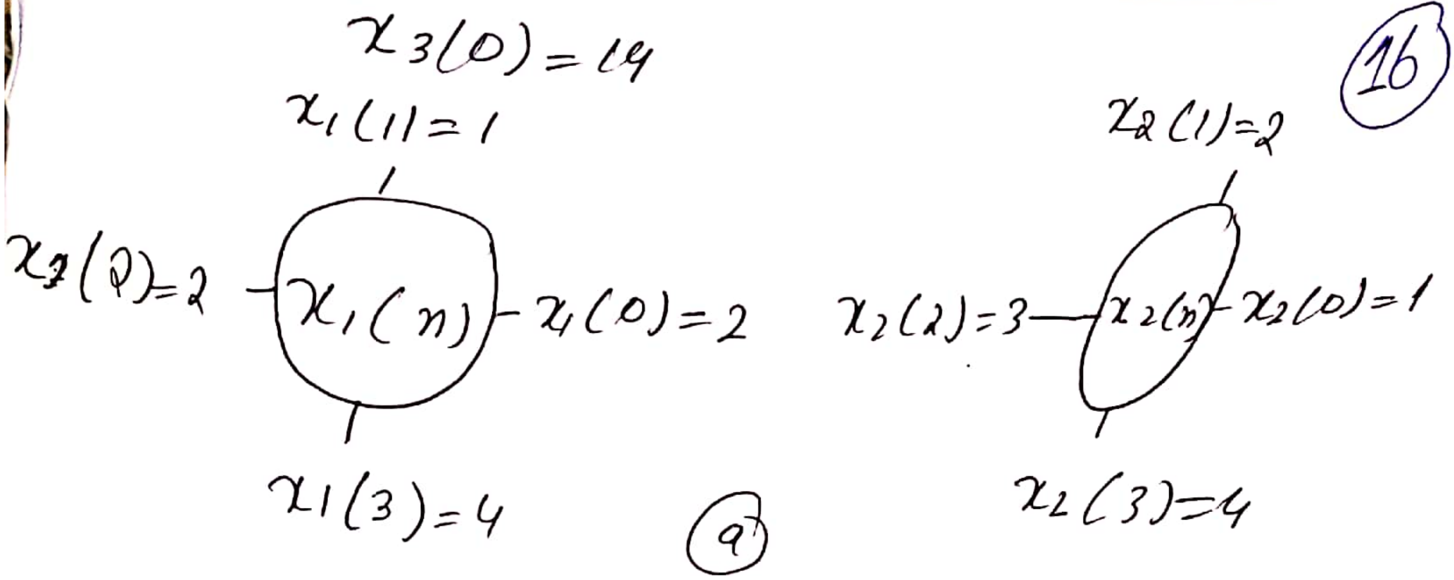
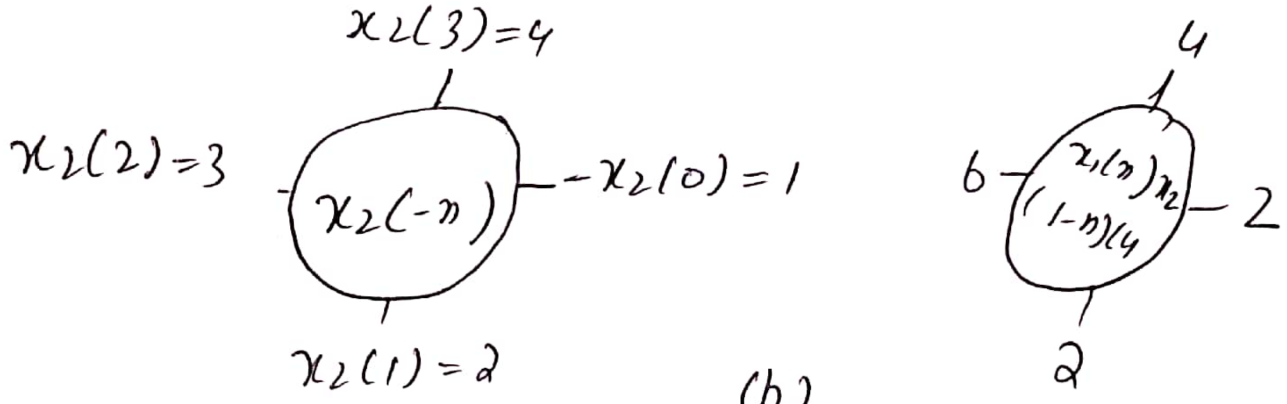


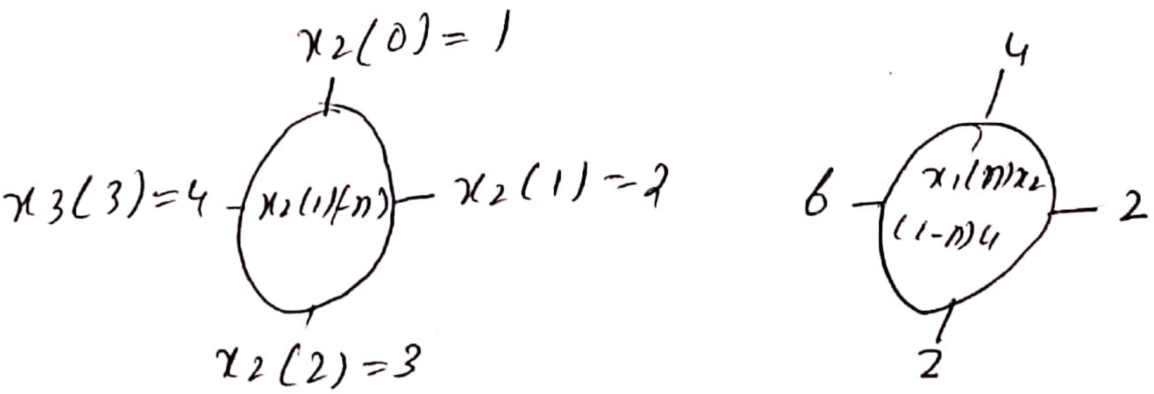
(a)



(b)

folded Sequence

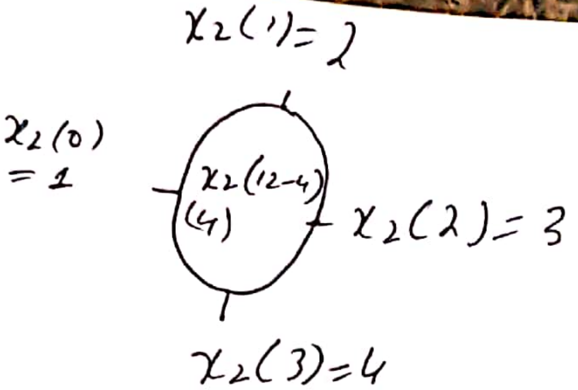
product Sequence



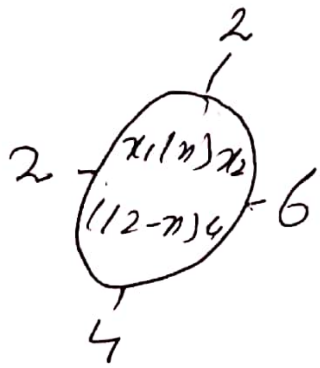
Folded Sequence by one unit in time

(c)

product Sequence

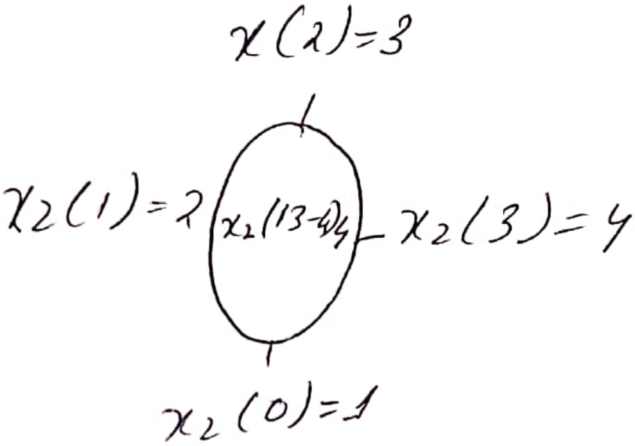


folded sequence in rotated by unit in time

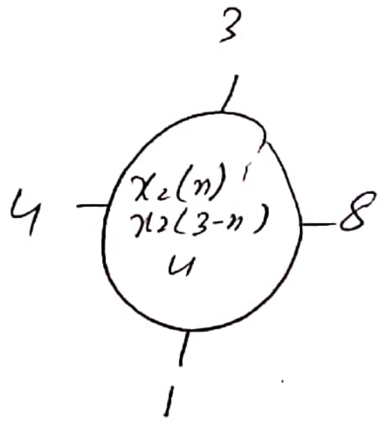


product sequence.

(d)



folded sequence rotated by there unit in time



product sequence

(e

for $m=1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\ x_2((1-n))_4$$

It is easily verified that $x_2((1-n))_4$ is simply the sequence $x_2((1-n))_4$ rotated counter clockwise by one unit in Time

$$x_3(1) = 6$$

for $m=2$ we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4.$$

Now $x_2((2-n))_4$ is the folded sequence rotated Two unit of time in The counter clock wise direction. the product sequence $x_1(n)\, x_2(2-n)_4$

$$x_3(2) = 4$$

for $m=3$ we have

$$x_3(3) = \sum_{n=0} x_1(n)\, x_2((3-n))_4$$

The folded sequence $x_2((-n))_4$ is now rotated by three unit in time to yield $x_2((3-n))_4$ and the resultant sequence is multiplied by $x_1(n)$ to yield the product sequence

$$x_3(3) = 16$$

We observe that the computation above is continued beyond $m=3$ we simply repeat the sequence of four values obtained above. Therefore, the circular convolution of the two sequence $x_1(n)$ and $x_2(n)$ yield the sequence

$$x_3(n) = \{14, 16, 14, 16\}$$