ARSALAN KHAN

I.D NO 7614

PAPER NUMERICAL

SUBMITTED TO. SHUMAILA

DATE 23/Sep/2020

QNO 1:

Apply both Eulers method and the improved Euler method to the solution of

$$\frac{dy}{dx} = 2x \quad ; \quad y(0) = 1$$

For $0 \leq x \leq 0.5$. Using $h = 0.1$. Compare your answer with the analytic solution. Work through out to three decimal places.

ANSWER:

By using Euler's Method

$$y(0) = 1 \quad , \quad h = 0.1 \quad , \quad x_0 = 0$$

By formula

$$y_{n+1} = y_n + hf(x_n, y_n)$$

$$y_{n+1} = y_n + h[2x_n]$$

FIRST ITERATION:

$$y_1 = y_0 + h(2x_0)$$

$$y_1 = 1 + 0.1\,(2(0))$$

$$y_1 = 1 + 0.1$$

$$\boxed{y_1 = 1.1}$$

→ y_1 $x_{n+1} = x_n + h$

$$x_1 = x_0 + h$$

$$x_1 = 0 + 0.1$$

$$\boxed{x_1 = 0.1}$$

2ND ITERATION:

$$n = 1$$

$$y_2 = y_1 + h\,(2x_1)$$

$$y_2 = 1.1 + 0.1\,(2(0.1))$$

$$\boxed{y_2 = 1.12}$$

→ $x_{n+1} = x_n + h$

$$x_2 = x_1 + h$$

$$\boxed{x_2 = 0.2}$$

3RD ITERATION:

$n = 2$

$y_3 = y_2 + h(2x_2)$

$y_3 = 1.12 + 0.1(2(0.2))$

$\boxed{y_3 = 1.16}$

$\rightarrow \quad x_{n+1} = x_n + h$

$x_3 = x_2 + 0.1$

$x_3 = 0.2 + 0.1$

$\boxed{x_3 = 0.3}$

B) BY Modified Euler Method:

$$\frac{dy}{dx} = 2x$$

As

$y_0 = 1 \quad , \quad x_0 = 0 \quad , \quad h = 0.1$

By using formula

$$y_{n+1} = y_n + h\left[f(x_n)\right]$$

$$y_{n+1} = y_n + h(2x_n) \quad \text{—— (1)}$$

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_{n+1}, y_{n+1})\right]$$

$$= y_n + \frac{h}{2}\left[2x_n + 2x_n\right]$$

$$= y_n + \frac{h}{2}\left[4x_n\right]$$

First Iteration:

$n = 0$

$x_{n+1} = x_n + h$

$x_1 = x_0 + h$

$x_1 = 0 + 0.1$

$\boxed{x_1 = 0.1}$

$y_1 = y_0 + \frac{h}{2}(4x_0)$

$y_1 = 1 + \frac{0.1}{2}(4(0))$

$\boxed{y_1 = 1}$

2nd Iteration:

$n = 1$

$x_2 = x_1 + h$

$x_2 = 0.1 + 0.1$

$\boxed{x_2 = 0.2}$

$y_2 = y_1 + \frac{h}{2}(4x_1)$

$y_2 = 1 + \frac{0.1}{2}(4(0.1))$

$\boxed{y_2 = 1.02}$

3RD Iteration

$n = 2$

$x_3 = x_2 + h$

$x_3 = 0.2 + 0.1$

$\boxed{x_3 = 0.3}$

$y_3 = y_2 + \frac{h}{2}(4x_2)$

$= 1.02 + \frac{0.1}{2}(4(0.2))$

$\boxed{y_3 = 1.06}$

Q NO 2: Use the fourth order Runge-kutta method to obtain a solution of

$$\frac{dy}{dx} = x^2 + x - y$$

Subject to $y=0$ when $x=0$, for $0 \le x \le 0.6$ with $h=0.2$, work through out to four decimal places.

Answer: As

$y=0$, $x=0$, $h=0.2$ $0 \le x \le 0.6$

$y_{n+1} = y_n + K$

FIRST ITERATION:

$y_1 = y_0 + K$, $K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$

$K_1 = hf(x_n, y_n)$

$K_1 = h(x_0^2 - x_0 - y_0)$

$K_1 = 0.2(0^2 - 0 - 0)$

$$\boxed{K_1 = 0}$$

$\rightarrow$ $k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}\right)$

$= 0.2 f\left(x_0 + h/2, y_0 + h/2\right)$

$= 0.2 f\left(0 + \frac{0.2}{2}, 0 + \frac{0.2}{2}\right)$

$= 0.2 f(0.1, 0.1)$

$= 0.2(0.1 + \cancel{0.1} - \cancel{0.1})$

$\boxed{k_2 = 0.0020}$

$\rightarrow$ $k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$

$= 0.2 f\left(0 + \frac{0.2}{2}, 0 + \frac{0.002}{2}\right)$

$= 0.2 f(0.1, 0.001)$

$= 0.2(0.12 + 0.1 - 0.001)$

$\boxed{k_3 = 0.0218}$

$\rightarrow$ $k_4 = hf(x_n + h, y_n + k_3)$

$= 0.2 f(0 + 0.2, 0.0218)$

$= 0.2 f(0.2, 0.0218)$

$= 0.2(0.22 + 0.2 - 0.0218)$

$$\boxed{k_4 = 0.0436}$$

$$k = \frac{1}{6}(0 + 2(0.002) + 2(0.0218) + 0.0436)$$

$k = 0.0152$

$y_1 = 0 + 0.0152$

$y_1 = 0.052$

or $\boxed{y_1 = 0.0152}$

Q NO 3: A rocket is released and travels at a variable speed, and the value is sampled by an on-board computer as 1 second intervals. The computer is required to calculate the distance travel by the rocket and relay the value to a ground station at regular Intervals. Record values of measumment taken by computer during the first ten second. calculate the value that the computer will relay to the ground station after 10 seconds.

TIME	0	1	2	3	4	5	6	7	8	9	10
SPEED	10.1	17.2	24.4	29.2	34.6	41.2	50.9	57.8	60.3	61.2	62.1

ANSWER: As

$$a = 0 \quad , \quad b = 10 \quad , \quad n = 10$$

$$h = \frac{b-a}{n}$$

$$= \frac{10-0}{10}$$

$$\boxed{h = 1}$$

NOW;

x	0	1	2	3	4	5	6	7	8	9	10
$f(x_0)$	10.1	17.2	24.4	29.2	34.6	41.2	50.9	57.8	60.3	61.2	62.1

Using formula

$$f(x)\,dx = \frac{h}{2}\left[f(x_0) + 2\,f(x_1) + f(x_2) + f(x_3) \ldots\ldots f(x_9) + f(x_{10})\right]$$

$$= \frac{1}{2}\left[10.1 + 2\begin{pmatrix}17.2 + 24.4 + 29.2 + 34.6 + 41.2 + 50.9 + \\ 57.8 + 62.1\end{pmatrix}\right.$$

$$\boxed{f(x)\,dx = 412.9}$$

Q NO 4:

Estimate the values of the following Integral using Simpson's Rule.

$$\int_2^3 \ln\left(x^3+1\right)\,dx$$

Use 10 strips

Answer: As we know that

$$n = 10$$

$$h = \frac{3-2}{10} = 0.1$$

	x_0	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9
x	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
$f(x)$	0.693	0.846	1.003	1.162	1.320	1.476	1.628	1.777	1.922	2.062

Now using formula

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[f(x_0) + 4\left(f(x_1) + f(x_3)\dots\right)\right] + 2$$

$$\left[f(x_2) \dots + f(x_n)\right]$$

$$= \frac{0.1}{3}\Big[0.693 + 4\,(0.846 + 1.162 + 1.476 + 1.777)$$

$$+ 2\,(1.003 + 1.320 + 1.628 + 1.922)$$

$$+ 2.062\Big]$$

$$\boxed{\int_a^b f(x)\,dx = 1.184}$$