NAME:-

NAVEED AHMAD

I.D:

7880

SUBJECT:- DIFFERENTIAL EQUATION.

DEPTT: CIVIL.

QUISTION : 1

$$\frac{dy}{dt} = e^{y-t} \sec(y)(1+t^2) \qquad y(0)=0$$

By seperating variables

$\therefore \cos = 1/\sec$

$$\int e^{-y} \cos y \, dx = \int (1+t^2) e^{-t} \, dt$$

integrating by parts ①

$u = e^{-y}$ $\quad$ $dv = \cos y dy$

$du = -e^{-y} dy$ $\quad$ $v = \sin y$

integrating by parts ⓘⓘ

$u = e^{-y}$ $\quad$ $dv = \sin y \, dy$

$du = -e^{-y} dy$ $\quad$ $V = -\cos y$

by integrating 1^{st} part

$$LHS = e^{-y} \sin y + \int e^{-y} \sin y \, dy.$$

by integrating part 2

$= e^{-y} \sin y - e^{-y} \cos y - \int e^{-y} \cos y \, dy$.

Since the last integral is same as L.H.S

$L.H.S = e^{-y}(\sin y - \cos y)$

by adding L.H.S

$2(LHS) = e^{-y}(\sin y - \cos y)$

$\div$ ing by 2

$L.H.S = \frac{e^{-y}}{2}(\sin y - \cos y)$

integrating part (3)

$u = 1+t^2 \qquad dv = e^{-t} dt$

$du = 2t\,dt \qquad v = e^{-t}$

Scanned with CamScanner

integrating by part (4)

$u = 2t \qquad dv = e^{-t}\,dt.$

$du = 2\,dt \qquad v = -e^{-t}$

Let us evaluate R.H.S

by integrating part 3

$$R.H.S = -(1+t^2)\,e^{-t} + \int 2te^{-t}\,dt.$$

by integrating part 4

$$= -(1+t^2) - 2te^{-t} + \int 2e^{-t}\,dt.$$

$$= -(t^2+2t+1)\,e^{-t} - 2e^{-t} + C$$

$$= -(t^2+2t+3)\,e^{-t} + C$$

Comparing L.H.S and R.H.S

$$\frac{e^{-y}}{2}(\sin y - \cos y) = -(t^2+2t+3)e^{-t}c$$

Since $y(0)=0$ we have

$$\frac{1}{2}(0-1) = -3+c$$

$$c = 5/2$$

Hence The Solution is implicitly expressed as

$$\frac{e^{-y}}{2}(\sin y - \cos y) = -(t^2+2t+3)e^{-t} + 5/2.$$

QUESTION- 2

$$(\sqrt{x+y} + \sqrt{x-y})\,dx - (\sqrt{x+y} - \sqrt{x-y})\,dy = 0$$

$$\frac{dy}{dx} = \frac{\sqrt{x+y} + \sqrt{x-y}}{\sqrt{x+y} - \sqrt{x-y}} \quad \text{—} (1)$$

This is homogeneous differential eq in x and y to solve this put.

$$y = vx$$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

This eq (1) becomes.

$$v + x\frac{dv}{dx} = \frac{\sqrt{x+vx} + \sqrt{x-vx}}{\sqrt{x+vx} - \sqrt{x-vx}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v} + \sqrt{1-v}}{\sqrt{1+v} - \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}} \times \frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}+\sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{1+\cancel{v}+1-\cancel{v}+2\sqrt{1-v^2}}{2v}$$

$$\cancel{v + x\frac{dv}{dx} = \sqrt{1+v} + \sqrt{1-v}}$$

$$v + x\frac{dv}{dx} = \frac{2\,1+\sqrt{1-v^2}}{2v}.$$

$$v + \frac{x\,dv}{dx} = \frac{1+\sqrt{1-v^2}}{v}$$

$$x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v} - v$$

$$\frac{x\,dv}{dx} = \frac{1+\sqrt{1-v^2}-v^2}{v}.$$

$$x\frac{dv}{dx} = \frac{\sqrt{1-v^2}\,(1+\sqrt{1-v^2})}{v}.$$

Scanned with CamScanner

$$\frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)} = \frac{dx}{x}.$$

taking integral on b/s.

$$\int \frac{v\,dv}{\sqrt{1-v^2}\left(1+\sqrt{1-v^2}\right)} = \int \frac{dx}{x}.$$

put $1+\sqrt{1-v^2} = t$

$$\frac{1}{2}\left(1-v^2\right)^{-1/2}(-2v)\,dv = dt.$$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt.$$

$$\int \frac{-dt}{t} = \int \frac{dx}{x}.$$

$$-\ln t = \ln x + \ln c.$$

$$-\ln\left(1+\sqrt{1-v^2}\right) = \ln cx.$$

$$\ln\left(1+\sqrt{1-v^2}\right) = -\ln cx$$

$$\ln\left(1+\sqrt{1-v^2}\right) = \ln (cx)^{-1}$$

$$1+\sqrt{1-v^2} = 1/cx.$$

$$1+\sqrt{1-\frac{y^2}{x^2}} = \frac{1}{cx}.$$

$$1+\sqrt{\frac{x^2-y^2}{x^2}} = 1/\cancel{c}a.$$

$$x+\sqrt{x^2-y^2} = 1/c$$

$$x+\sqrt{x^2-y^2} = c_1 \qquad \therefore \; 1/c = c_1$$

Which is required.

Scanned with CamScanner

QUISTION- 3

$$(D^4+D^2)y = 3x^2+4\sin x - 2\cos x$$

SOLUTION:-

$$(D^4+D^2)y = 3x^2+4\sin x - 2\cos x.$$

$$\Rightarrow f(D)y = f(x)$$

As it is non-homogenous linear equation.

So solution will be.

$$y = y_c + y_p \quad \text{——} ①$$

Complementry Solution y_c.

$$D^4+D^2 = 0 \Rightarrow D^2(D^2+1)=0$$

Either $D^2=0 \Rightarrow D=0$

$$D^2+1=0 = D^2 = -1.$$

$$D=\sqrt{-1} \Rightarrow D=i \text{ or } D=0+i$$

Scanned with CamScanner

Roots are real and complex.

$y_c = c_1 e^{0x} + e^{0x}(c_2 \cos x + c_3 \sin x)$

$y_c = c_1 + c_2 \cos x + c_3 \sin x.$

$y_p = \frac{1}{F(D)} F(x)$

$y_p = \frac{1}{D^4+D^2}(3x^2 + 4\sin x - 2\cos x)$

$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}.$

$f(D) = D^4 + D^2$

at $D = 0 \Rightarrow f(D) = 0$

So $f'(D) = 4D^3 + 2D$

Now also for $D=0$ $f'(D) = 0$

again differentiating

$f''(D) = 12D+2.$

So for $D = 0$

So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$$y_p = \frac{x^2\, 3x^2}{12D+2} + \frac{x^2}{12D+2} \cdot 4\sin x - \frac{x^2}{12D+2} \cdot 2\cos x$$

So putting $D=0$ in all.

$$y_p = \frac{x^2 \cdot 3x^2}{12(0)+2} + \frac{x^2 \cdot 4\sin x}{12(0)+2} - \frac{2x^2\cos x}{12(0)+2}.$$

$$y_p = \frac{3x^4}{2} + \frac{4x^2\sin x}{2} - \frac{2x^2\cos x}{2}$$

$$= \frac{3}{2}x^4 + 2x^2\sin x - x^2\cos x$$

So putting in equation (1)

$$y = c_1 + c_2\cos x + c_3\cos x + 3/2\, x^4 + 2x^2\sin x$$
$$- x^2\cos x.$$

$$y = c_1 + (c_2 - x^2)\cos x + (c_3 + 2x^2)(\sin x + 3/2\, x^4)$$