Abdul Aziz 13741

Q1 (a) $y' = (x+2)y^2$

Solution

$$y' = (x+2)y^2$$

$$\frac{dy}{dx} = (x+2)y^2$$

$$\int \frac{1}{y^2}\,dy = \int (x+2)\,dx$$

$$\int y^{-2}\,dy = \int (x+2)\,dx$$

$$\frac{y^{-2+1}}{-2+1} = \frac{x^2}{2} + 2x + c$$

$$y^{-1}/_{-1} = x^2/_2 + 2x + c$$

Multiplying both sides -1

$$y^{-1} = -\left(\frac{x^2}{2} + 2x + c\right)$$

$$y = -\left(\frac{1}{x^2/_2 + 2x + c}\right)$$

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Q1 (b)

sol:

let $y + 9x = u$

$$\frac{dy}{dx} + 9 = \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{du}{dx} - 9$$

sol

$$\frac{du}{dx} - 9 = u^2$$

$$\frac{du}{dx} = u^2 + 9$$

$$\int \frac{1}{u^2+9}\, du = \int dx$$

$$\int \frac{1}{(3)^2 + (4)^2}\, du = \int dx$$

$$\frac{1}{3} \tan^{-1}\left(\frac{u}{3}\right) = x + c_1$$

$$\tan^{-1}\left(\frac{u}{3}\right) = 3x + 3c_1$$

$$\tan^{-1}(u/3) = 3x + c$$

$$u/3 = \tan(3x + c)$$

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$$y + 9x = 3\tan(3x + c)$$

$$y = -9x + 3\tan(3x+c)$$ Answer

Q# 2

$$x^3 du + y^3 dy = 0$$

$$M du + N dy = 0$$

$$M = x^3 \quad , \quad N = y^3$$

$$\frac{\partial M}{\partial y} = 0 \quad , \quad \frac{\partial N}{\partial x} = 0$$

$$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad \text{so exact}$$

$$U = \int M du + K(y)$$

$$U = \int x^3 du + K(y)$$

$$U = \frac{x^4}{4} + K(y) \quad \text{—— (1)}$$

$$\frac{\partial U}{\partial y} = 0 + \frac{d}{dy} K(y)$$

$$\frac{\partial U}{\partial y} = \frac{d}{dy} K(y)$$

Since we know that

$$\frac{\partial U}{\partial y} = N = y^3$$

$$y^3 = \frac{d}{dy} K(y) \Rightarrow \int y^3 = \int d\,K(y)$$

$$\Rightarrow K(y) = y^4/4 + C_1 \quad \text{put in } 1$$

$$U = \frac{x^4}{4} + \frac{y^4}{4} + C_1$$

$$C_2 = \frac{x^4}{4} + \frac{y^4}{4} + C_1$$

$$C_2 - C_1 = \frac{x^4}{4} + \frac{y^4}{4}$$

$$\Rightarrow C = \frac{x^2}{4^2} + \frac{y^4}{y} = \text{U Answer.}$$

Q3# (a) $4y'' - 20y' + 25y = 0$

sol:- The Axilary equation is

$$4\lambda^2 - 20\lambda + 25 = 0$$

$$4\lambda^2 - 10\lambda - 10\lambda + 25 = 0$$

$$2\lambda(2\lambda - 5) - 5(2\lambda - 5) = 0$$

$$(2\lambda - 5)(2\lambda - 5) = 0$$

$$\lambda_1 = 5/2 \;,\; \lambda_2\, 5/2$$

The roots are real and equal

so $y = (c_1 + c_2)e^{\lambda x} = (c_1 + c_2)e^{5/2}$

Q 3 (b)

$4y'' - 6y' - 7y = 0$

sol:- $4y'' - 6y' - 7y = 0$

Auxilary eq is

$$4\lambda^2 - 6\lambda - 7 = 0$$

Here

$a = 4,\; b = -6,\; c = -7$

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$$\lambda = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\lambda = \frac{(-6) \pm \sqrt{(-6)^2-4(4)(-7)}}{2(4)}$$

$$\lambda = \frac{6 \pm \sqrt{36+112}}{8}$$

$$\lambda = \frac{6 \pm \sqrt{148}}{8}$$

2 as common

$$\lambda = \frac{3 \pm \sqrt{37}}{4}$$

so $\lambda = \frac{3+\sqrt{37}}{4}$

$$\lambda = \frac{3-\sqrt{37}}{4}$$

Roots are real so

$$y = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$$

$$\boxed{y = c_1 e^{\frac{3+\sqrt{37}}{4}t} + c_2 e^{\frac{3-\sqrt{37}}{4}t}}$$