Name :- Muhammad Usama

ID :- 7731

Section :- A

Be (C)

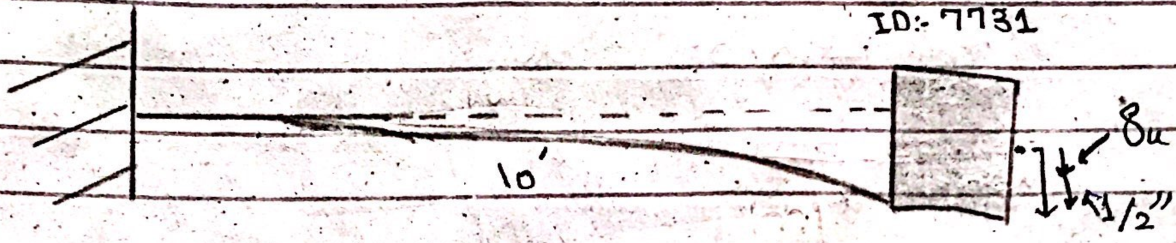
Subject :- Intro to structural dynamics & Earthquick Engineering.

Instructor :- Sir Yaseen

Date :- 29/06/2020

Question (1)

Solution :-



The General E.O.M for SDoF System is

$$Ku + C\dot{u} + m\ddot{u} = P(t)$$

In this Case System is in nondamage Condition ($C=0$) undergoing free vibration ($P_t=0$)

Hence general EOM becomes $Ku + m\ddot{u} = 0$ ..... (i)

$$K = 3EI/L^3$$

$$K = \frac{3 * 29000\ k/in^2 * 150\ in^4}{(10 * 12\ in)^3}$$

$$K = 7.55\ k/in$$

In order to eliminate the changes of mistake during calculation, it is more appropriate to use fondamental units like lb, ft sec or kg, m sec,

$$K = 7.55\ k/in = 90625\ lb/ft$$

$m = \frac{7731 \text{ lb}}{32.2 \text{ ft/sec}^2}$

$m = 240.09$ slug

$\therefore \omega_n = \sqrt{k/m}$

$\omega_n = \sqrt{\frac{90625}{240.09}}$

$\omega_n = 19.42$ rad/sec.

$T_n = 2\pi/\omega_n = 2\pi/19.42$

$T_n = 0.323$ sec.

Substituting the corresponding values in eq.-I.

$90625u + 240.09\ddot{u} = 0$

where 'k' is in lb/ft & 'm' is in lb sec/ft^2.

General solution to the EOM for undamped free vibration is;

$u(t) = u(0)\cos(\omega_n t) + \dot{u}(0)/\omega_n \sin(\omega_n t)$.

$u(0) = 1/2'' = 1/24$ ft and $u(0) = 0$

$u(t) = (1/24) * \cos(19.42t) + 0$

$u(t) = (1/24) * \cos(19.42\,t)$

Equivalent static force at any time "t" is

$f_s(t) = k \cdot u(t)$

$$f_s(t) = \frac{90625 * \cos(19.42\,t)}{24}$$

$f_s(t) = 3776.04 \cos(19.42\,t)$

Amplitude of dynamic displacement u_o for undamped free vibration is

$$u_o = \sqrt{(u(0))^2 + \left(\frac{u(0)}{\omega_n}\right)^2}$$

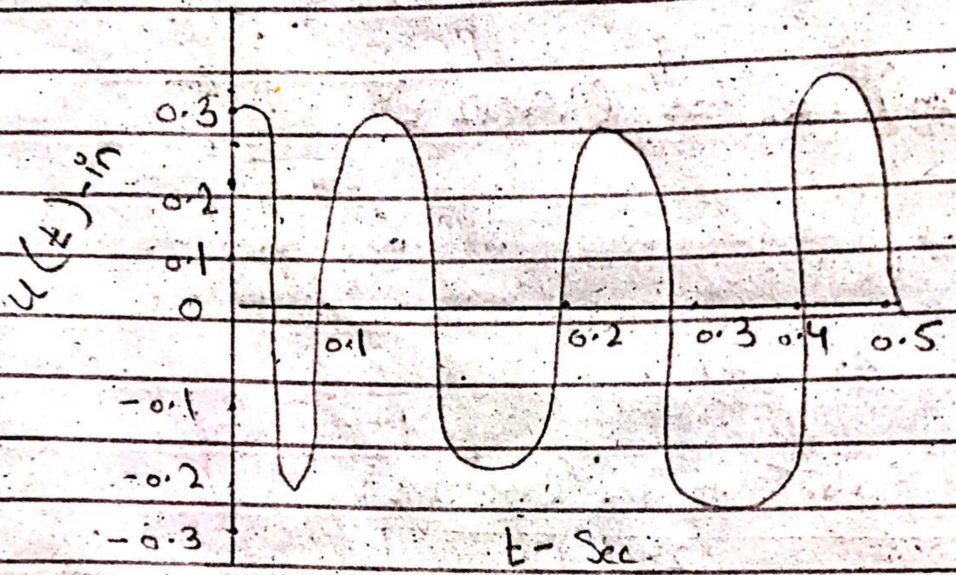
$$u_o = \sqrt{\left(\frac{1}{24}\right)^2 + 0}$$

$u_o = 1/24$ ft

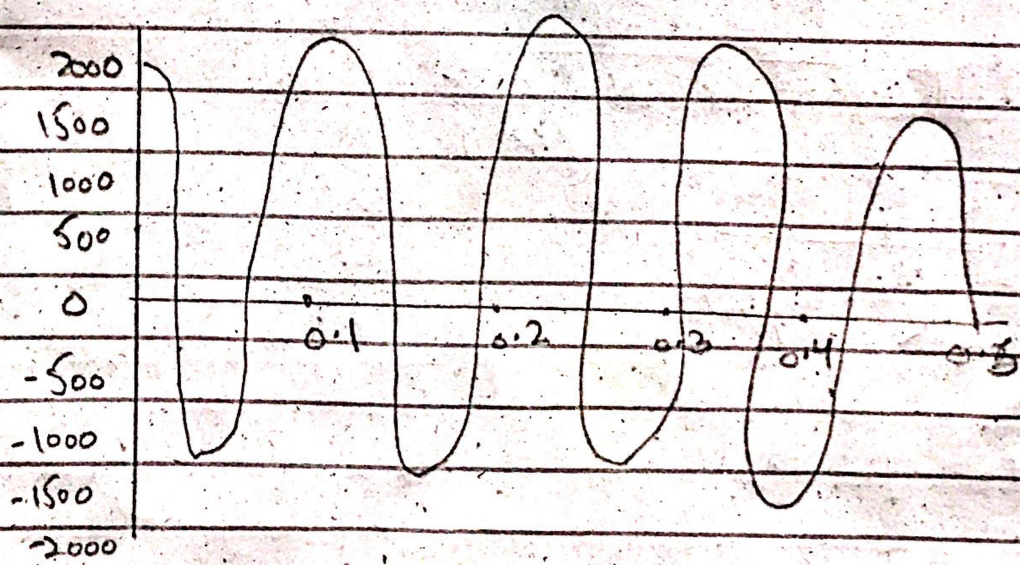
Amplitude of equivalent static force,

$k u_o = 90625 * 1/24$

$k u_0 = 3776.04$ lb



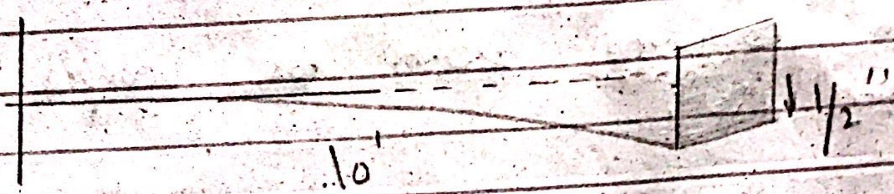
Undamped force vibration.



Undamped force vibration.

Question (2)

Solution:-



E.O.M for damped free vibration is;

$ku + c\dot{u} + m\ddot{u} = 0$ .... (i)

* It is known from (question (1))

$k = 90625$ lb/ft and $m = 240.09$ slug

$\omega_n = 19.42$ $\qquad m = 240.09$ sec/ft

$\therefore c = \zeta * 2m\omega_n = 2 * 240.09 * 19.42 * \zeta$

($\zeta = 0.03 - 0.05$ with considerable cracking the damping ratio)

$\Rightarrow c = 2 * 240.09 * 19.42 * 0.05$

$c = 466.25$ lb.sec/ft

By substituting values of k, c & m in

and m in eq^{n} i we get

$$90625u + 466.25\dot{u} + 240.09\ddot{u} = 0$$

Solution to the E.O.M for damped free vibration is ;

$$u(t) = e^{-\xi\omega_n t}\left(u(0)\cos(\omega_D t) + \frac{1}{\omega_D}\left(\dot{u}(0) + u(0)\xi\omega_n\right)\sin(\omega_D t)\right).$$

$\omega_D = 19.42$ rad/sec

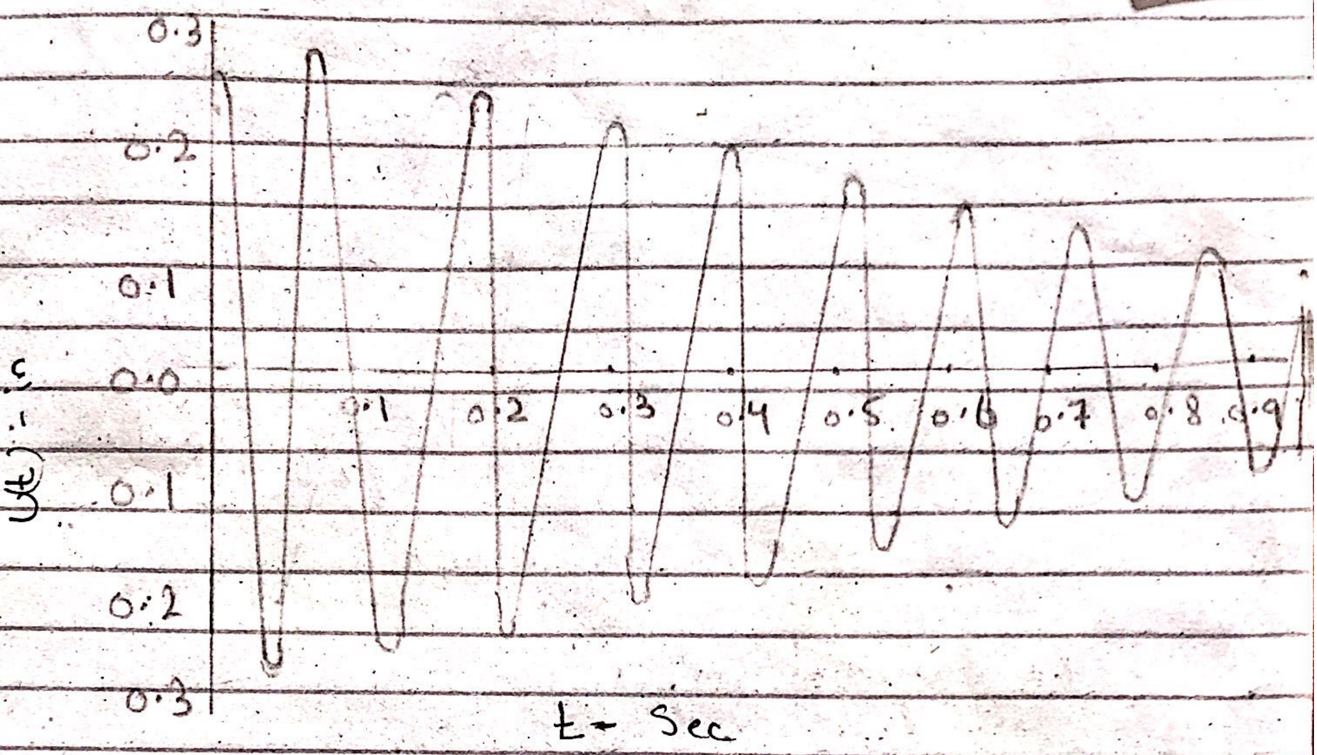
$$u(t) = e^{-0.05 * 19.42}\left(\frac{1}{24} * \cos(19.42 + \frac{1}{19.42} * 0 + \frac{1}{24} * 0.05 * 19.42 * \sin(19.42t)\right)$$

$$u_t = e^{-0.097t}\left(0.0417 * \cos(19.42t) + \right.$$

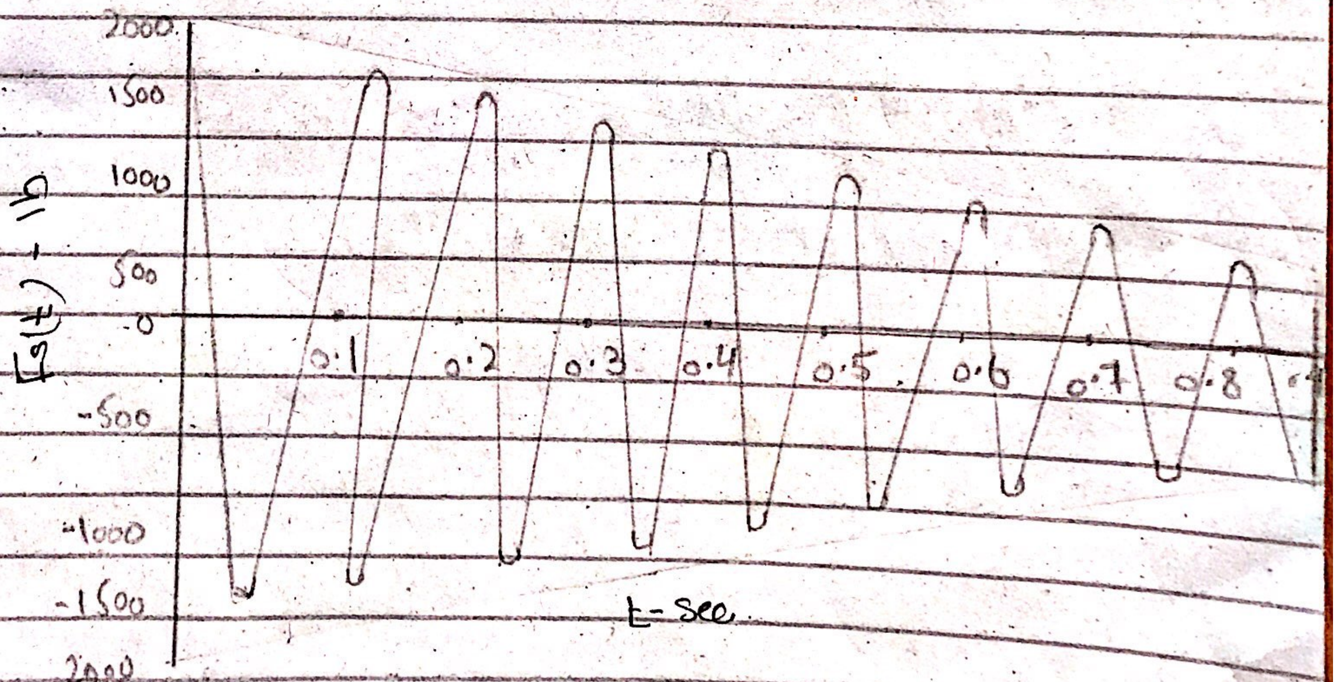
$$f_s(t) = k.u(t) = 90625 * u(t)$$

$$f_s(t) = e^{-1.3773t}\left(3776.04 * \cos(19.42t) + 1903.12 * \sin(19.42)\right)$$

Damped Free vibration :-

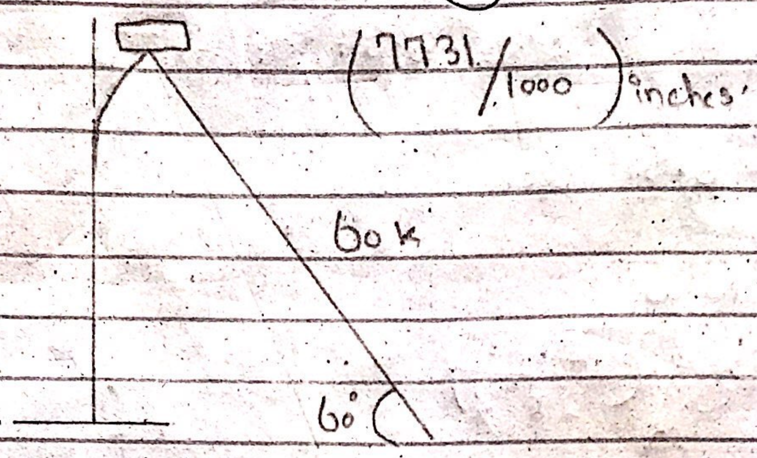


Variation of displacement with time



Variation of Equivalent static force with time.

Question 3



Solution :-

$u_1 = 7731/1000 = 7.73''$

After $j = 7$, $u_{j+1} = u_6 = 2.286$ cm

$= 0.9''$

(a) ζ = Damping ratio = ?

$$j = \frac{1}{2\pi\zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$7 = \frac{1}{2\pi\zeta} \ln(7.73/0.9)$$

$\zeta = 0.0488 = 4.88\%$

b) :- T_n = ?

7 Cycles of vibrations are completed in 3.57 sec.

Time required to complete one cycle

$= 3.57/7 = T_D$

$T_D = 0.51$ sec

Now

$\omega_D = \omega_n \sqrt{(1-\xi^2)}$

$2\pi/\omega_D = 2\pi / (\omega_n \sqrt{1-\xi^2})$

$\Rightarrow T_D = \dfrac{T_n}{\sqrt{1-\xi^2}}$

$T_n = T_D * \sqrt{1-\xi^2}$

$\Rightarrow T_n = 0.51 * \sqrt{1-(0.048)^2}$

$\Rightarrow T_n = 0.5094 = 0.51$ sec

(c) $k = ?$

$k = \dfrac{60 * \cos 60°}{7.73} = 3.88$ k/in

$k = 46560$ lb/ft

D):- Weight of the tank $W = ?$

$$W_n = \sqrt{k/m} = \sqrt{\frac{k}{(w/g)}} = \sqrt{\frac{k*g}{w}}$$

$$\Rightarrow W_n^2 = k*g/w$$

$$w = k*g / w_n^2$$

Also

$$W_n = 2\pi / T_n$$

$$W = kg\left(\frac{4\pi^2}{T_n^2}\right) = kg * T_n^2/4\pi^2$$

$$W = \frac{46.560 * 32.2 * (0.51)^2}{4\pi^2}$$

$$W = 9877.5 \text{ lb}$$

$$W = 9.87 \text{ k}$$

e). $c = ?$

It is known that $\zeta = c/2mW_n$

$$\Rightarrow c = \zeta * 2mW_n = \zeta * 2m * (2\pi/T_n).$$

$$c = 0.0488 * 2 * 2 * (\pi/0.51)\left(\frac{9878.5}{32.2}\right)$$

C = 368.85 lb. sec/ft.

(F) No. of Cycles to reduce displacement amplitude from 7.73 in to 0.5" j = ?

$$j = \frac{1}{2\pi \zeta} \ln\left(\frac{u_1}{u_{j+1}}\right)$$

$$\Rightarrow j = \frac{1}{2 * \pi * 0.0488} \ln\left(\frac{7.73}{0.5}\right)$$

$\Rightarrow j = 8.93$ or 9 Cycles.